Trust-region methods for rectangular systems of nonlinear equations

Margherita Porcelli (University of Florence)
joint work with
Maria Macconi (University of Florence),
Benedetta Morini (University of Florence)

We address the numerical solution of the bound-constrained nonlinear system

$$\Theta(x) = 0, \quad x \in \Omega. \tag{1}$$

Here $\Theta : X \to \mathbb{R}^m$ is a continuously differentiable mapping, $X \subseteq \mathbb{R}^n$ is an open set containing the feasible region Ω and Ω is an n-dimensional box, $\Omega = \{x \in \mathbb{R}^n : \ l \leq x \leq u\}$. These inequalities are meant component-wise and $l \in (\mathbb{R} \cup -\infty)^n$, $u \in (\mathbb{R} \cup \infty)^n$. Taking into account the variety of applications yielding the problem (1), we allow any relationship between m and n.

The relevance of this problem is well known. It arises in equality-constrained optimization, restoration feasibility for nonlinear programming, parameter identification problems, see [1, 2, 3]. Moreover, the general class of problems given by nonlinear system of equalities and inequalities can be cast as in (1).

We propose two trust-region methods for bound-constrained nonlinear systems. The first method is based on a Gauss-Newton model, the second one is based on a regularized Gauss-Newton model and results to be a Levenberg-Marquardt method. The globalization strategy uses affine scaling matrices arising in bound-constrained optimization. Under reasonable assumptions, these methods globally converge to a solution of (1) or to a first-order stationary point for the bound-constrained least-squares problem

$$\min_{x \in \Omega} \theta(x) = \frac{1}{2}\|\Theta(x)\|_2^2. \tag{2}$$

Further, locally quadratic convergence properties are established under standard assumptions. The algorithmic options of the methods are discussed and numerical results on several test problems are presented.

References

[1] J.E. Dennis, M. El-Alem, K. Williamson, *A trust-region approach to nonlinear systems of equalities and inequalities*, SIAM J. Optim., 9 (1999), pp. 291-315.

[2] N.I.M. Gould, Ph.L. Toint, *FILTRANE: a Fortran 95 filter-trust-region package for solving nonlinear least-squares and nonlinear feasibility problems*, ACM Trans. Math. Software, 33 (2007), pp. 3-25.

[3] H.F. Walker, L.T. Watson, *Least-squares secant update methods for underdetermined systems*, SIAM J. Numer. Math., 27 (1990), pp. 1227-1262.