Adjoint-based sensitivity and feedback control of noise emission

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Active flow control strategies

- In addition to the passive flow control and the shape optimization, it is a necessity to control flows to enhance performances in transportation vehicles: drag, lift, noise emission, flow instabilities, separation, ...

- My challenge: to propose a methodology based on theoretical and numerical approaches for actuation law design dealing with large system (DoF $\geq 10^6$)

1. **Open loop control**:
   - optimization problem with full PDE (DNS, LES, ...)
   - expensive and time consuming
   - low robustness

2. **Feedback control**: more efficient, used in real flow and systems, robustness can be a parameter or an issue, first step towards adaptive control.
Feedback control: to manage the huge size of fluid flow configurations

Need a Reduced Order Model

1. ROM based on global stability modes: laminar flow $\rightarrow$ turbulent flow

2. ROM based on POD modes: laminar flow $\rightarrow$ moderate turbulent flow.
   Rowley (Balanced POD, 2012), Airiau & Cordier (2013), ...

3. POD analysis + heuristic feedback law based on physical considerations
   Pastoor et al (2008), ...

Usual feedback loop
(sensors $\rightarrow$ state estimate $\rightarrow$ actuation
Done with ROM

Present work: with DNS,
direct output feedback law
sensor outputs $\rightarrow$ actuation law
Introduction and testcase

General methodology: control of a ROM

- Application to any flow control as soon as a POD is relevant
- 8 steps: large developments and programming
- DNS: large DOF ($>10^6$)
- ROM/LINEAR CONTROL: very low DOF ($<10$)
- Computational cost: $2 \text{ DNS} + \text{ROM}$
- Many parameters, options and choices...
- what else?

Issues for an efficient feedback law:

- Optimal position and type of actuators (controlability)
- Optimal position and type of sensors (observability)
- Well capturing the response of the flow (computed by DNS) to any actuation: actuation mode(s)

POD: Proper Orthogonal Decomposition
1a - Testcase: 2D compressible cavity flow

- $M_\infty = 0.6$, $R_\theta = 688$, $Re_L = 2981$, noise control is a good (severe) testcase

- Self-sustained instability due to a feedback effects with the impingement of the shear layer on the downstream cavity corner
- Instantaneous pressure, acoustic wave directivity Rowley’s case
- $O_1$ and $O_3$: center point of the observation domain for sensitivity
- $O_2$: probes for spectral analysis (SPL)
Actuator position and type provided by the sensitivity analysis

1. Observed quantity (functional)

\[ J(q, f) = \int_{\Omega} \int_{0}^{T} j_{\text{observed}} d\Omega dt, \quad j_{\text{observed}} = \frac{1}{2}(p - \bar{p})^2 \]

\( q \): state vector, \( f \) forcing/actuation vector

2. Variational problem and Fréchet derivative: sensitivity \( S(x, t) \)

\[ \delta J = \langle \frac{\partial j_{\text{observed}}}{\partial q_k}, \delta q_k \rangle_\Omega = \langle S_{fi}, \delta f_i \rangle_\Omega \]

3. Adjoint Navier-Stokes equations: sensitivity is related to the adjoint state: \( S_{fi} = q_i^* \)

4. true for wall localized forcing and global volume forcing
0 - Actuators settings

- 2D Fourier modes, adjoint state (case 10) from adjoint DNS

  - Stationary mode sensitivity: steady actuation
  - 2nd Rossiter mode sensitivity: unsteady actuation

**Modes localized in space associated to the controllability property**

**Figure 3** - Modes at $St_1$ for $\hat{m}_x^*/U_\infty$ (blue: 0.48, red: 7.22) and $\hat{p}^*/U_\infty^2$ (blue: 0.23, red: 3.71)
Need to define actuation mode in the ROM from actuated DNS

Wall normal velocity forcing

- Distributed actuation centered at $x_f = 2.72X/D$

$$f_w(x, t) = \gamma(t) \exp[-r^2/\sigma^2], \quad r^2 = ||x - x_{for}||^2, \quad \sigma = 50\Delta y$$

- Large frequency bandwidth actuation $A_1(t)$:

$$\gamma(t) = A_1 \sin(2\pi St_1 t) \times \sin(2\pi St_2 t - A_2 \sin(2\pi St_3 t))$$

To excite and therefore later to control all possible physical unstable perturbations
Proper Orthogonal Decomposition of unactuated flow field $\rightarrow \phi_i^u(x)$

POD mesh size $<\text{DNS mesh} \Rightarrow$ gain in CPU time and accuracy

Optimal size?

$$q^a(x, t) = \bar{q}^a(x) + \sum_{i=1}^{N} a_i^a(t) \phi_i^u(x) + \gamma(t) \psi(x)$$ \hspace{1cm} (1)

- $(\phi_{i=1,N}, \psi)$ orthogonal basis, truncation to $N$ modes
- Assumption : $\bar{q}^a(x) \approx \bar{q}^u(x)$
- $q = (\zeta = 1/\rho, u, v, p)$
3 - Actuation mode, $A_1 = 0.001$

Sine forcing, $u$ velocity, Vigo’s and isentropic models

Distributed actuation along the wall and in the shear layer

Chirp, $u$ velocity, Vigo’s model

Actuation close to the noise source location
Direct feedback output control

5, 6a, 6b - ROM-Galerkin projection

- **Projection** of NSE (formulation with $\zeta = 1/\rho$, *Vigo-98, Bourguet-09*) on $(\phi_i^u, \phi_i^v, \phi_i^w)$. Nonlinear forced dynamical system of low order

$$\dot{a} = C + La + a^t Qa + \gamma \hat{L}a + \gamma \hat{C} + \gamma^2 \hat{Q}$$

(2)

- **Calibration** of ROM (find $C$ and $L$ for $a(t)_{POD} = a(t)_{ROM}$)

- **Equilibrium (steady) state** (many states can exist):

  Physical domain:
  $$q^e(x) = \bar{q}(x) + \sum_{i=1}^{N} a_i^e \phi_i(x)$$

  Equilibrium state of the NS eq.

- **Linearization** with $\tilde{a} = a - a^e$:

  $$\begin{align*}
  \dot{\tilde{a}} &= L\tilde{a} + \tilde{a}^t Qa + a^t Q\tilde{a} + (\hat{L}a + \hat{C})\gamma \\
  \tilde{a} &= \tilde{A}\tilde{a} + \tilde{B}\gamma \quad \text{State equation}
  \end{align*}$$

(3)
Required for the feedback control law design

- Unsteady pressure sensors \((\tilde{y}_i = \tilde{p}_i)\):

\[ \tilde{y} = \tilde{C}\tilde{a} + \tilde{D}\gamma, \quad \tilde{y} = y - \bar{y} - \tilde{C}a^e \quad (4) \]

- Sensor \(y_i\) is located on the POD mesh at \(x_k\):

\[ \tilde{C}_{ij} = \phi_j^u(x_k) \text{ and } \tilde{D}_i = \psi_k = \psi(x_k). \]

\(N_s = 6 \ (1 \rightarrow 6)\) sensors are used to build the actuation law.

Optimal positions: physical considerations, observability
direct output feedback control law design

- linear state space model: \( \dot{\tilde{a}} = \tilde{A}\tilde{a} + \tilde{B}\gamma \)
- feedback control law: \( \gamma = -K_c \tilde{a} \)
- minimization of \( J = \int_0^T (\tilde{a}^T\tilde{a} + \ell^2\gamma^2) \, dt \).
- Ricatti equation: \( K_c = \frac{1}{\ell^2} \tilde{B}^T X \cdot \left( \tilde{A}^T X + X\tilde{A} - \frac{1}{\ell^2} X\tilde{B}\tilde{B}^T X + I_d = 0 \right) \).
- outputs: \( \tilde{y} = \tilde{C}\tilde{a} + \tilde{D}\gamma \)
- direct feedback output control: \( \gamma(t) = \alpha(y(t) - \bar{y}) + \beta \) \hspace{1cm} (5)

- \( N_{POD} = N_{Sensors} \): \( \gamma(t) = -K_c(\tilde{C} - \tilde{D}K_c)^{-1}(y - \bar{y} - \tilde{C}a^e) \)
- \( \beta \) imposes the mean actuation velocity, \( \alpha \) imposes the damping of the time variation of the actuation
- implementation in DNS code
application: efficiency & robustness

- Efficient and robust feedback control law: decay of pressure fluctuation levels

\[ \gamma(t) = \alpha(y(t) - \bar{y}) + \beta, \]

- tuning \( \beta \) to improve the efficiency

A) \( \beta \approx 5 \text{m/s} \),

B) \( \beta \approx 10 \text{m/s} \),

C) \( \beta \approx 17 \text{m/s} \)

- Robustness, feedback law
  - time window: \( \text{it}=25000 \)
  - spatial window: red box
  - near and far field noise damping
  - subharmonic and harmonics: weakly nonlinear effects
Noise reduction

Fig. 1: $SPL_{ac} - SPL_{unac}$

Fig. 2: mean pressure

Fig. 3: SPL levels

Case C: maximum of -10 dB
- Global noise reduction
- Few areas with increase, but SPL remains low
- Wavy SPL contours: weakly nonlinear effect
- Actuation modifies the mean pressure: nonlinearity
Actuation drastically modifies the flow dynamic: \( \mathbf{q}(x, t) = \bar{\mathbf{q}}(x) + \sum_{i=1}^{N} a_i(t) \phi_i(x) \)

- Relative Information Content (RIC): more relevant modes are required
- Eigen values distribution: new spectra
- Time coefficients \( a_i(t) \rightarrow \text{constante} \) for \( t \rightarrow \infty \), time damping
POD time coefficients

- Phase portrait: convergence towards a steady (equilibrium) state?

**POD decompositions:**

1) with unsteady actuation

\[
q^a(x, t) = \tilde{q}^a(x) + \sum_{i=1}^{N} a_i^a(t) \phi_i^u(x) + \gamma(t) \psi(x)
\]

2) with stabilization \((t \to \infty)\):

\[
q^\infty(x) = \tilde{q}^{ac}(x) + \sum_{i=1}^{N} a_i^\infty \phi_i^{ac}(x)
\]

Next step: determine the final state \(q^\infty(x)\)
Summary and perspectives

• Some concluding remarks
  1. Feedback output control law implemented in DNS (2 DNS + R0M → low cost)
  2. Efficient and quite robust (when time → ∞)
  3. Several dB of noise reduction (up to -10 dB)
  4. Possible to tune the feedback law to improve efficiency, towards nonlinearity
  5. Independent on the DNS code: → LES?

• Current works
  1. Linear Quadratic Gaussian control with ROM with state estimate
  2. Feedback law: \( \gamma(t) = \int_{t-t_c}^{t} \sum_{i=1,N_s} G_i(t-\tau)y_i(\tau) \, d\tau \)

• Some improvements and perspectives
  1. Sensitivity analysis to many parameters included in the approach: \( N_{POD}, N_s \), actuation location and type, option of the ROM or POD, ...
  2. To test other flow decomposition to better take into account of actuation (with \( \dot{\gamma} \))
  3. To increase physical parameters (Re, Ma) and to test other flows
  4. Nonlinear feedback analysis and robustness analysis (\( H_2, H_\infty \))
  5. Validation/comparisons with experiments on low Reynolds number reference flows
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