Enabling Petascale Science through Combinatorial Algorithms

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www.cscapes.org
CSC Community Thanks CERFACS

- CSCAPES Institute is a natural outgrowth of the CSC community, which has been fostered by CERFACS for the past 20 years.
- Thank you to CERFACS researchers, who have led the way on sparse solvers, parallel computing, matchings, mathematical software, and many other topics!
- Thanks for organizing the CSC05 Workshop at CERFACS!
- Thanks for organizing the annual Sparse Days conferences, a remarkable forum for discussions, good food and wine!
Parallelization, Load Balancing

Graph Coloring

Performance

Automatic Differentiation

Graph Matching

Combinatorial problems?
Load Balancing

...enabling parallelization and fast run-times for irregular applications
Partitioning and Load Balancing

- **Goal**: assign data (and tasks) to processors to
  - minimize application runtime
  - maximize utilization of computing resources
- **Metrics**:
  - minimize processor idle time (balance workloads)
  - keep inter-processor communication costs low
- **Impacts performance of a wide range of simulations**
  - Accelerator code speeded up 3X with a geometric partitioner
- **Several partitioning and load balancing algorithms**

![Diagram](image1)

Linear solvers & preconditioners

Adaptive mesh refinement

Contact detection

Particle simulations
Zoltan Toolkit: Data Services for Dynamic Applications

- Dynamic Load Balancing
- Data Migration
- Graph Coloring
- Matrix Ordering
- Unstructured Communication
- Distributed Data Directories

Matrix:

\[
\begin{array}{ccc}
A & B & C \\
0 & 1 & 0 \\
\end{array}
\quad
\begin{array}{ccc}
D & E & F \\
2 & 1 & 0 \\
\end{array}
\quad
\begin{array}{ccc}
G & H & I \\
1 & 2 & 1 \\
\end{array}
\]
Zoltan 3.0 is now available (www.cs.sandia.gov/Zoltan). New features use hypergraphs for modeling communications accurately:

• Hypergraph repartitioning
  – Reduces total communication in dynamic applications.

• Hypergraph refinement
  – Quickly improves an existing parallel distribution (partitioning)

• Hypergraph partitioning with fixed vertices
  – Allows application to fix certain data to specific processors.

• Hierarchical partitioning
  – 2-level partitioning, possibly using different algorithms, cost metrics
  – Useful for complex computer architectures (e.g., multi-core)
Automatic Differentiation

...enabling the solution of nonlinear differential equations, optimization, sensitivity analysis, uncertainty quantification, etc.
AD: Introduction

- Transforms code for computing a function into code for differentiating it.
- Function computed from intrinsic operations, and modeled by a directed acyclic graph (DAG).
- Compute derivatives by composing partial derivatives for each operation, using the chain rule on the DAG.
- Efficiency of generated code depends on sophistication of compiler analysis and combinatorial algorithms.
AD: Combinatorial Problems

- Parallel algorithms for differentiating reduction operations
- Reduce operations and storage needed to compute the derivatives by evaluating the DAG in suitable orders
  - Two extreme modes: Forward and Reverse
  - Modeled as vertex and edge elimination in DAG
  - Stop at some intermediate stage to find minimum storage
- Location of checkpoints in reverse mode
- Graph coloring for computing many derivatives in one AD pass through the DAG
- Integration with PETSc and Zoltan toolkits
Sensitivity analysis in climate model

- Sensitivity of flow through Drake Passage to ocean bottom topography (P. Heimbach, MIT)
  - Finite difference approximations: 23 days
  - Naïve automatic differentiation: 2 hours 23 minutes
  - Smart automatic differentiation: 22 minutes
AD: Current Capabilities

- **Fortran 77: ADIFOR 2.0/3.0**
  - Robust, mature tool with excellent language coverage
  - Excellent compiler analysis
  - Efficient forward mode; adequate reverse mode

- **C/C++: ADIC 2.0**
  - Semi-mature tool with full C language coverage
  - Sophisticated differentiation algorithms
  - Efficient forward mode

- **Fortran 90: OpenAD/F**
  - New tool with partial language coverage
  - Sophisticated differentiation algorithms
  - Accurate and novel compiler analysis
  - Innovative templating mechanism
  - Efficient forward and reverse modes
Graph Coloring

...reducing work in Automatic Differentiation; and discovering parallelism in computations
Coloring and Jacobian Computation

Original Jacobian

Compressed representation (Structurally orthogonal columns packed together)

D1 coloring formulation on column inter. graph

D2 coloring bipartite graph
Coloring and Derivatives: The Big Picture

- **Scenarios and coloring models:**
  - unsymmetric vs symmetric matrix
  - direct vs substitution method
  - uni- vs bi-directional partitioning

- **Developed novel sequential algorithms**

- **Future plans**
  - Develop parallel versions
  - integrate with AD tools

<table>
<thead>
<tr>
<th></th>
<th>1d partition</th>
<th>2d partition</th>
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</thead>
<tbody>
<tr>
<td><strong>Jacobian</strong></td>
<td>Distance-2 coloring</td>
<td>Star bicoloring</td>
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<td><strong>Hessian</strong></td>
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<td>Acyclic bicoloring</td>
</tr>
<tr>
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</tbody>
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**Diagrams:**
- D2 coloring
- Star coloring
- Acyclic coloring
Framework for parallel coloring

- Essential ingredients of framework:
  - Partition graph on processors, and speculatively color subgraphs in rounds
  - Exchange color info after a superstep (coloring a specified no. of vertices)
  - Detect conflicts after each round, resolve using randomization, recolor when needed

- Applied to D-1 and D-2 coloring, implemented in MPI; available in Zoltan

- Extending the framework to
  - Tera- and peta-scale machines
  - Other graph problems

*Weak scalability* on two families of graphs: random (unstructured); planar (structured).
Matchings in Graphs

...enabling load balancing and linear solvers
**Matchings in Graphs**

- **Matching** is a pairing of vertices; a vertex is paired with one neighboring vertex or none.

- **Applications**
  - Place large elements on diagonals of matrices for solvers.
  - Block triangular form to reduce work in solvers, improve condition number.
  - Coarsening step in multilevel graph and hypergraph partitioners.
Block Upper Triangular Form (BTF)

Circuit model from Xyce (Hoekstra, Day; Sandia) 683K rows, 2M nnz, 584K diag blocks
Solved 200 times faster! 100M problem waits.
Outreach and Training

- Organized the SIAM Workshop on CSC in Feb. 2007. 100 attendees, 12 early career researchers supported. SIAM News article in May 2007. URL: [www.cscapes.org, click on CSC07](http://www.cscapes.org)
- International collaborations with CERFACS, AD groups in Germany, CSC groups in Norway, Netherlands, and others.
- 5 Postdoctoral researchers, 4 PhD students, and an undergraduate are involved in CSCAPES research, and are co-mentored by Lab scientists.
- Working with several enabling technology and applications groups to integrate CSC software and solve their combinatorial problems.
- We welcome application kernels where CSC issues are significant; tell us about your combinatorial problems!
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