Constrained Aero-elastic Multi-Point Optimization
Using the Coupled Adjoint Approach

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Motivation

- During the flight, the wing deforms due to the aero-elastic effects.
- The aero-elastic deformation significantly modifies the wing shape (twist and bending) and impacts the aerodynamic coefficients => need to take them into account in the design phase.

![Diagram of Jig Shape and Flight Shape](image)

![Graphs showing aerodynamic coefficients](image)

Courtesy of Stefan Keye
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The effect becomes more evident towards the wing tip

Courtesy of Stefan Keye
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- The aero-elastic equilibrium is obtained after several couplings between aerodynamics and structure and incurs high-computational cost
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- Gradient-based optimization algorithms are known to be efficient but computing the gradients is expensive with the standard finite differences approach.

Traditional Finite Differences Approach

1. Design Variables
2. Loop over number of design variables
3. Aerodynamics
   - Deformations
   - Loads
4. Structure
5. Gradients
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  => in case of optimization efficient strategies are required

- Gradient-based optimization algorithms are known to be efficient but computing the gradients is expensive with the standard finite differences approach.
  => need for an efficient approach to determine the gradients: the coupled aero-structural adjoint approach

Traditional Finite Differences Approach

Coupled Adjoint Approach

Adjoint approach is independent on the number of design variables
Formulation of the Coupled Adjoint Approach
Components of the Coupled System

- Loose aero-structural coupling is employed at DLR
Components of the Coupled System

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![Diagram showing TAU and ANSYS components of the coupled system.](image)
Components of the Coupled System

- Loose aero-structural coupling is employed at DLR

TAU

ANSYS
Components of the Coupled System

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Transfer loads from CFD mesh to CSM mesh using:
Linear Interpolation Tool
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5 Couplings
Formulation of the Coupled Adjoint Equations

- **Motivation:** Efficient computation of the gradient of a cost function \( I \) w.r.t the design parameters \( D \).
- The aero-structural coupled system is defined by:

  \[
  I = I(W,D) ; \quad R = R(W,D) = 0
  \]

- Define the Lagrange: \( L = I + \Psi R \)

  \[
  \begin{align*}
  I & = \{w, u\} \\
  R & = \{Rs, Ra\} \\
  W & = \{w, u\} \\
  D & = \{A, T\}
  \end{align*}
  \]

  \[
  \begin{align*}
  I = & I(W,D) & R = & [Ra] \\
  R = & Rs & \text{Aerodynamic residual} \\
  & Rs & \text{Structural residual} \\
  W = & [w] & \text{Flow variables} \\
  & u & \text{Structural deformation} \\
  D = & [A] & \text{Shape design variables} \\
  & T & \text{Structural thickness}
  \end{align*}
  \]

  \[
  \begin{bmatrix}
  \frac{\partial Ra}{\partial w} & \frac{\partial Ra}{\partial u} \\
  \frac{\partial Rs}{\partial w} & \frac{\partial Rs}{\partial u}
  \end{bmatrix}
  \begin{bmatrix}
  \psi a \\
  \psi s
  \end{bmatrix}
  =
  -
  \begin{bmatrix}
  \frac{\partial I}{\partial w} \\
  \frac{\partial I}{\partial u}
  \end{bmatrix}
  \]

  \[
  \begin{bmatrix}
  \frac{dI}{dA} \\
  \frac{dI}{dT}
  \end{bmatrix}
  =
  \begin{bmatrix}
  \frac{dL}{dA} \\
  \frac{dL}{dT}
  \end{bmatrix}
  +
  \begin{bmatrix}
  \psi a \\
  \psi s
  \end{bmatrix}
  \begin{bmatrix}
  \frac{\partial Ra}{\partial A} & \frac{\partial Ra}{\partial T} \\
  \frac{\partial Rs}{\partial A} & \frac{\partial Rs}{\partial T}
  \end{bmatrix}
  \]

  \[
  \text{The Coupled Adjoint Equation}
  \]

  \[
  \text{The Gradients}
  \]

Where \( \psi a \) and \( \psi s \) are the aerodynamic and the structure Lagrange multipliers.
Test case description
Test Case Description

- Wing-Body configuration based on the DO728 geometry used as a test case.
- Reynolds number: 21e06
- Lift is kept constant by varying angle of attack (implicit lift constraint → requires gradient correction)

- CFD
- One-Equation turbulence model Spalart-Allmaras
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  - Parameterization: 30 parameters fixing the body
    150/2 FFD parameters on the wing (implicit thickness constraint)
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- Reynolds number : 21e06
- Lift is kept constant by varying angle of attack (implicit lift constraint \( \rightarrow \) requires gradient correction)
  \[
  \frac{d(C_D)@constant	ext{ Lift}}{dA} = \frac{dC_D}{dA} - \left( \frac{dC_D}{d\alpha} \times \frac{d\alpha}{dA} \right) \frac{dC_L}{dA}
  \]

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- Wing-Body configuration based on the DO728 geometry used as a test case.
- The Structure model:
  - 27 Ribs
  - 2 Spars
  - Lower & Upper Skin
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- Wing-Body configuration based on the DO728 geometry used as a test case.
- The Structure model:
  - 27 Ribs
  - 2 Spars
  - Lower & Upper Skin
- CSM
- The CSM mesh:
  - 4000 nodes.
  - Modeled using rectangular shell elements,
    each node has 6 DOFs.
Optimizations
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Three optimizations were performed
Optimizations

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- 1. Unconstrained Single-point optimization at the design point

  - Mach
    - 0.417
    - 0.500
    - 0.340

  - CL
    - 0.417
    - 0.500
Three optimizations were performed

1. Unconstrained Single-point optimization at the design point
2. Constrained Single-point optimization at the design point

<table>
<thead>
<tr>
<th>Mach</th>
<th>CL</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.417</td>
<td>0.340</td>
</tr>
<tr>
<td>0.500</td>
<td>0.80</td>
</tr>
<tr>
<td>0.82</td>
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</tr>
</tbody>
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Three optimizations were performed:

1. Unconstrained Single-point optimization at the design point
2. Constrained Single-point optimization at the design point
3. Constrained Multi-Point Optimization
Three optimizations were performed

1. Unconstrained Single-point optimization at the design point
2. Constrained Single-point optimization at the design point
3. Constrained Multi-Point Optimization

The objective is drag reduction at constant CL and wing thickness. (Aerodynamic objective with elastic effects taken into account)
**Unconstrained Single-point Optimization**

- The optimization employed a quasi-Newton gradient-based algorithm.
Unconstrained Single-point Optimization

- The optimization employed a quasi-Newton gradient-based algorithm.
- Optimization converged after 41 aero-structural couplings and 25 coupled adjoint computations.
- The optimization reduced the drag by 15 drag counts while keeping the lift and the thickness constant.
Unconstrained Single-point Optimization

- The optimization employed a quasi-Newton gradient-based algorithm.
- Optimization converged after 41 aero-structural couplings and 25 coupled adjoint computations.
- The optimization reduced the drag by 15 drag counts while keeping the lift and the thickness constant.
Unconstrained Single-point Optimization

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- Optimization converged after 41 aero-structural couplings and 25 coupled adjoint computations.
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Unconstrained Single-point Optimization

\[ \eta = 0.25 \]

\[ \eta = 0.45 \]

\[ \eta = 0.55 \]

\[ \eta = 0.75 \]

Baseline

Optimized
Unconstrained Single-point Optimization

- **Baseline**
- **Optimized**

**Chart 44**
Unconstrained Single-point Optimization

This (at constant lift) increases the bending moment at the Wing’s root

Keep CMx constant during the aero-elastic optimization
Constrained Single-point Optimization

- The optimization employed SQP gradient-based algorithm.
- Optimization converged after 31 aero-structural couplings and 19 coupled adjoint computations.
- The optimization reduced the drag by 13 drag counts while keeping the lift and the thickness constant.
Constrained Single-point Optimization

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Constrained Multi-point Optimization

- The points were equally weighted.

\[
\text{Cost Function} = \sum_{i=1}^{5} 0.2 \times C_{Di}
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- 25 design iterations
- 20 gradient computations
- 260 hrs on 48 processors
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Drag counts:
-5, -7, -22, -32
Constrained Multi-point Optimization

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Constrained Multi-point Optimization

![Graphs showing Constrained Multi-point Optimization at different Cl values: Cl=0.340, Cl=0.417, Cl=0.500, Mach=0.78, Mach=0.80, Mach=0.82.](image)
Constrained Multi-point Optimization
Future Work: Gradient Correction due to Trimming

- Trimming the flight with a horizontal tailplane (to reach a target pitching moment) will be considered during the Optimization.
- The gradients of our cost function need to be corrected if the flight is trimmed using horizontal tailplane. (similar to correcting gradients of drag when running for target lift).
- Use the Lagrange formulation to predict the correction term in the gradients.
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- The gradients of our cost function need to be corrected if the flight is trimmed using horizontal tailplane. (similar to correcting gradients of drag when running for target lift).
- Use the Lagrange formulation to predict the correction term in the gradients.
- If our cost function is drag then:

\[
\frac{dC_D}{dD}_{\text{trim}} = \frac{C_D}{dD} + \frac{\partial C_D}{\partial \alpha_W} [\frac{1}{\partial C_L/\partial \alpha_W} \left( - \frac{\partial C_L}{dD} - \frac{\partial C_L}{\partial \alpha_t} \delta \alpha_t \right) + \frac{\partial C_D}{\partial \alpha_t} \delta \alpha_t]
\]

\[
\delta \alpha_t = \left( \frac{\partial C_{my}}{\partial C_L/\partial \alpha_W} \frac{dC_L}{dD} - \frac{dC_{my}}{dD} \right) / \left( \frac{\partial C_{mv}}{\partial \alpha_t} - \frac{(\partial C_{my}/\partial \alpha_W)(\partial C_L/\partial \alpha_t)}{\partial C_L/\partial \alpha_W} \right)
\]

\(\alpha_W\): far-field angle of attack
\(\alpha_t\): tail's angle of incidence
Future Work: Gradient Correction due to Trimming

➢ Tested on 2D Euler case
Future Work: Gradient Correction due to Trimming

- Tested on 2D Euler case
Conclusions

- The coupled aero-structural adjoint approach was employed to efficiently obtain the gradients employed in single- and multi-point aero-elastic optimizations.

- The approach employs DLR’s TAU code to solve the flow equations and ANSYS Mechanical to solve the structure equations, and deals with inviscid as well as viscous turbulent flows.

- The coupled adjoint approach could save around 75% of computational time, which makes it now possible to perform aero-elastic optimizations using the gradient-based techniques, even for multi-point optimizations.

- A single- and multi-point optimizations with constrained rolling moment were performed and expected to have better effect on the structure (less weight).

- Future optimizations will include aerodynamic to structure cross sensitivities

- Future optimizations will include (horizontal tail) trimming effect
THANK YOU FOR YOUR ATTENTION