Adjoint-based sensitivity and feedback control of noise emission

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Plan

Active flow control strategies

- In addition to the passive flow control and the shape optimization, it is a necessity to control flows to enhance performances in transportation vehicles : drag, lift, noise emission, flow instabilities, separation, ...
- My challenge : to propose a methodology based on theoretical and numerical approaches for actuation law design dealing with large system ($DoF \geq 10^6$)

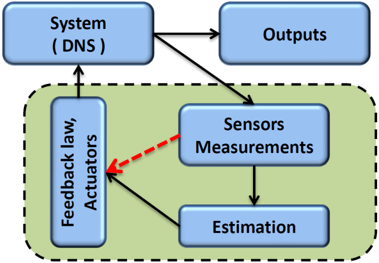
1. Open loop control :
    - optimization problem with full PDE (DNS, LES, ...)
    - expensive and time consuming
    - low robustness
    - boundary layers, shear layer, jet flows Airiau et al (2003), Wei & Freund (2006), Spagnoli & Airiau (2008), Sesterhenn & al (2012)
2. Feedback control : more efficient, used in real flow and systems, robustness can be a parameter or an issue, first step towards adaptive control.

Direct feedback ouptut control

Feedback control : to manage the huge size of fluid flow configurations

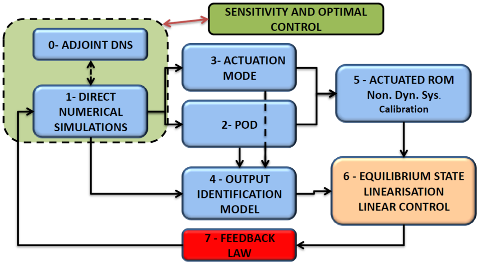
☞ Need a Reduced Order Model

1. ROM based on global stability modes : laminar flow → turbulent flow
   Bargagallo, Sipp et al (2012), Raymond et al (2013) , Rowley, ... , DoF $\approx 10^5$
2. ROM based on POD modes : laminar flow → moderate turbulent flow. Rowley (Balanced POD, 2012), Airiau &Cordier (2013), ...
3. POD analysis + heuristic feedback law based on physical considerations Pastoor et al (2008), ...



- Usual feedback loop
  (sensors → state estimate → actuation
  Done with ROM
- Present work : with DNS,
  direct output feedback law
  sensor outputs → actuation law

General methodology : control of a ROM



POD : Proper Orthogonal Decomposition

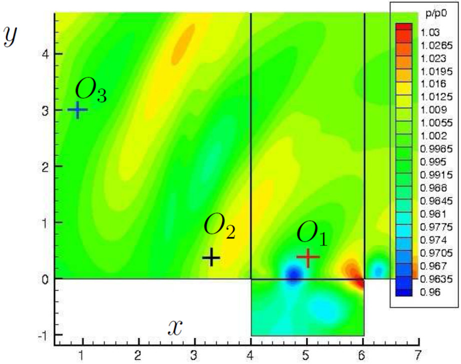
- Application to any flow control as soon as a POD is relevant
- 8 steps : large developments and programming
- DNS : large DOF ($> 10^6$)
- ROM/LINEAR CONTROL : very low DOF (< 10)
- Computational cost : 2 DNS + ROM
- Many parameters, options and choices ...
- what else ?

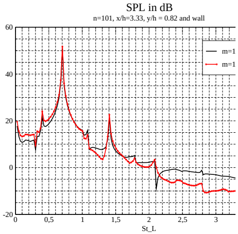
☞ Issues for an efficient feedback law:

- Optimal position and type of actuators (controlability)
- Optimal position and type of sensors (observability)
- Well capturing the response of the flow (computed by DNS) to any actuation : actuation mode(s)

1a - Testcase : 2D compressible cavity flow

☞ $M_\infty = 0.6$, $R_\theta = 688$, $Re_L = 2981$, noise control is a good (severe) testcase





Tescase L/H =2

(Rowley, JFM 2002)

$$St_L = \frac{n - 0.25}{M_\infty + 1.754}$$

$St_L = 0.703$

Rossiter 2 : $St_L = 0.74$

75 000 Δt

- Self-sustained instability due to a feedback effects with the impingement of the shear layer on the downstream cavity corner
- Instantaneous pressure, acoustic wave directivity Rowley's case
- O_1 and O_3 : center point of the observation domain for sensitivity
- O_2 : probes for spectral analysis (SPL)

0 - Actuators : sensitivity analysis

☞ Actuator position and type provided by the sensitivity analysis

1. Observed quantity (functional)

$$J(\mathbf{q},\mathbf{f}) = \int_\Omega \int_0^T j_{observated} \, d\Omega dt, \qquad j_{observated} = \frac{1}{2}(p - \bar{p})^2$$

   $\mathbf{q}$: state vector, $\mathbf{f}$ forcing/actuation vector

2. Variational problem and Fréchet derivative : sensitivity $S(\mathbf{x}, t)$

$$\delta J = \langle \frac{\partial j_{observated}}{\partial q_k}, \delta q_k \rangle_\Omega = \langle S_{f_i}, \delta f_i \rangle_\Omega$$

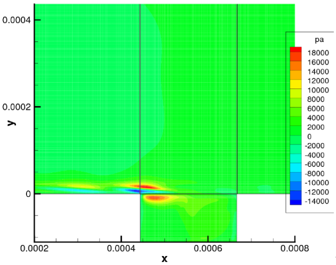
3. Adjoint Navier-Stokes equations : sensitivity is related to the adjoint state : $S_{f_i} = q_i^*$

4. true for wall localized forcing and global volume forcing

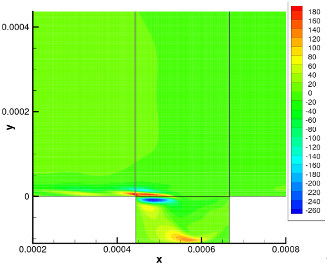
0 - Actuators settings

☞ 2D Fourier modes, adjoint state (case 10) from adjoint DNS (Continuous)

- stationnary mode sensitivity : steady actuation



$\leftarrow \hat{p}^*$



$\leftarrow \hat{m}_x^*$

- 2nd Rossiter mode sensitivity : unsteady actuation

x-momentum forcing mass forcing

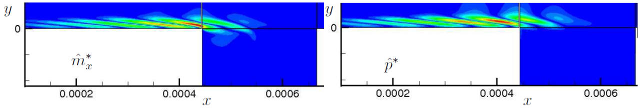


FIGURE 3 – Modes at St_1 for $\hat{m}_x^*/U_\infty$ (blue : 0.48, red : 7.22) and $\hat{p}^*/U_\infty^2$ (blue : 0.23, red : 3.71)

Modes localized in space associated to the controllability property

1b - DNS response to a generic actuation

☞ Need to define actuation mode in the ROM from actuated DNS

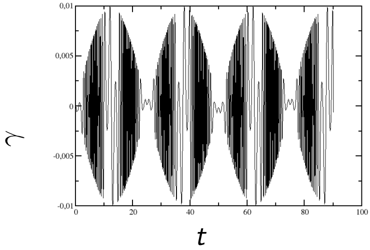
Wall normal velocity forcing

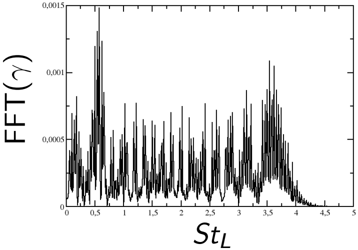
- Distributed actuation centered at $x_f = 2.72X/D$

$$f_w(x,t) = \gamma(t)\exp[-r^2/\sigma^2], \qquad r^2 = ||\mathbf{x} - \mathbf{x}_{for}||^2, \sigma = 50\Delta_y$$

- Large frequency bandwidth actuation $A_1(t)$:

$$\gamma(t) \quad = \quad A_1 \sin(2\pi St_1 t) \times \sin\left(2\pi St_2 t - A_2 \sin(2\pi St_3 t)\right)$$



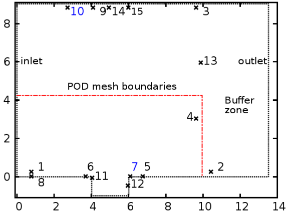


$St_{cavity} \approx 0.7$

☞ To excite and therefore later to control all possible physical unstable perturbations

3 -From DNS to POD & ROM - actuation mode

☞ Proper Orthogonal Decomposition of unactuated flow field → $\phi_i^u(\mathbf{x})$



POD mesh size < DNS mesh =>
gain in CPU time and accuracy
Optimal size ?

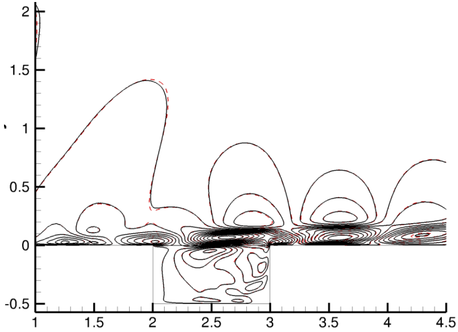
$$\mathbf{q}^a(\mathbf{x},t) = \bar{\mathbf{q}}^a(\mathbf{x}) + \sum_{i=1}^{N} a_i^a(t)\phi_i^u(\mathbf{x}) + \gamma(t)\psi(\mathbf{x}) \quad (1)$$

- $(\phi_{i=1,N}^u, \psi)$ orthogonal basis, truncation to N modes
- Assumption : $\bar{\mathbf{q}}^a(\mathbf{x}) \approx \bar{\mathbf{q}}^u(\mathbf{x})$
- $\mathbf{q} = (\zeta = 1/\rho, u, v, p)$

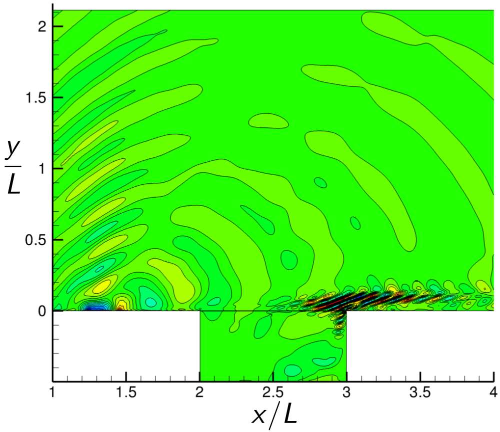
3 - Actuation mode, $A_1 = 0.001$

Chirp, ,u velocity, Vigo's model

Sine forcing, u velocity, Vigo's and isentropic models



Distributed actuation along the wall and in the shear layer



Actuation close to the noise source location

5, 6a, 6b - ROM-Galerkin projection

- Projection of NSE (formulation with $\zeta = 1/\rho$, *Vigo-98, Bourguet-09*) on $(\phi^u_{i=1,N})$: Nonlinear forced dynamical system of low order

$$\dot{a} = C + La + a^t Qa + \gamma \hat{L}a + \gamma \hat{C} + \gamma^2 \hat{Q} \quad (2)$$

- Calibration of ROM (find C and L for $a(t)_{POD} = a(t)_{ROM}$)
- Equilibrium (steady) state (many states can exist) :

$$\text{Physical domain:} \quad \mathbf{q}^e(\mathbf{x}) = \bar{\mathbf{q}}(\mathbf{x}) + \sum_{i=1}^{N} a_i^e \phi_i(\mathbf{x})$$

Equilibrium state of the NS eq.

- Linearization with $\tilde{a} = a - a^e$:

$$\begin{aligned} \dot{\tilde{a}} &= L\tilde{a} + \tilde{a}^t Qa + a^t Q\tilde{a} + (\hat{L}a + \hat{C})\gamma \\ \dot{\tilde{a}} &= \tilde{A}\tilde{a} + \tilde{B}\gamma \qquad \text{State equation} \end{aligned} \quad (3)$$

4 - Use of sensors : output identification model

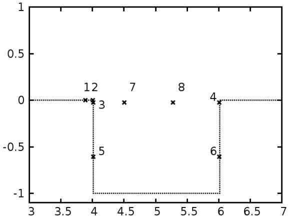
☞ Required for the feedback control law design

- Unsteady pressure sensors ($\tilde{y}_i = \tilde{p}_i$):

$$\tilde{y} = \tilde{C}\tilde{a} + \tilde{D}\gamma, \qquad \tilde{y} = y - \bar{y} - \tilde{C}a^e \tag{4}$$

- sensor y_i is located on the POD mesh at $\mathbf{x}_k$:

$$\tilde{C}_{ij} = \phi_j^u(\mathbf{x}_k) \text{ and } \tilde{D}_i = \psi_k = \boldsymbol{\psi}(\mathbf{x}_k).$$



$N_s = 6$ $(1 \rightarrow 6)$ sensors are used to build the actuation law.
Optimal positions : physical considerations, observability

6c - ROM-Linear Quadratic Regulator control

☞ direct ouput feedback control law design

- linear state space model : $\dot{\tilde{a}} = \tilde{A}\tilde{a} + \tilde{B}\gamma$
- feedback control law : $\gamma = -K_c\ \tilde{a}$
- minimization of $\mathcal{J} = \int_0^T \left(\tilde{a}^t\tilde{a} + \ell^2\gamma^2\right)\ dt.$
- Ricatti equation : $K_c = \frac{1}{\ell^2}\tilde{B}^t X$. $\left(\ \tilde{A}^T X + X\tilde{A} - \frac{1}{\ell^2}X\tilde{B}\tilde{B}^T X + Id = 0\ \right).$
- outputs : $\tilde{y} = \tilde{C}\tilde{a} + \tilde{D}\gamma$
- direct feedback output control :

$$\gamma(t) = \alpha(y(t) - \bar{y}) + \beta \qquad (5)$$

- $N_{POD} = N_{Sensors}$: $\gamma(t) = -K_c(\tilde{C} - \tilde{D}K_c)^{-1}(y - \bar{y} - \tilde{C}a^e)$
- β imposes the mean actuation velocity, α imposes the damping of the time variation of the actuation
- implementation in DNS code

application: efficiency & robustness

- Efficient and robust feedback control law : decay of pressure fluctuation levels

Fig. 1 : control law

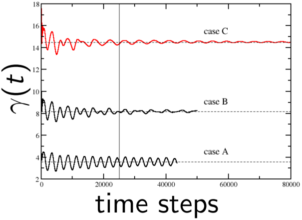


Fig. 2 : probes values

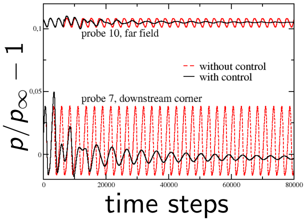


Fig. 3 : spectra

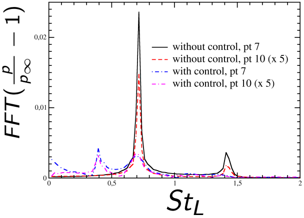
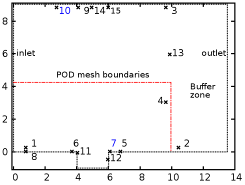


Fig. 4 : Probes location



☞ $\gamma(t) = \alpha(\mathrm{y}(t) - \bar{\mathrm{y}}) + \beta$,

tuning β to improve the efficiency

☞ A) $\beta \approx 5m/s$,

B) $\beta \approx 10m/s$,

C) $\beta \approx 17m/s$

☞ Robustness, feedback law

time window : it=25000

spatial window : red box

☞ near and far field noise damping

☞ subharmonic and harmonics :

weakly nonlinear effects

Noise reduction

Fig. 1 : $SPL_{ac.} - SPL_{unac.}$

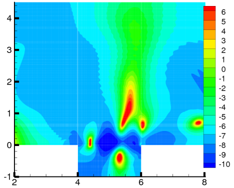


Fig. 2 : mean pressure

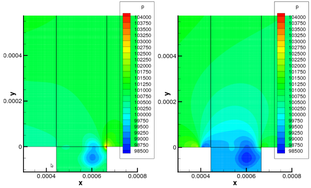
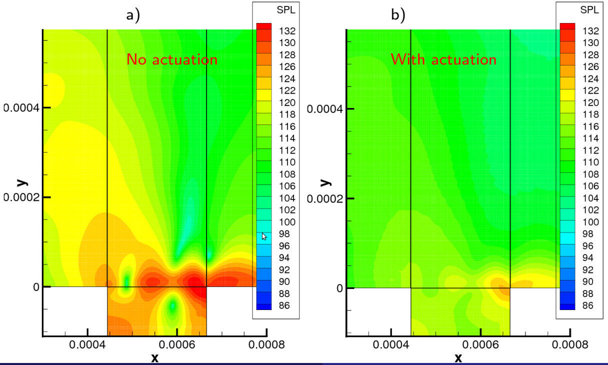


Fig. 3 : SPL levels



- Case C : maximun of -10 dB
- Global noise reduction
- Few areas with increase, but SPL remains low
- wavy SPL contours: weakly nonlinear effect
- Actuation modifies the mean pressure : nonlinearity

Deeper analysis : RIC content & POD eigen values

- Actuation drastically modifies the flow dynamic: $\mathbf{q}(\mathbf{x},t) = \bar{\mathbf{q}}(\mathbf{x}) + \sum_{i=1}^{N} a_i(t)\phi_i(\mathbf{x})$

Fig. 1 : POD eigenvalues

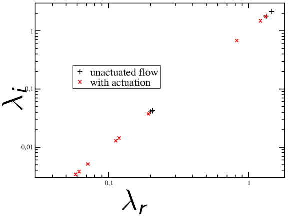


Fig. 2 : mode relevance

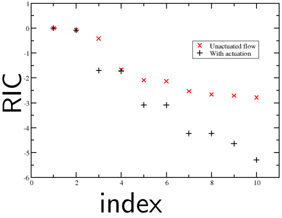


Fig. 3 : $a_i(t)$, no actuation

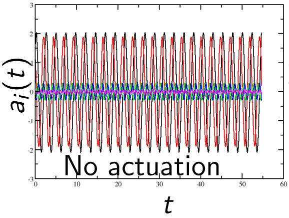
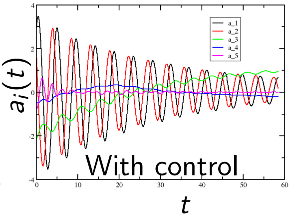


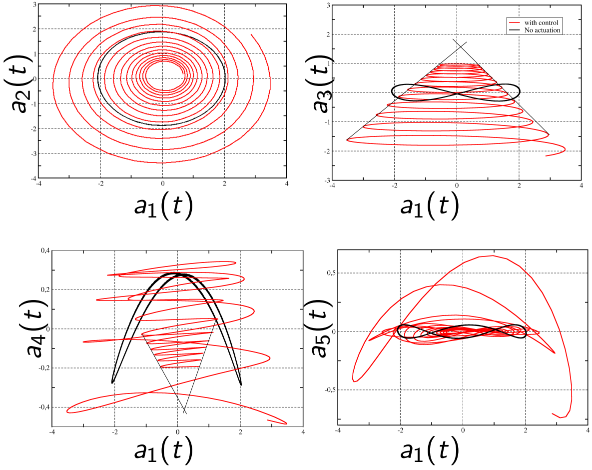
Fig. 4 : $a_i(t)$, with control



- ☞ Relative Information Content (RIC) : more relevant modes are required
- ☞ Eigen values distribution : new spectra
- ☞ time coefficients $a_i(t) \to$ constante for $t \to \infty$, time damping

POD time coefficients

- Phase portrait : convergence towards a steady (equilibrium) state ?



☞ POD decompositions :

☞ 1) with unsteady actuation

$$\mathbf{q}^a(\mathbf{x},t) = \bar{\mathbf{q}}^a(\mathbf{x}) + \sum_{i=1}^{N} a_i^a(t)\phi_i^u(\mathbf{x}) + \gamma(t)\psi(\mathbf{x})$$

☞ 2) with stabilization ($t \to \infty$) :

$$\mathbf{q}^\infty(\mathbf{x}) = \bar{\mathbf{q}}^{ac}(\mathbf{x}) + \sum_{i=1}^{N} a_i^\infty \phi_i^{ac}(\mathbf{x})$$

☞ Next step : determine the final state $\mathbf{q}^\infty(\mathbf{x})$

Summary and perspectives

- Some concluding remarks
  1. Feedback output control law implemented in DNS (2 DNS + R0M $\rightarrow$ low cost)
  2. Efficient and quite robust (when time $\rightarrow \infty$)
  3. Several dB of noise reduction (up to -10 dB)
  4. Possible to tune the feedback law to improve efficiency, towards nonlinearity
  5. Independent on the DNS code : $\rightarrow$ LES ?
- Current works
  1. Linear Quadratic Gaussian control with ROM with state estimate
  2. Feedback law : $\gamma(t) = \int_{t-t_c}^{t} \sum_{i=1,N_s} G_i(t-\tau) y_i(\tau)\, d\tau$
- Some improvements and perspectives
  1. Sensitivity analysis to many parameters included in the approach : N_{POD}, N_s, actuation location and type, option of the ROM or POD, ...
  2. To test other flow decomposition to better take into account of actuation (with $\dot{\gamma}$)
  3. To increase physical parameters (Re, Ma) and to test other flows
  4. Nonlinear feedback analysis and robustness analysis (H_2, H_∞)
  5. Validation/comparisons with experiments on low Reynolds number reference flows

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