Computing the trace of the inverse of sparse matrices using modified moments

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We consider the problem of computing the trace of the inverse of a large sparse symmetric matrix of order $n$. Bai and Golub gave analytic bounds using the three first moments

$$
\mu_0 = n, \quad \mu_1 = \text{tr}(A) = \sum_{i=1}^{n} a_{i,i}, \quad \mu_2 = \text{tr}(A^2) = \sum_{i,j=1}^{n} a_{i,j}^2 = \|A\|_F^2.
$$

However, there are many cases for which these bounds are not sharp. We propose to use more moments to compute an estimate of the trace of the inverse. Since working with the moments can be highly unreliable, we use the modified Chebyshev algorithm which relies on so-called modified moments which we compute using Chebyshev polynomials as auxiliary polynomials. Numerical experiments will be given which show that this technique is effective and gives good approximations of the trace of the inverse. Similar techniques can be used to compute the determinant of sparse matrices.