Normal Degree and Krylov Sequences

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The normal degree $n(A)$ for a normalizable matrix $A$ is the degree of the minimum degree polynomial $n(z)$ for which $A' = n(A)$, where $A'$ is the dual of $A$ in some H-inner product space in which $A$ becomes normal. Because the recurrence lengths of orthogonalizing Krylov subspace bases are exactly the normal degrees, normal degree is important for sparse matrix solvers $A x = b$. Minimal normal degree may be looked at for individual data $b$, for given sets $b$ to be treated in parallel, or for the whole operator $A$. Recent papers by Liesen, Saylor, Strakos, Faber, Tichy, perhaps others, apparently did not know that in a paper [1] I showed the following, which I will elaborate and bring up to date in my lecture.

- Normal degree is independent of the H-inner product. This corrected incompleteness in the literature, and removes certain vagueness one still finds in current papers.

- If $A$ is unitary, then $n(A) = m - 1$. Here $m$ is the degree of $A$’s minimum polynomial.

- I emphasized how normal degree is fundamentally a complex interpolation problem which needed a corresponding new theory of “complex dipoles analysis”. This theory has been found, is quite interesting, and I will comment on its implications.

References
