The History of UMFPACK and AMD

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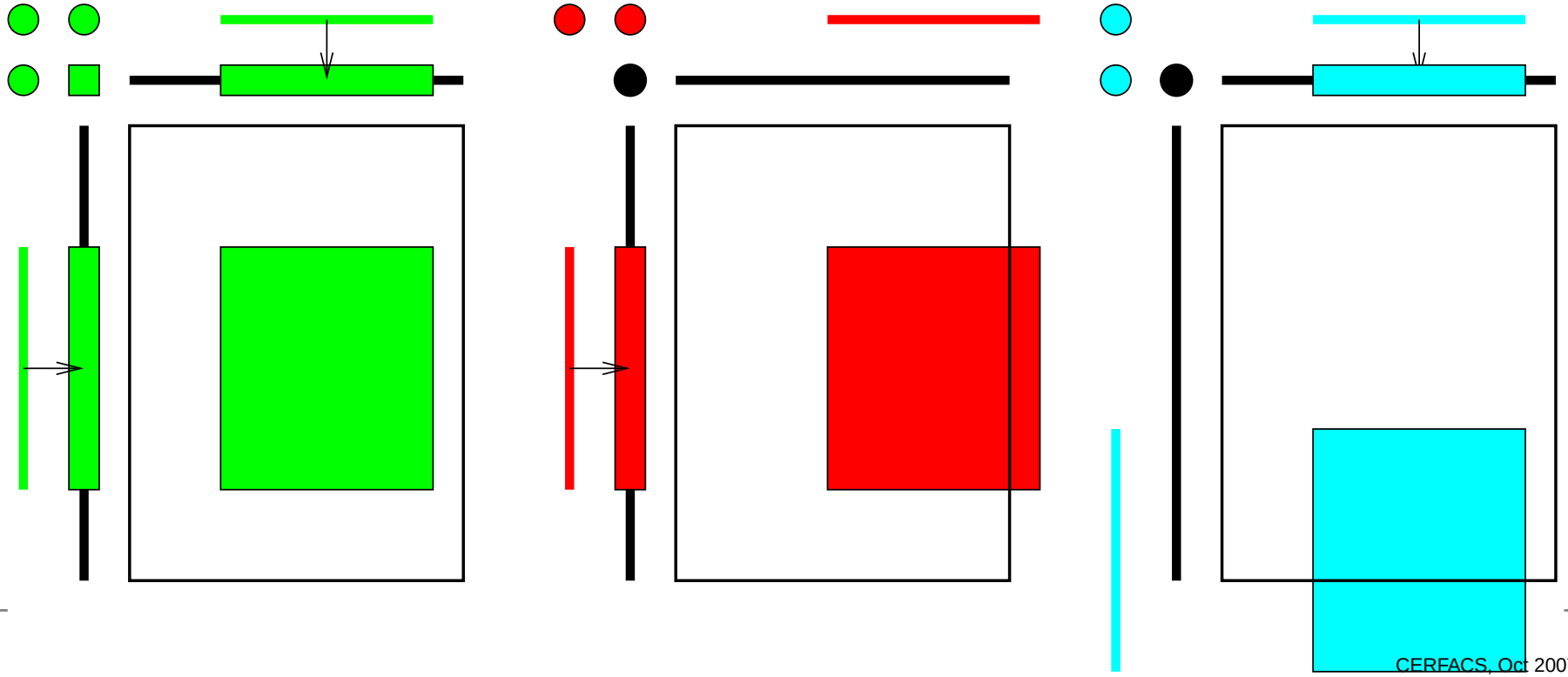
The road to CERFACS

- May 1989, SIAM Symposium on Sparse Matrices, Gleneden Beach, Oregon
- Aug 1989: PhD Thesis, future work:

  Can some of the ideas of the clique tree and clique graph methods be used in the unsymmetric case, without resorting to forcing a symmetric structure on the matrix?
- CERFACS post-doc, Sept 1989 to Dec 1990
- Univ. of Florida, Jan 1991 to date

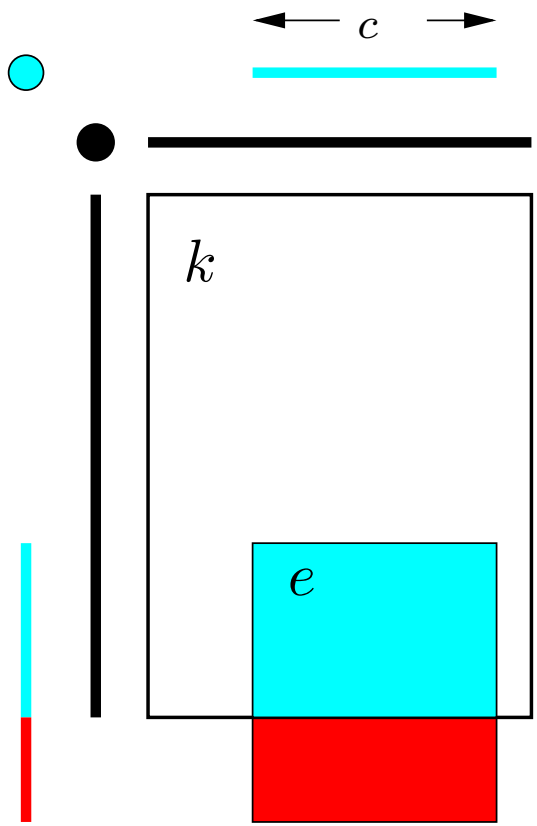
UMFPACK : right-looking multifrontal

- fill-in as rectangular frontal matrices
- *element lists* to reconstruct rows/cols
- pivot ordering on the fly (1990 to 1995)
- preordering + adjustment on the fly (1996 to date)



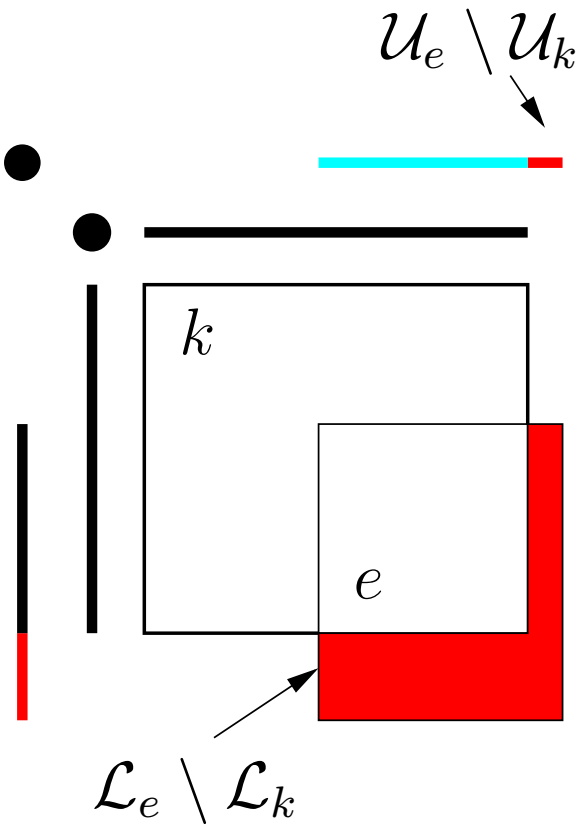
Aggressive frontal matrix assembly

- scan the row/col elements lists twice
- front e is r-by-c
- current front is k
- if e appears in c times in columns of k, then any row in e and k can be assembled from e to k
- ditto for columns



Degrees for Markowitz search (1990)

- degree is $\leq \sum$ *external* element sizes
- *external* element size: if front e is $|\mathcal{L}_e|$-by-$|\mathcal{U}_e|$ and appears in $t < |\mathcal{U}_e|$ columns of front k, then its external row size is $|\mathcal{U}_e \setminus \mathcal{U}_k| = |\mathcal{U}_e| - t$

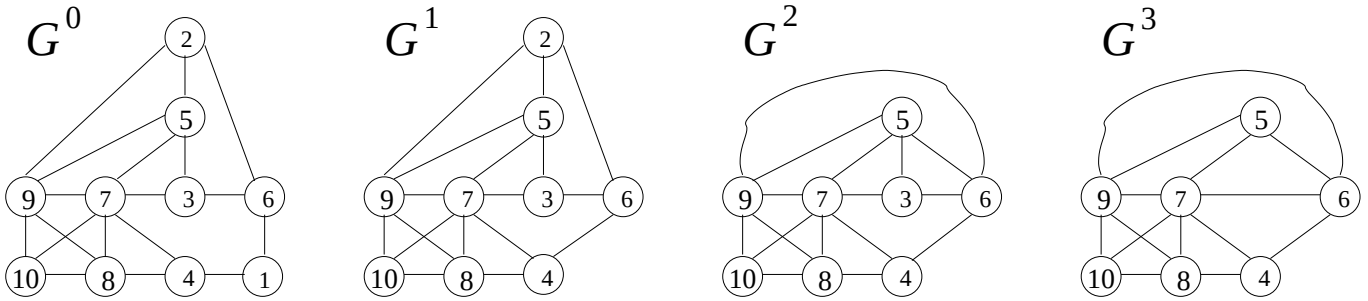


- column j degree $\leq |\mathcal{A}_{*j}| + |\mathcal{L}_k| + \sum |\mathcal{L}_e \setminus \mathcal{L}_k|$
- row i degree $\leq |\mathcal{A}_{i*}| + |\mathcal{U}_k| + \sum |\mathcal{U}_e \setminus \mathcal{U}_k|$

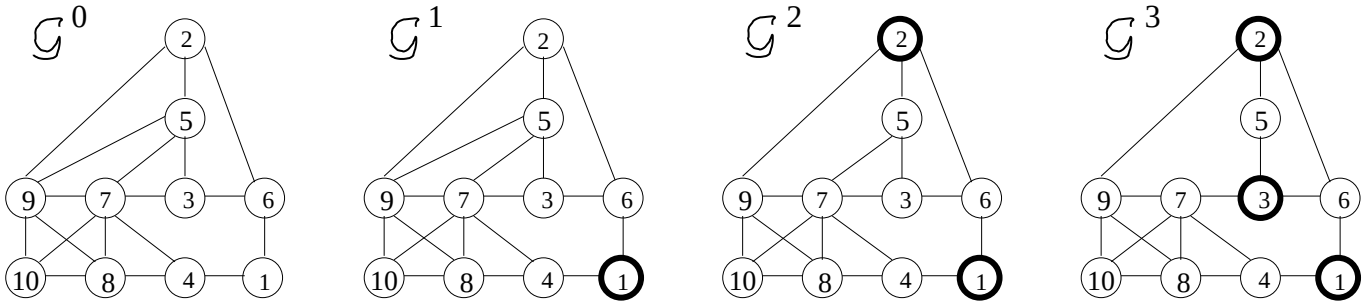
AMD : the UMFPACK degree update

- *approximate* minimum degree
- find a node ordering for sparse Cholesky
- right-looking symbolic factorization
- Patrick Amestoy, Tim Davis, Iain Duff (1994, St. Girons)
- starting with MA27 (Iain Duff and John Reid)
- replaced exact degrees with approximate (upper bound) degrees

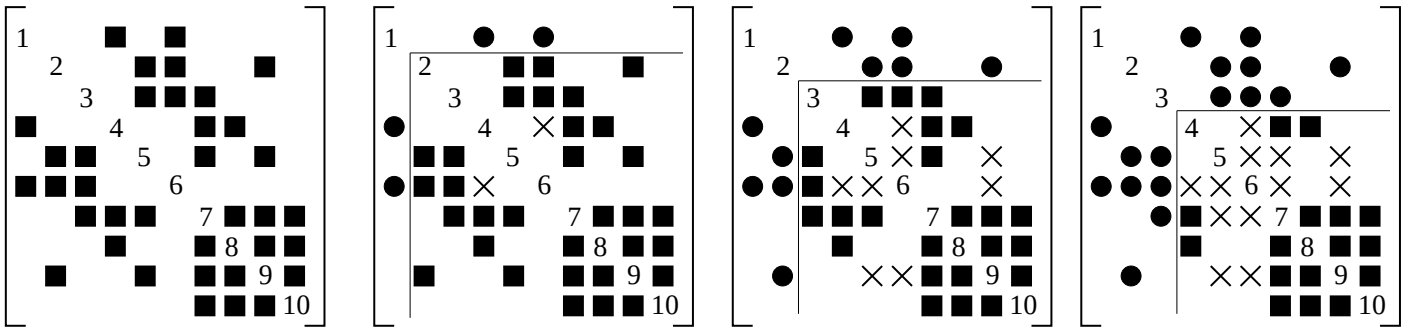
AMD : the UMFPACK degree update



(a) Elimination graph



(b) Quotient graph

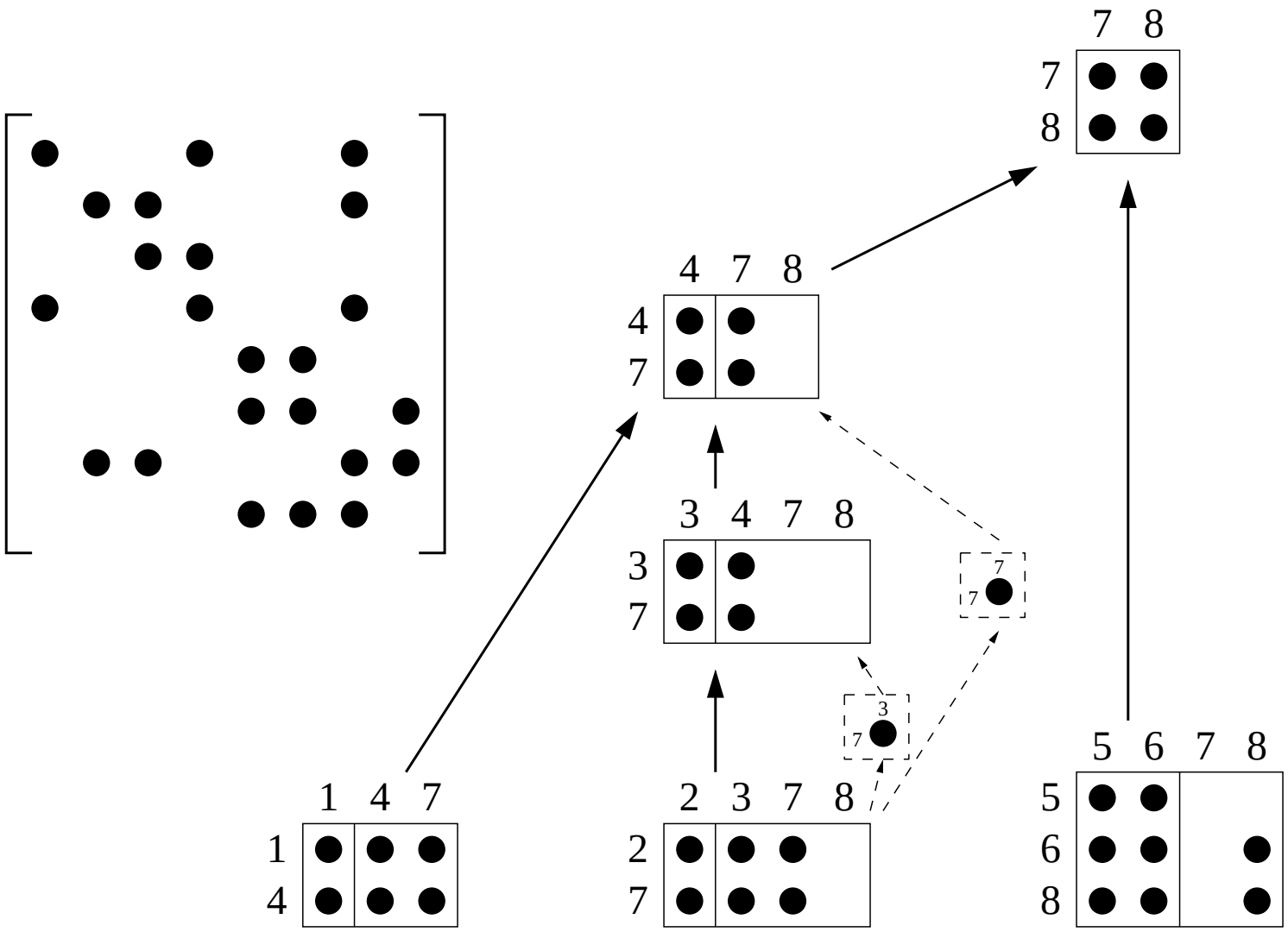


(c) Factors and active submatrix

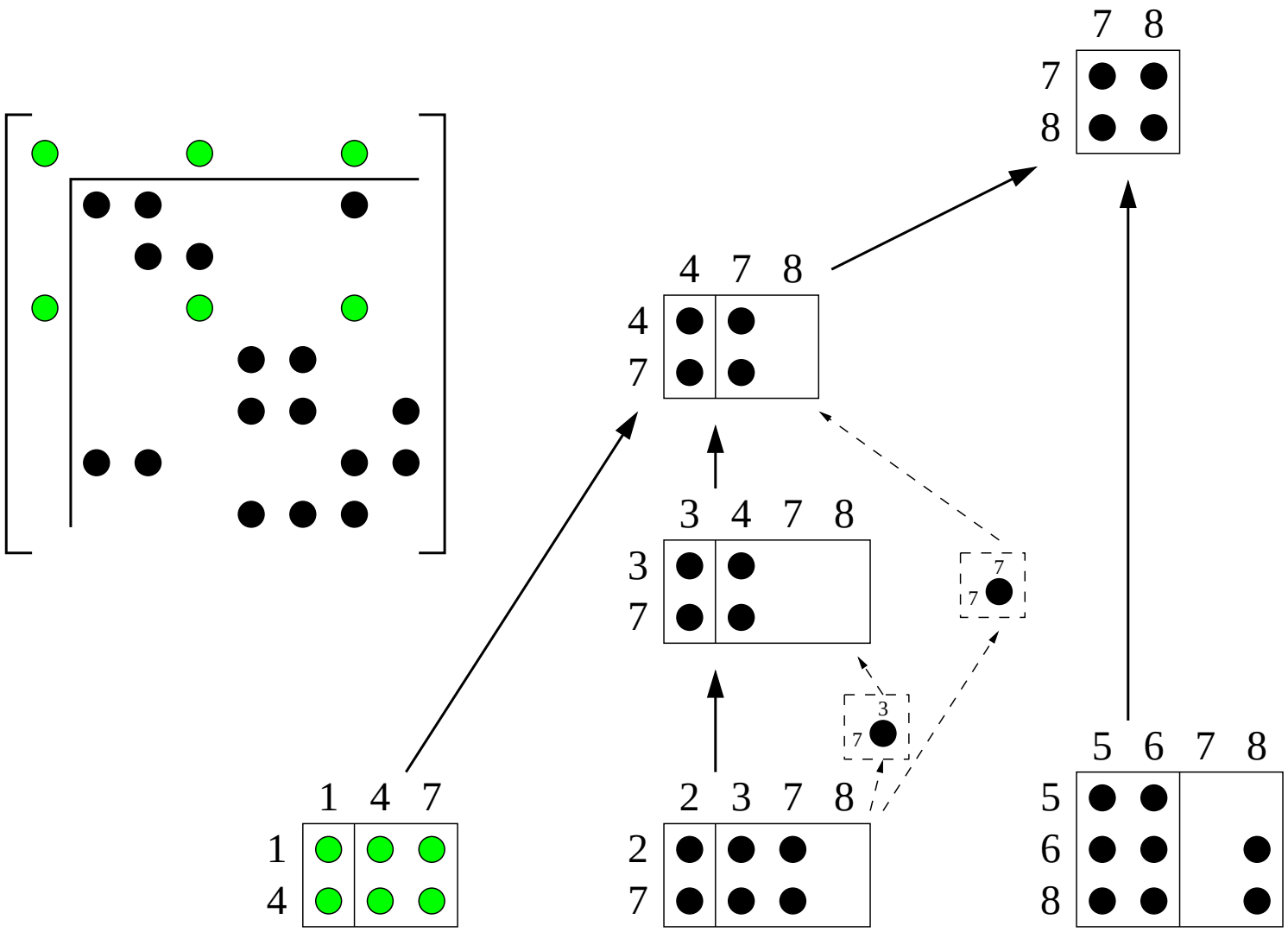
UMFPACK 2007

- symbolic LU factorization, $PAQ = LU$
  - ordering (COLAMD, AMD, ...) for Q
  - column elimination tree / row-merge tree
  - QR upper bound
  - supercolumns
- right-looking multifrontal with partial pivoting
  - unsymmetric frontal matrices
  - unifrontal chains in the column elimination tree
  - numerical assembly / approximate degree update
  - local pivot search, find P and modify Q

Column etree with frontal matrices



Column etree with frontal matrices



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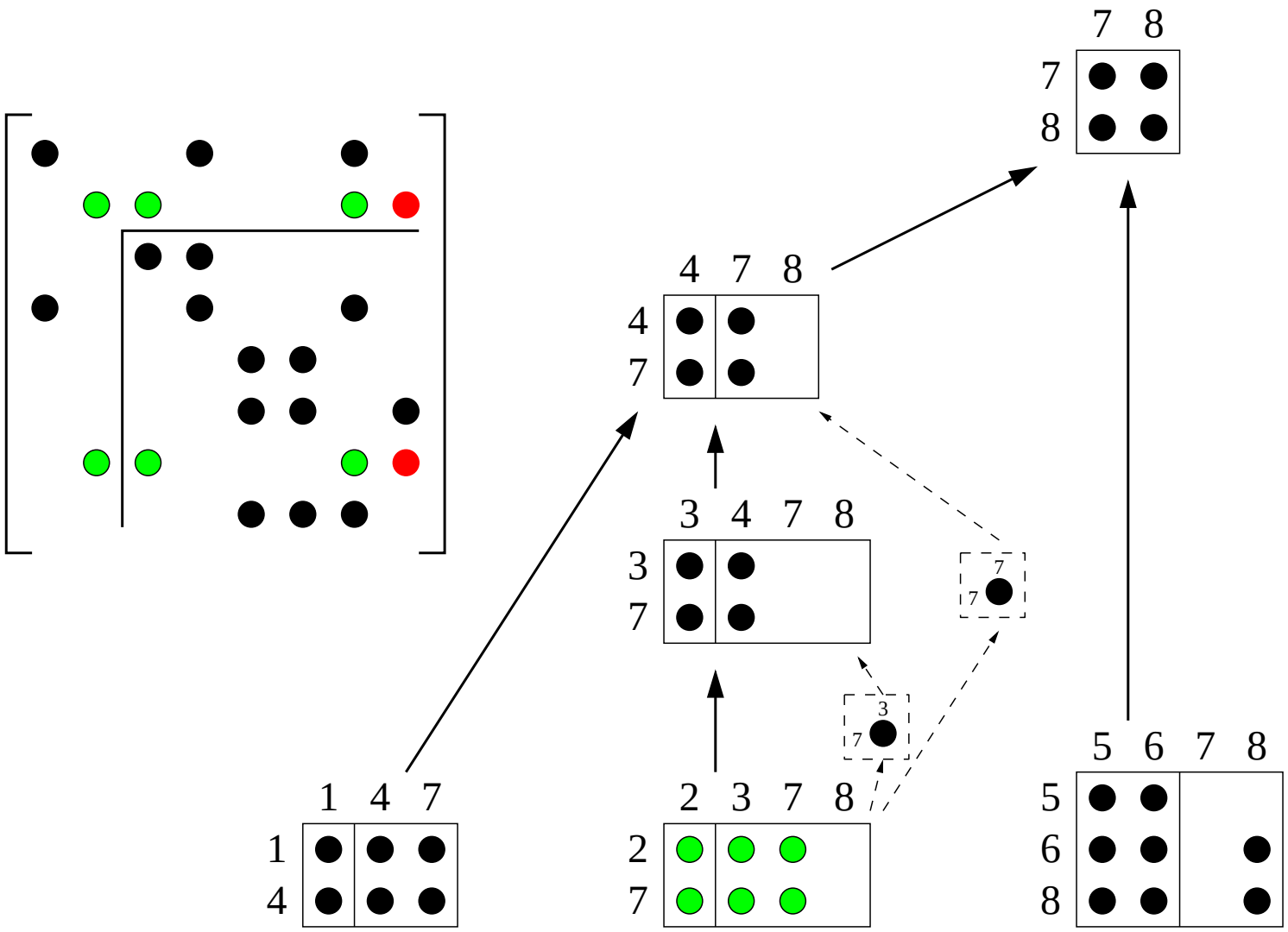
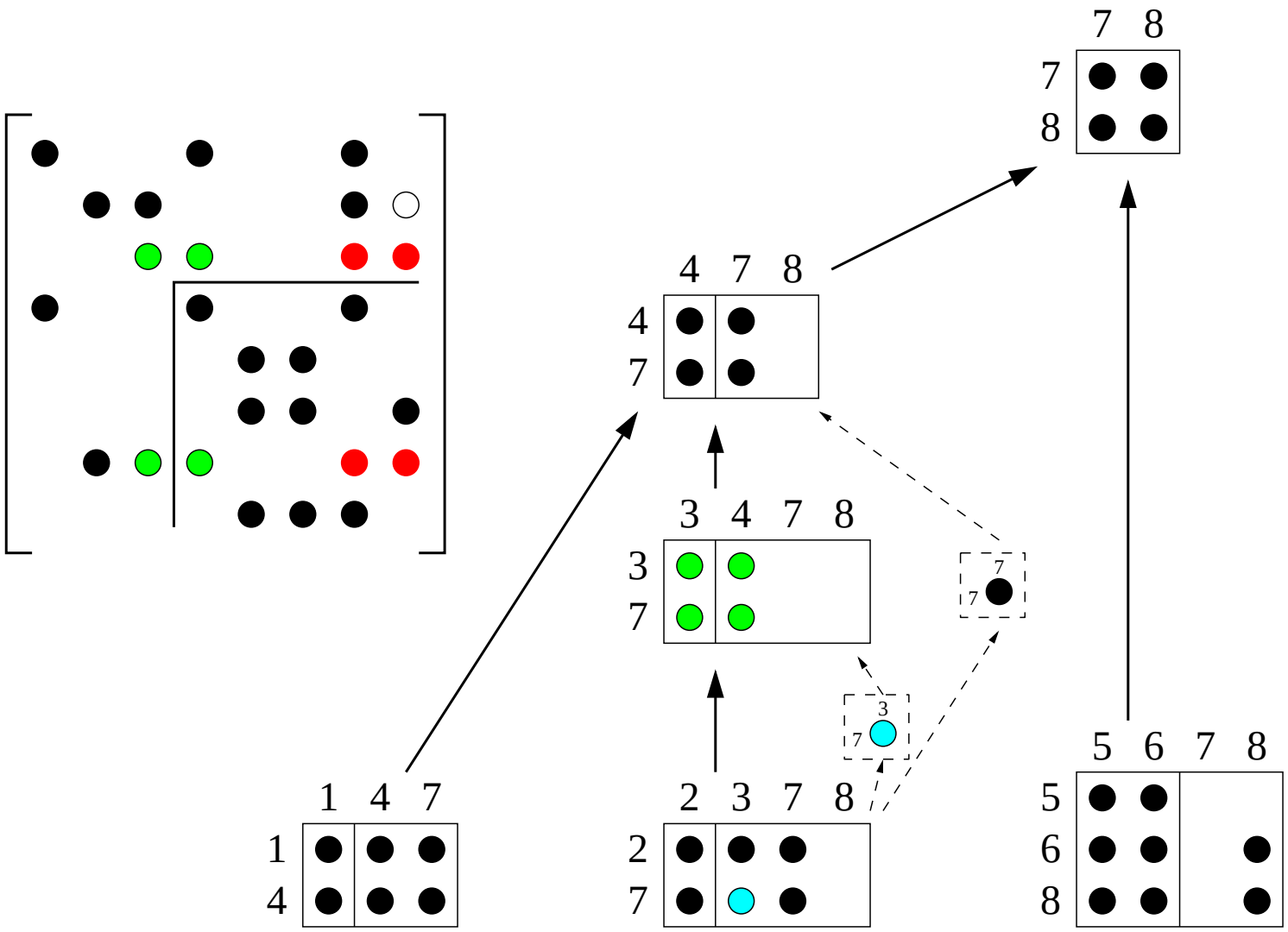
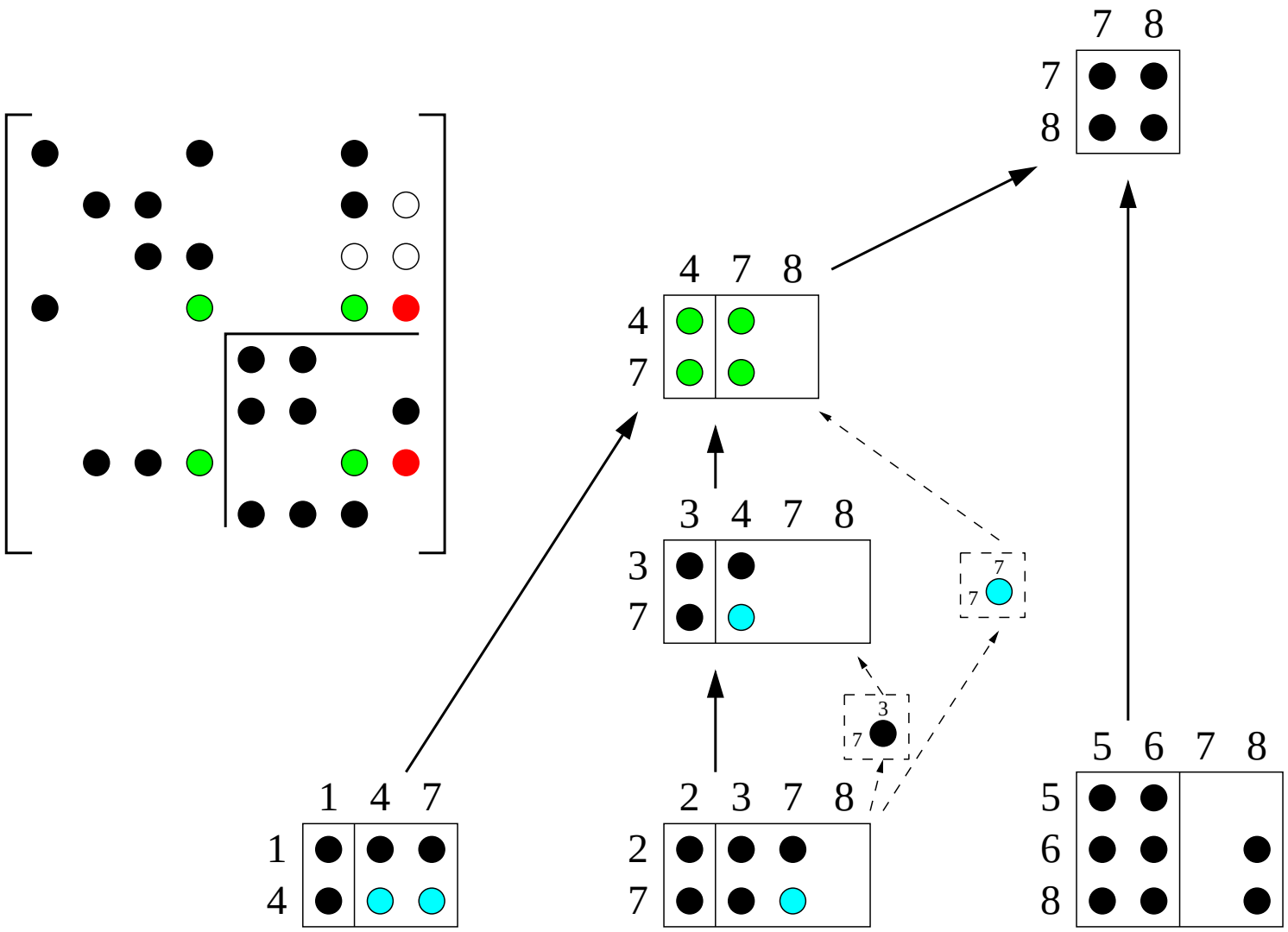


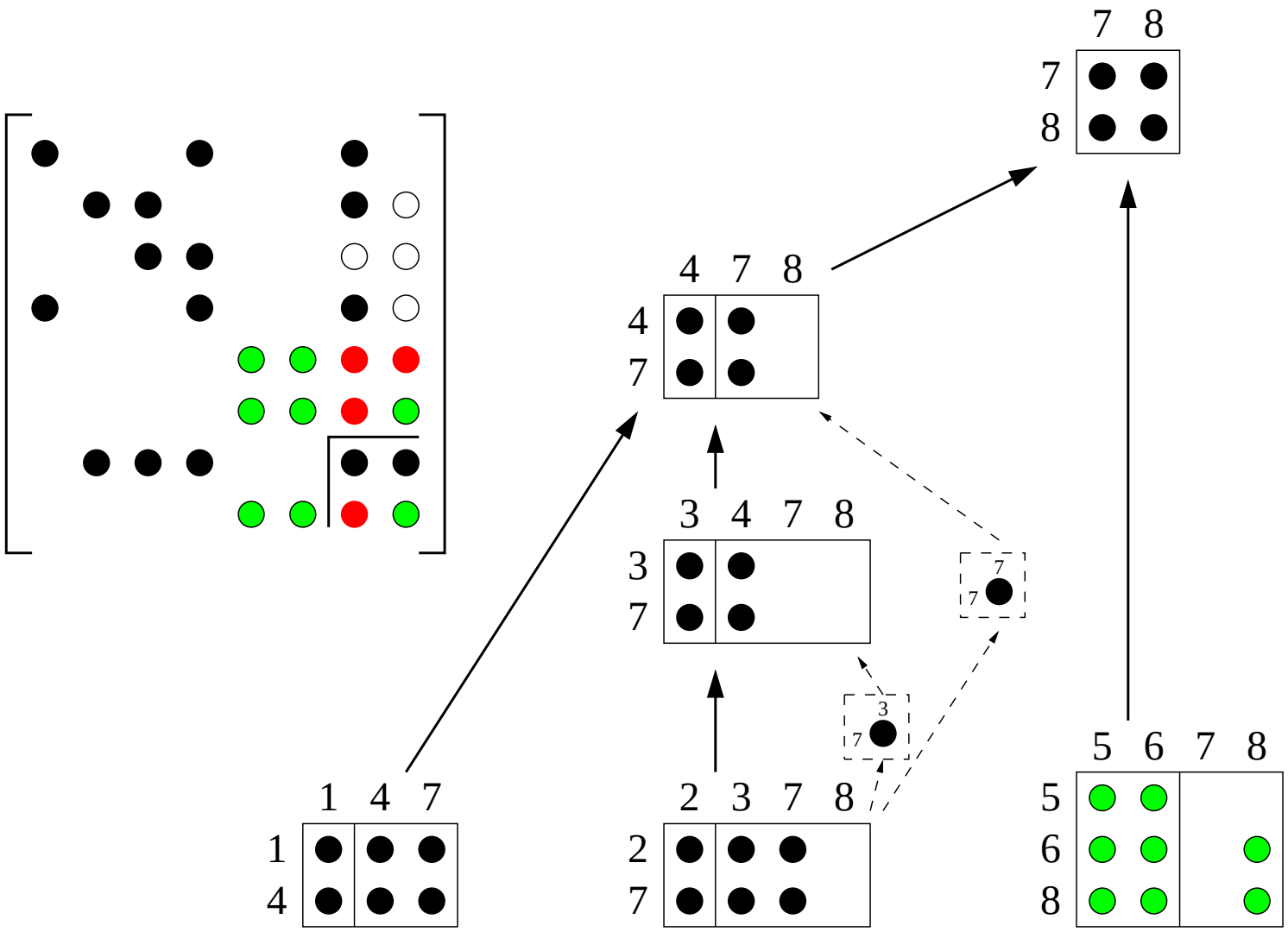
Column etree with frontal matrices



Column etree with frontal matrices



Column etree with frontal matrices



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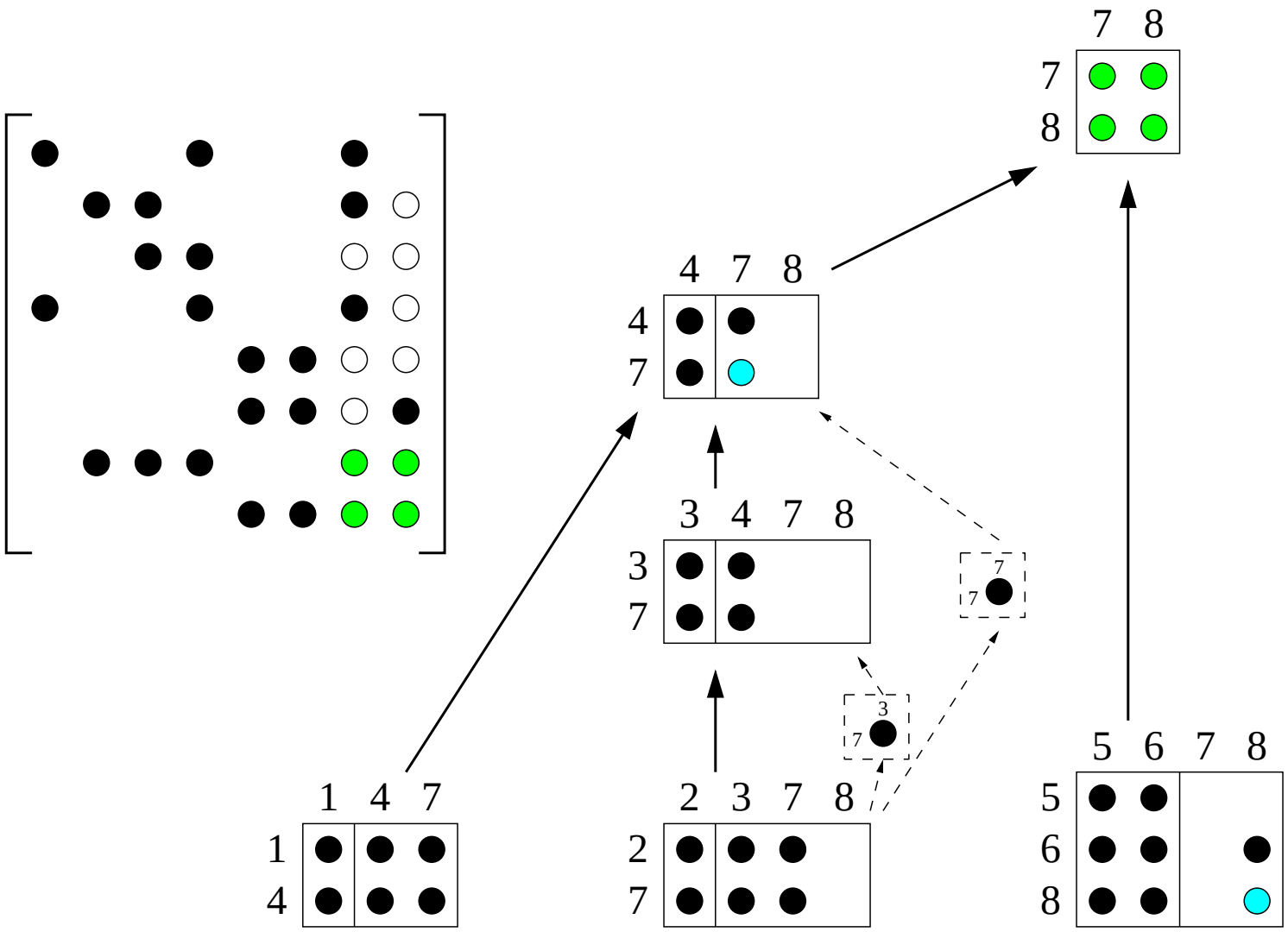
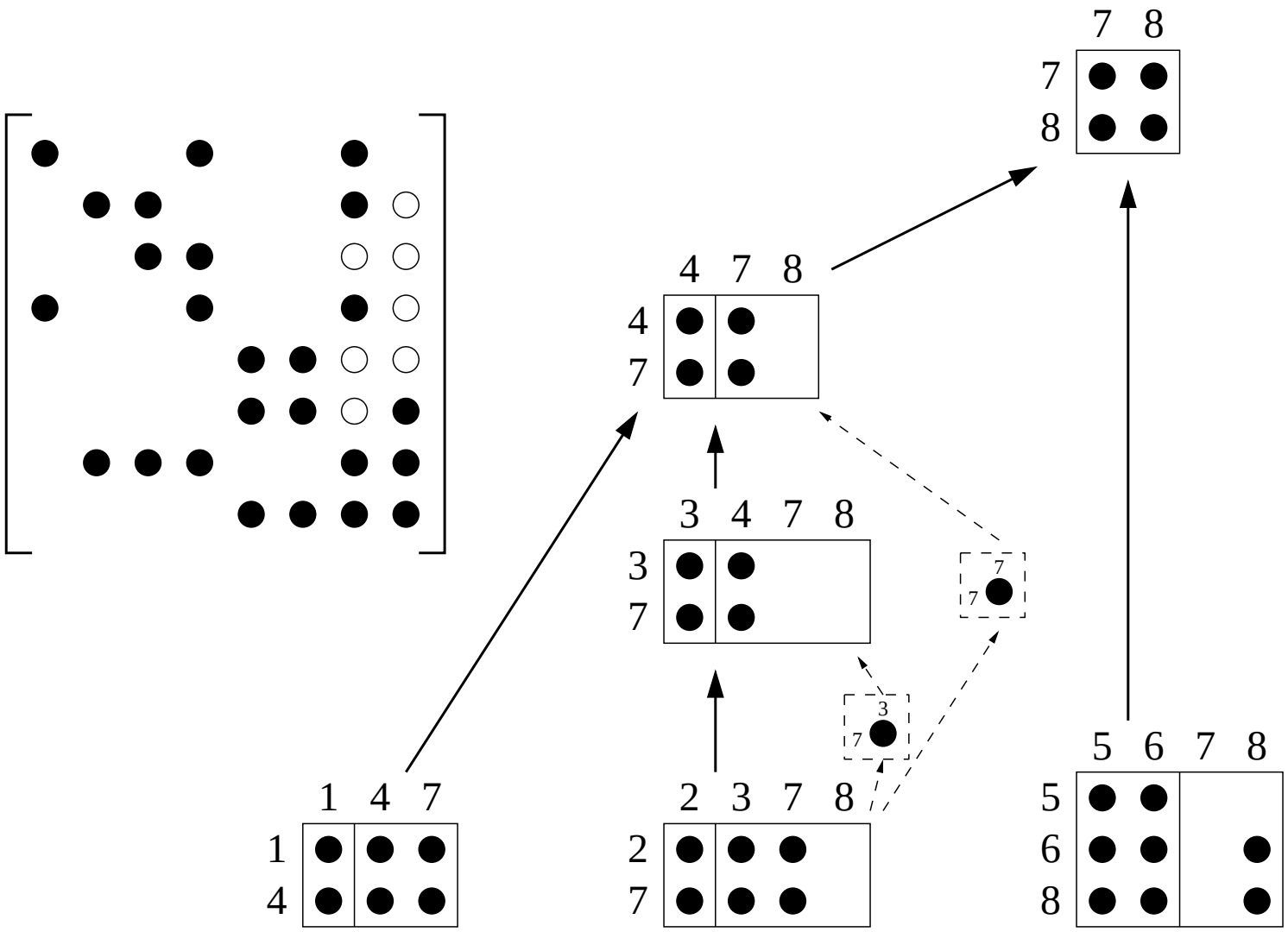


Column etree with frontal matrices



UMFPACK / AMD history

- 1989: future work in PhD thesis; Sept '89 to CERFACS
- 1990: CERFACS, UMFPACK 1.0 (with Duff)
- 1994: AMD (St. Girons, Amestoy and Duff)
- 1995: MA38 (UMFPACK 2.2) in HSL (with Duff)
- 1998: COLAMD (with Gilbert, Ng, Larimore)
- 2000: COLAMD in MATLAB 6.0
- 2001: UMFPACK 3 (in C, with Duff) using COLAMD
- 2002: UMFPACK 4 in MATLAB 6.5 (x=A\b)
- 2004: AMD in MATLAB 7.0, UMFPACK 4.3 uses AMD
- 2006: CHOLMOD in MATLAB 7.2 (sparse Cholesky)
- 2007: MA57 in MATLAB 7.5 (sparse LDL^T)

MATLAB sparse x=A\b

- if diagonal: scale
- if square and banded: tridiagonal, or LAPACK
- if upper/lower triangular: forward/backsolve
- if morally triangular: permute, triangular solve
- if symmetric:
  - real positive diagonal: Cholesky (CHOLMOD)
  - else: LDL^T with 2-by-2 pivots (MA57)
- if square (or Cholesky failed): LU (UMFPACK)
- if not square: QR (Givens-based)

Any technical questions?

Les Traductions Faibles

Une leçon francaise, et de logiciel :

Fabulous Translations

Une leçon francaise, et de logiciel :

Lessen my French; I'll eat my words.

Or in plain language to my ear,
I shall reduce each word, I fear,
from French to mangled English here,
first word-for-word then sentence clear.

Les Traductions Faibles

Nous créons bien des théories de math :

Fabulous Translations

Nous créons bien des théories de math :

No cryin' band-aids for theories of math.

Which is to say in English clear,
my theory of math has lemmas unclear;
no quick fix here; I shed a tear.
My journal paper won't pass my peer.

Les Traductions Faibles

Nous sommes heureuses d’être a Toulouse :

Fabulous Translations

Nous sommes heureuses d'être a Toulouse :

your sum's a ruse, a debt to lose.

Your math's a mess, your sum's obtuse,
your lemmas are lost, you've nothing to lose,
but in your proof, some miracle use,
or yet another method choose.

Les Traductions Faibles

CERFACS est une place pour les logiciels math :

Fabulous Translations

CERFACS est une place pour les logiciels math :

Sure, facts have their place *in the logic of math.*

In other words, that is to say,
mathematical code with truth it can play.
If your code has a bug that is here to stay,
just call it a fact, or a feature, OK?

Les Traductions Faibles

Pour matrice creuse, nos codes sont vites :

Fabulous Translations

Pour matrice creuse, nos codes sont vites :

poor me, tries we cruise, no codes sound fit.

Alas I’ve cruised this road before,
my code once more has dumped it core,
My sorry attempts I’ll try once more,
to fix my code until I’m sore.

Les Traductions Faibles

Nous vous souhaitons un bon voyage :

Fabulous Translations

Nous vous souhaitons un bon voyage :

Don’t sweat it now, just clear out of town.

My poems of math and matrices sparse
encited them all to shout ’til they’re hoarse.
I’d better clear out and mount up my horse,
and ride out of town; just don’t be too harsh.

In Xanadu ...

In Xanadu did Kubla Khan
A stately pleasure-dome decree :
Where Alph, the sacred river, ran
Through caverns measureless to man
Down to a sunless sea.

by Coleridge

In Toulouse, France ...

In Xanadu did Kubla Khan
A stately pleasure-dome decree :
Where Alph, the sacred river, ran
Through caverns measureless to man
Down to a sunless sea.

In Toulouse, France, does Iain Duff
Solve matrix problems high an' tough.
Where sparse, the multifrontal, ran,
Writ down in parallel Fortran
upon a C-less Sun.

In Stanford U ...

In Xanadu did Kubla Khan
A stately pleasure-dome decree :
Where Alph, the sacred river, ran
Through caverns measureless to man
Down to a sunless sea.

In Stanford U. did Golub, Gene,
a matrix SVD decree :
where A the matrix rank is found
through $\mathbb{R}^n$'s measured in tight bound
from sigma 1 to n.