Intel® Direct Sparse Solver for Clusters, a research project for solving large sparse systems of linear algebraic equation

Alexander Kalinkin
Anders Anton
Anders Roman
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Agenda

• Intro
• Algorithm
• Reordering step
• Factorization step
• Experiments
• Conclusion
Problem statement

\[
Ax = b
\]

**Cons**
- No extra data available for matrix but some global properties (positive define, hermitian...)
- Huge size

**Pros**
- Clusters with modern Intel® CPUs
- Intel® MKL library with optimized BLAS, LAPACK, PARDISO functionality
**Algorithm (Ax=b)**

**Input:** matrix A, vector b; special parameters.

1. **Matrix reordering and symbolic factorization**
   - Reorder matrix A to reduce fill-in in factor L, create dependency tree representation of matrix A

2. **Numeric factorization**
   - Compute decomposition $A=LL^T$ or $LDL^T$ or $LU$
     - The most time-consuming part

3. **Forward and backward substitution**
   - Solve $Ly=b$ (forward step), $Dz=y$ (diagonal step), then $L^Tx=z$ (backward step)

**Output:** vector x.
Reordering step

Matrix A after reordering (example of 4 leafs/process)

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- non-zero block

Tree representation of matrix A after reordering
Factorization step

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- L-block updates R-block (or Right depends on Left)

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Tree representation of matrix A after reordering

A → B → D → E → F → C → G

Left block updates Right block (or Right depends on Left)
Factorization step

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Tree representation of matrix A after reordering

- Both tree and tree-node parallelization used
- All computations within the node are based on functionality from Intel® MKL
- Computation of leafs & updates of a block are independent on each process
- Data distributed between processes uniformly
Factorization step

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- Computation of leafs & updates of a block are independent on each process
- Data distributed between processes uniformly
Choosing one thread per process allow us to “mask” data transfer time under computational process.
Current status/interface

Supported as 2 additional libraries, Lnx & Win 64 bit only. Ported by different MPI via user-compiled wrapper.

C:

```c
{    
   ....
   PARDISO (pt, &maxfct, &mnum, &mtype,  
            &phase, &n, a, ia, ja, &idum, &nrhs,  
            iparm, &msglvl, b, x, &error);
   ...
}
```

Fortran:

```fortran
    ....
    Call PARDISO(pt, maxfct, mnum, mtype,  
                  phase, n, a, ia, ja, idum, nrhs,  
                  iparm, msglvl, b, x, error);
    ...
```
Experiments (scalability of time)
Experiments (scalability of time)

Additional processes reduce computational time!!!
Experiments (scalability of time)
Experiments (scalability of memory)

NDOF=398K, NNZ=15.7M
Absolute memory per node scalability
(Lower is better)

NDOF=1.7M, NNZ=12M
Absolute memory per node scalability
(Lower is better)
**Experiments (scalability of memory)**

- **NDOF=398K, NNZ=15.7M**
  - Absolute memory per node scalability
  - (Lower is better)

- **NDOF=1.7M, NNZ=12M**
  - Absolute memory per node scalability
  - (Lower is better)

*Additional processes decrease memory size per host!!!*
Conclusion

Intel® Direct Sparse Solver for Clusters based on Intel® MKL functionality results in

• Good scaling of computational time
• Good scaling of memory per node
Q & A
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