Abstract

Given the matrices A and E in $\mathbb{C}^{n\times n}$, we consider the coupling of the square matrices A and E by the complex parameter $t \in \mathbb{C}$ into $A(t) = A + tE$. The matrix E of rank $r \leq n$ is written in the form $E = UV^H$ where $U, V \in \mathbb{C}^{n\times r}$ of rank r represent a basis for $\operatorname{Im} E$, $\operatorname{Im} E^H$ respectively. We denote the *spectrum* of A by $\sigma(A)$.
In Homotopic Deviation theory, HD, t can be unbounded in $\hat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$. In this theory, two questions related to $A(t)$ are considered: i) existence and analyticity of the resolvent $t \in \hat{\mathbb{C}} \mapsto R(t,z) = (A(t) - zI)^{-1}$, $z \in \mathbb{C}$, and, if it exists, is it analytic and does it have a limit as $|t| \to \infty$? ii) the existence of the set Lim, of limits of the eigenvalues in the spectrum $\lim_{|t|\to\infty} \sigma(A(t))$ which stay at finite distance. The case of interest corresponds to $r = \operatorname{rank} E < n$: the deviation matrix E is singular.

This work on HD has two components.

1. The purely algebraic aspect of the theory introduces new kinds of singularities such as frontier and critical points. The key matrix is $M_z = V^H(zI - A)^{-1}U \in \mathbb{C}^{r\times r}$, defined for $z \in \mathbb{C}\backslash\sigma(A)$.

2. Computer experiments are used to perform a qualitative analysis of HD in finite precision. For this aim, several graphical tools are developed.

As an application of HD, the dependence of the structure of the **regular pencil** $P = (A - zI) + tE$, $t \in \mathbb{C}$ on the parameter z in $\mathbb{C}\backslash\sigma(A)$ is analysed by means of the notion of *frontier points*, which is a key notion in HD.

This work also performs a homotopic backward analysis and contrasts it with the classical normwise backward analysis. The thesis ends by an application of HD to Arnoldi's method.