Challenges in higher order extrapolations of transonic turbulent flows

Stéphane AUBERT Pascal FERRAND

`stephane.aubert@ec-lyon.fr`
Laboratoire de mécanique des fluides et d'acoustique
CNRS - Université de Lyon
École centrale de Lyon, France

MUSAF II Colloquium - Toulouse / 18th – 20th *September* 2013

Outline

1. Introduction
   - Context
   - Formulation
   - Numerics
2. NACA64A010
   - Configuration
   - Subsonic reference solution at $P_s = 145\mathrm{kPa}$
   - Transonic reference solution at $P_s = 130\mathrm{kPa}$
   - Reconstruction
3. Conclusion

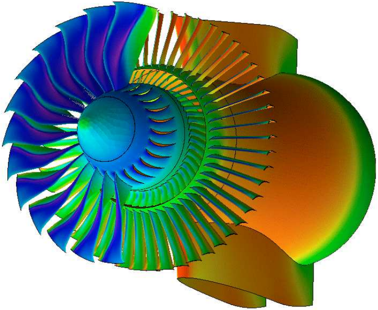
Outline

1. Introduction
   - Context
   - Formulation
   - Numerics
2. NACA64A010
   - Configuration
   - Subsonic reference solution at $P_s = 145\mathrm{kPa}$
   - Transonic reference solution at $P_s = 130\mathrm{kPa}$
   - Reconstruction
3. Conclusion

Outline

1. Introduction
   - Context
   - Formulation
   - Numerics
2. NACA64A010
   - Configuration
   - Subsonic reference solution at $P_s = 145\mathrm{kPa}$
   - Transonic reference solution at $P_s = 130\mathrm{kPa}$
   - Reconstruction
3. Conclusion

Transonic turbulent flows in turbomachinery



- Fans and HP compressors
- Blade design
- Operating conditions

Outline

1. Introduction
    - Context
    - Formulation
    - Numerics
2. NACA64A010
    - Configuration
    - Subsonic reference solution at $P_s = 145\mathrm{kPa}$
    - Transonic reference solution at $P_s = 130\mathrm{kPa}$
    - Reconstruction
3. Conclusion

Higher order extrapolations

If

$$\begin{aligned} p &: \text{design parameters} \\ q = q(p) &: \text{state variables} \\ F(q,p) = 0 &: \text{equilibrium constraints} \\ f(q,p) &: \text{objective functions} \end{aligned}$$

how to compute $q(p+\Delta p)$ knowing $\frac{\partial^n q}{\partial p^n}$, to evaluate $f(q(p+\Delta p), p+\Delta p)$?

- Taylor series expansion

$$q_{\mathscr{T}}(p+\Delta p) = q(p) + \frac{\partial q}{\partial p}\Delta p + \frac{1}{2}\frac{\partial^2 q}{\partial p^2}\Delta p^2 + \cdots$$

- Padé approximation

$$q_{\mathscr{P}}(p+\Delta p) = \frac{a_0 + a_1\Delta p + a_2\Delta p^2 + \cdots}{1 + b_1\Delta p + b_2\Delta p^2 + \cdots} \text{ such that } \frac{\partial^n q_{\mathscr{P}}}{\partial p^n} = \frac{\partial^n q_{\mathscr{T}}}{\partial p^n}$$

Calculation of $\frac{\partial^n q}{\partial p^n}$

Starting from $F(q,p)=0$,

$$\begin{aligned}\frac{\partial F}{\partial q}\cdot\frac{\partial q}{\partial p} &= -\frac{\partial F}{\partial p}\\ \text{i.e. } G\cdot q^{(1)} &= R\end{aligned}$$

Then,

$$\begin{aligned}G\cdot q^{(2)} &= R^{(1)} - G^{(1)}\cdot q^{(1)}\\ G\cdot q^{(n)} &= R^{(n-1)} - \sum_{i=1}^{n-1} C_{n-1}^{i} G^{(i)}\cdot q^{(n-i)}\end{aligned}$$

To compare to adjoint formulation,

$$\begin{aligned}\left[\frac{\partial F}{\partial q}\right]^t\cdot\lambda &= -\left[\frac{\partial f}{\partial q}\right]^t\\ \frac{df}{dp} &= \frac{\partial f}{\partial p}+\lambda^t\cdot\frac{\partial F}{\partial p}\end{aligned}$$

Outline

Numerics

- Steady compressible RANS equations :
  - Kok's $k - \omega$ turbulence model
  - Clipping of k production term
- Vertex centered finite volume method over structured meshes :
  - SRANS (Turb'Flow solver) : Roe's upwind scheme with van Albada's flux limiter
  - PRANS (Turb'Opty solver) : Jameson's centered scheme with pressure sensor
- Convergence :
  - SRANS (Turb'Flow solver) : Implicit time marching, 10 orders
  - PRANS (Turb'Opty solver) : GMRES(500), ILU(0) right preconditionning, 12 orders

Outline

1. Introduction
    - Context
    - Formulation
    - Numerics
2. NACA64A010
    - Configuration
    - Subsonic reference solution at $P_s = 145\mathrm{kPa}$
    - Transonic reference solution at $P_s = 130\mathrm{kPa}$
    - Reconstruction
3. Conclusion

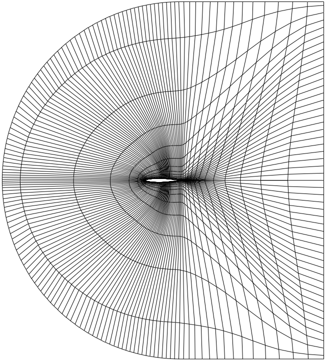
Outline

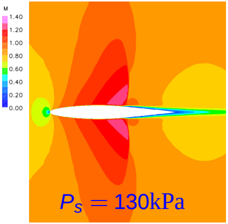
1. Introduction
   - Context
   - Formulation
   - Numerics
2. NACA64A010
   - Configuration
   - Subsonic reference solution at $P_s = 145\mathrm{kPa}$
   - Transonic reference solution at $P_s = 130\mathrm{kPa}$
   - Reconstruction
3. Conclusion

Mesh (1:2) and operating conditions

P_t, T_t, α

k, ω

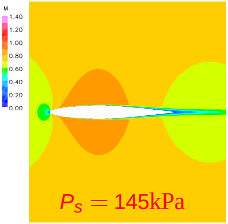




$P_s = 130\text{kPa}$

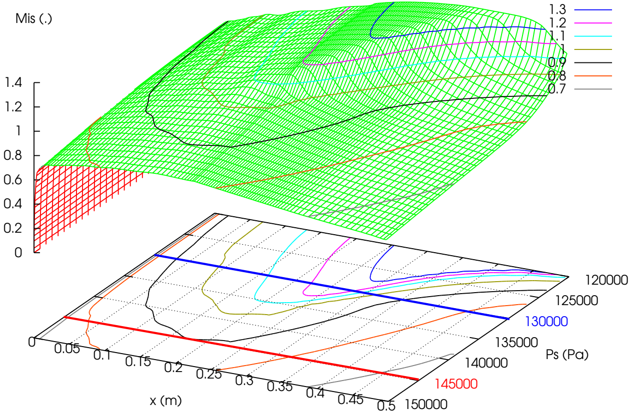
$P_s \in [120, 150]\text{kPa}$

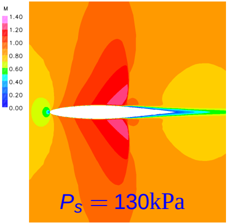
$\Rightarrow \mathrm{M}_\infty \in [0.454, 0.813]$



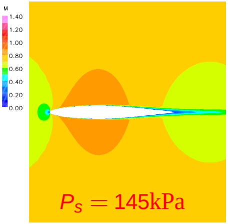
$P_s = 145\text{kPa}$

Suction side isentropic Mach number





$P_s = 130\text{kPa}$



$P_s = 145\text{kPa}$

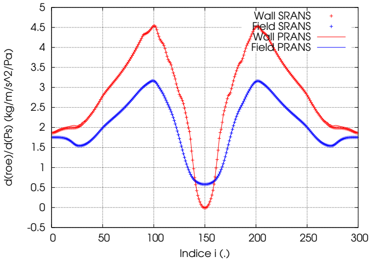
97 SRANS runs for $P_s \in [120, 150]\text{kPa}$

Outline

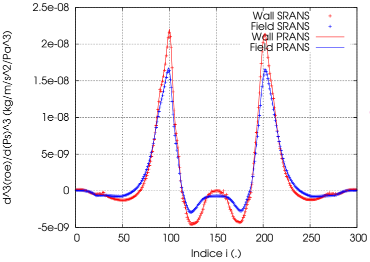
1. Introduction
   - Context
   - Formulation
   - Numerics
2. NACA64A010
   - Configuration
   - Subsonic reference solution at $P_s = 145\mathrm{kPa}$
   - Transonic reference solution at $P_s = 130\mathrm{kPa}$
   - Reconstruction
3. Conclusion

Flowfield derivatives : $\partial^n \rho E / \partial P_s^n$

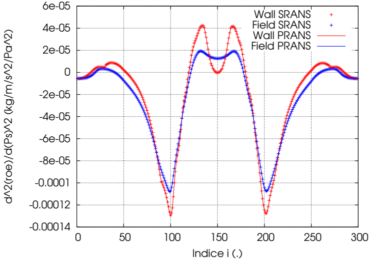
$n = 1$

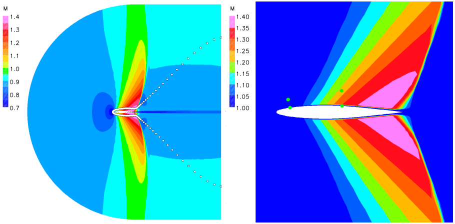


$n = 3$

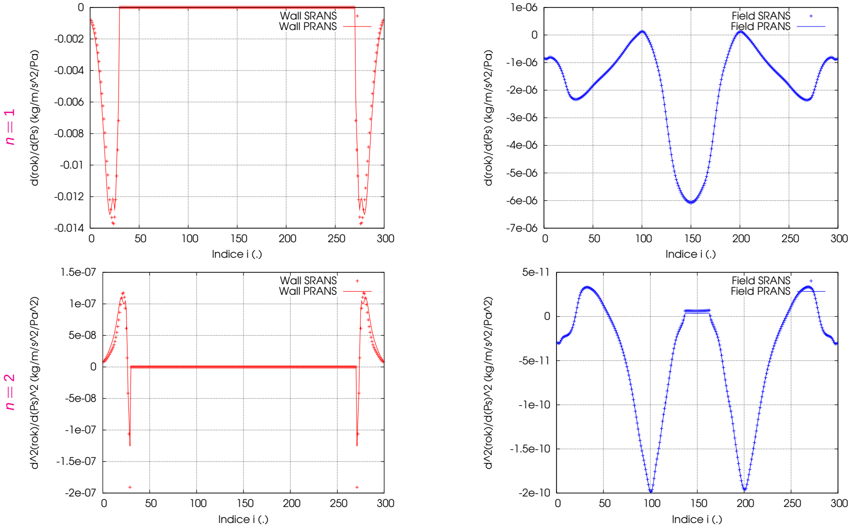


$n = 2$

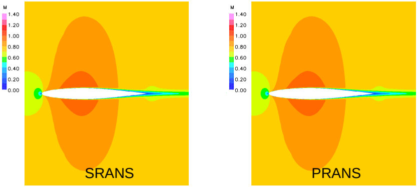


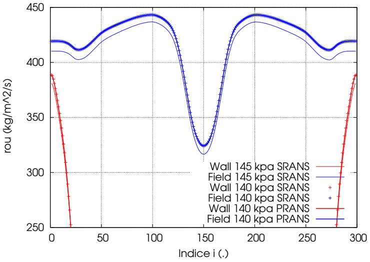


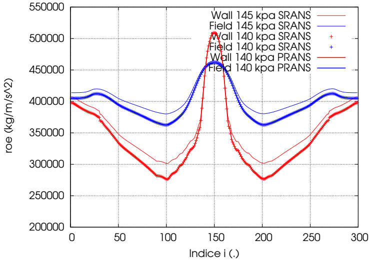
Flowfield derivatives : $\partial^n \rho k / \partial P_s^n$

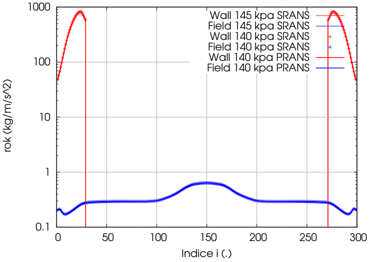


Flowfield extrapolation to $P_s = 140\mathrm{kPa}$

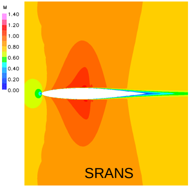


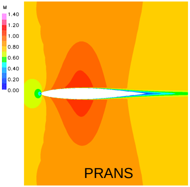


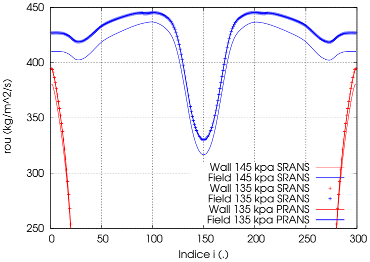


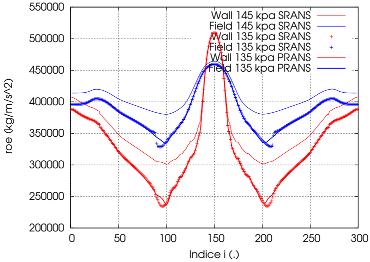


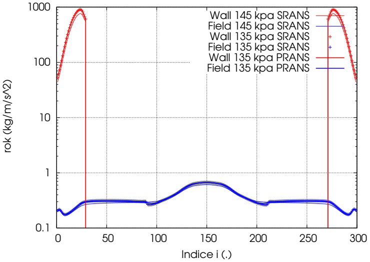
Flowfield extrapolation to $P_s = 135\text{kPa}$







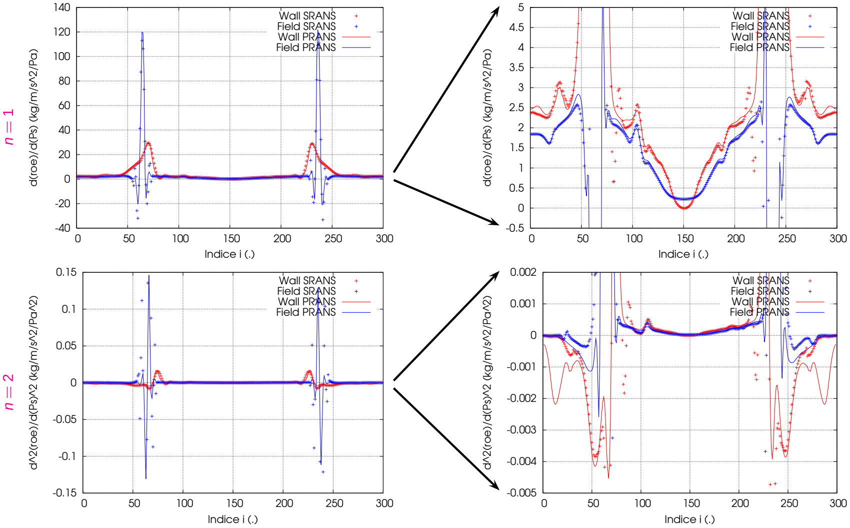




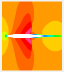
Outline

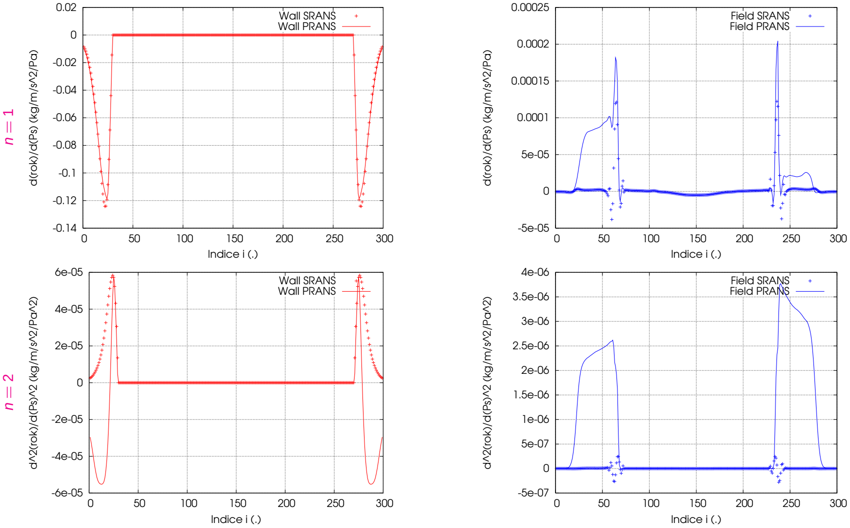
1. Introduction
    - Context
    - Formulation
    - Numerics
2. NACA64A010
    - Configuration
    - Subsonic reference solution at $P_s = 145\mathrm{kPa}$
    - Transonic reference solution at $P_s = 130\mathrm{kPa}$
    - Reconstruction
3. Conclusion

Flowfield derivatives : $\partial^n \rho E / \partial P_s^n$

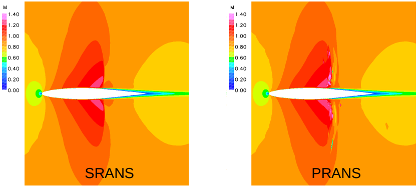


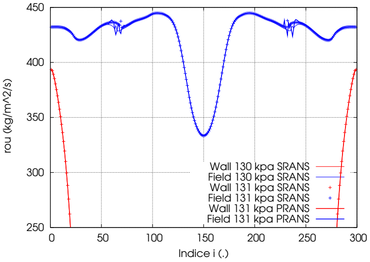
Flowfield derivatives : $\partial^n \rho k / \partial P_s^n$

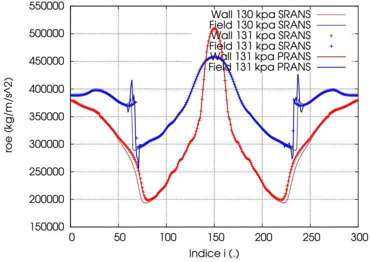


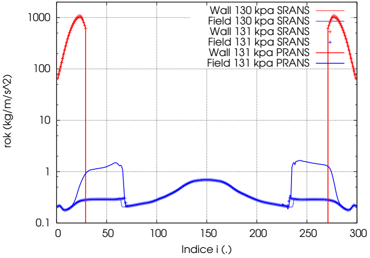


Flowfield extrapolation to $P_s = 131$kPa









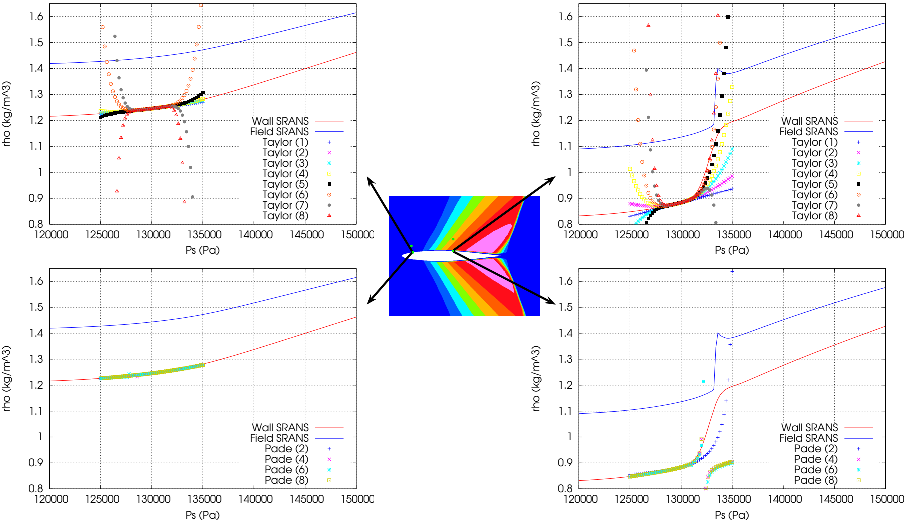
Outline

1. Introduction
   - Context
   - Formulation
   - Numerics

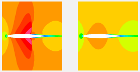
2. NACA64A010
   - Configuration
   - Subsonic reference solution at $P_s = 145\mathrm{kPa}$
   - Transonic reference solution at $P_s = 130\mathrm{kPa}$
   - Reconstruction

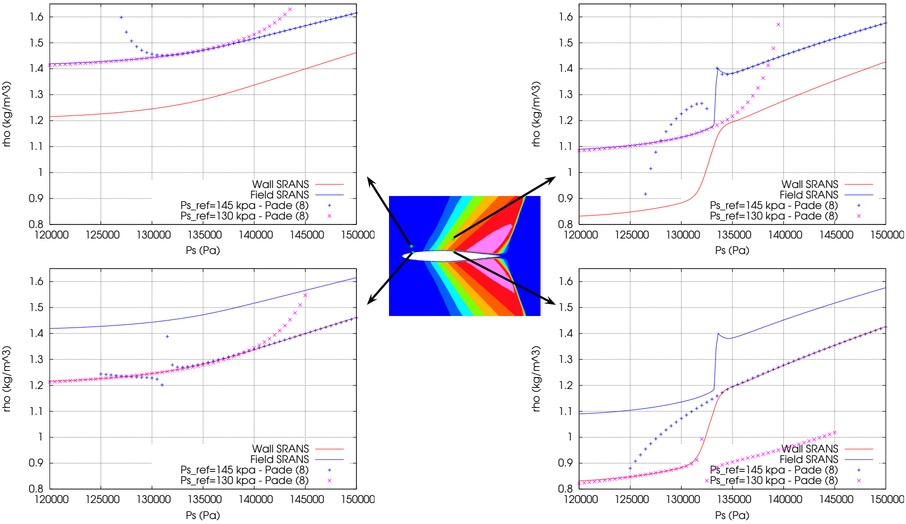
3. Conclusion

Padé vs Taylor



Two-point reconstruction

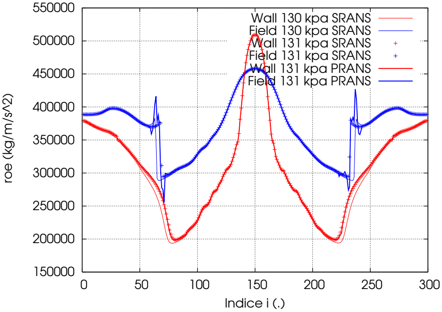


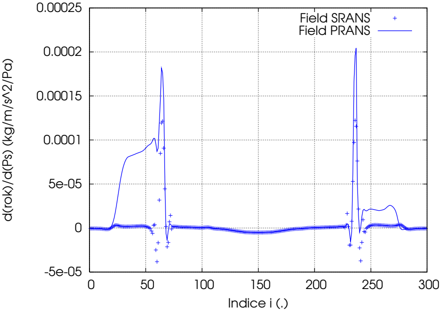


Outline

1. Introduction
    - Context
    - Formulation
    - Numerics
2. NACA64A010
    - Configuration
    - Subsonic reference solution at $P_s = 145\mathrm{kPa}$
    - Transonic reference solution at $P_s = 130\mathrm{kPa}$
    - Reconstruction
3. Conclusion

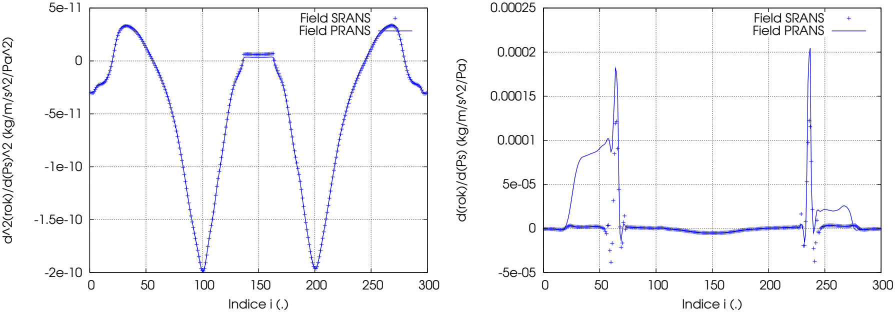
Challenges in numerics





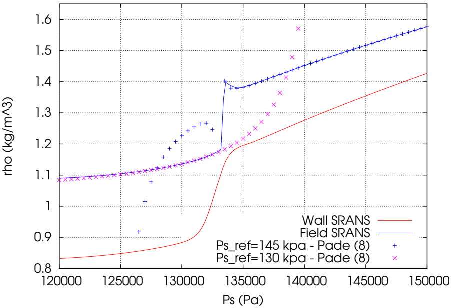
- Substitute to the pressure sensor/flux limiter in $\frac{\partial F}{\partial q}$ computation.
- Turbulence modeling/spatial scheme interaction across shock waves.
- . . . and, always and forever, linear system resolution!

Challenges in turbulence modeling



- Higher orders behaviour of standard turbulence models.
- Turbulence modeling/spatial scheme interaction across shock waves.

Challenges in response modeling



- Triggering function for two-point reconstruction.
- Physical information embedded in derivatives, poles of Padé approximation, . . .