NATIONAL TECHNICAL UNIVERSITY OF ATHENS
Parallel CFD & Optimization Unit
Laboratory of Thermal Turbomachines

The Continuous Adjoint Method for the Computation of First- and Higher-Order Sensitivities

(Activities, Recent Findings, Tips & Suggestions)

MUSAF-II
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Kyriakos C. GIANNAKOGLOU, Professor NTUA
kgianna@central.ntua.gr
http://velos0.ltt.mech.ntua.gr/research/

Introduction

Research Activities:

(A) CFD (running on GPUs)

(B) Evolutionary Algorithms ("plus")

(C) Adjoint Methods

Acknowledgements:

The "Adjoint" Group of the PCOpt/LTT/NTUA

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Dr. Varvara Asouti

Ioannis Kavvadias (PhD Student)

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(A) European Industries (turbomachinery, automotive, etc)

(B) EU-funded Projects

(C) CFD Software Developers

Activities related to the Adjoint Methods

- Development of both <u>continuous</u> and discrete adjoint methods.
- For <u>compressible</u> fluids (in-house, primitive variable solver, GPU-enabled).
- For <u>incompressible</u> fluids (OpenFOAM or in-house code).
- For <u>steady</u> & <u>unsteady</u> flows (check-pointing, storage of approximates).
- Internal (turbomachinery) & external aerodynamics (wings, cars).
- Development of Adjoint Methods for:
  - Shape Optimization,
  - Optimization of Flow Control systems,
  - Robust-design Optimization,
  - Topology Optimization.

Selected Topics for MUSAF II - Outline

- Continuous adjoint method for widely-used turbulence models, including wall functions. Recent findings.
- Computation of high-order sensitivities, using both continuous & discrete adjoint, for:
  - Exact Newton methods,
  - Truncated Newton methods,
  - Robust Design-Optimization methods.
- Continuous Adjoint for Flow-Control. Steady & unsteady problems.
- Various (Topology Optimization), On-going Research

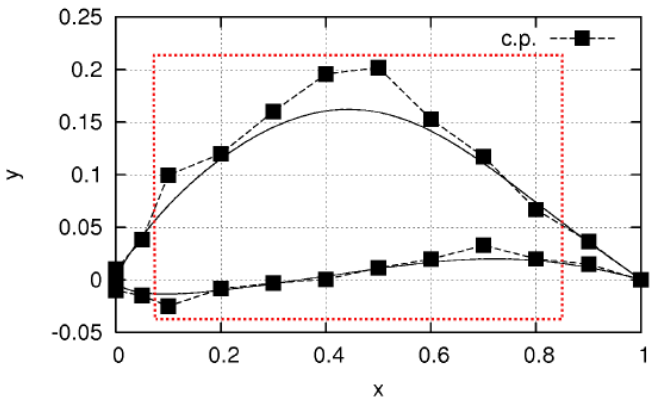
Starting Point: A reliable adjoint for Laminar Flows

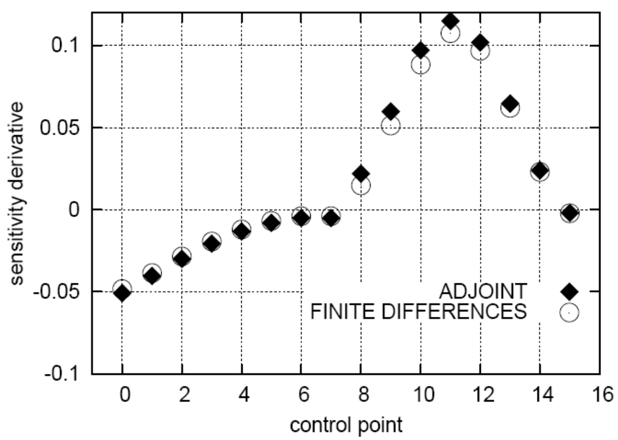
Computation of Sensitivity Derivatives on the starting airfoil

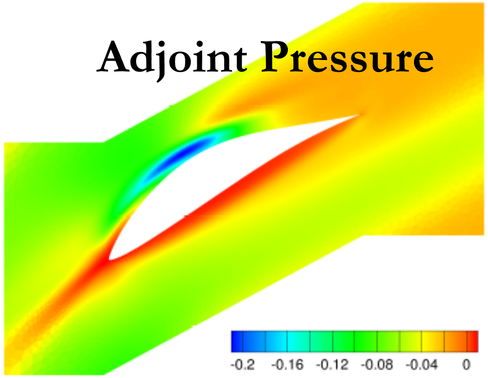
Laminar Subsonic Flow in a 2D Compressor Cascade, Fixed stagger angle & solidity.

Without running the Optimization Loop

$$F = \int_{S_{in}} \rho V_n p_t dS - \int_{S_{out}} \rho V_n p_t dS$$







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The commonly used approach - The "frozen turbulence assumption"

Demonstrated for incompressible flows, exists & runs also for compressible flows

- **State Equations**

$$R^p = \frac{\partial v_j}{\partial x_j} = 0$$

$$R_i^v = v_j \frac{\partial v_i}{\partial x_j} + \frac{\partial p}{\partial x_i} - \frac{\partial}{\partial x_j}\left[(\nu + \nu_t)\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)\right] = 0$$

(plus the turbulence model eqs.)

- **Development of the Adjoint Equations & Boundary Conditions**

For any objective function F:

$$F_{aug} = F + \int_\Omega u_i R_i^v d\Omega + \int_\Omega q R^p d\Omega$$

Differentiate F_{aug} w.r.t. to b_m, where b_m are the N design variables…

- **Adjoint Equations**

$$R^q = \frac{\partial u_j}{\partial x_j} = 0$$

$$R_i^u = -v_j \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - (\nu + \nu_t)\frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \frac{\partial q}{\partial x_i} = 0$$

No adjoint equation to the turbulence model!

Adjoint to the Spalart-Allmaras (SA) Turbulence Model

Exact Differentiation of the Turbulence Model Eqs.

Demonstrated for incompressible flows, exists & runs also for compressible flows

Demonstrated for the Spalart-Allmaras model. Exists for k- **ε** *& k-* **ω** *SST.*

$$R^p = \frac{\partial v_j}{\partial x_j} = 0$$

$$R_i^v = v_j \frac{\partial v_i}{\partial x_j} + \frac{\partial p}{\partial x_i} - \frac{\partial}{\partial x_j}\left[(\nu + \nu_t)\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)\right] = 0 \qquad \nu_t = \widetilde{\nu} f_{v_1}$$

$$R^{\widetilde{\nu}} = \frac{\partial (v_j \widetilde{\nu})}{\partial x_j} - \frac{1}{\sigma}\frac{\partial}{\partial x_j}\left[(\nu + \widetilde{\nu})\frac{\partial \widetilde{\nu}}{\partial x_j}\right] - \frac{c_{b2}}{\sigma}\left(\frac{\partial \widetilde{\nu}}{\partial x_j}\right)^2 - \widetilde{\nu} P(\widetilde{\nu}) + \widetilde{\nu} D(\widetilde{\nu}) = 0$$

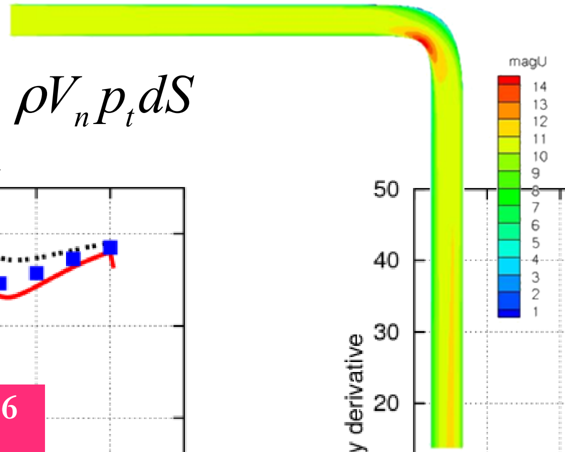
$$F_{aug} = F + \int_\Omega u_i R_i^v d\Omega + \int_\Omega q R^p d\Omega + \int_\Omega \widetilde{\nu_a} R^{\widetilde{\nu}} d\Omega$$

p	pressure	q	Adjoint pressure
v_i	velocities	u_i	Adjoint velocities
$\widetilde{\nu}$	turbulence variable	$\widetilde{\nu_a}$	Adjoint turbulence variable

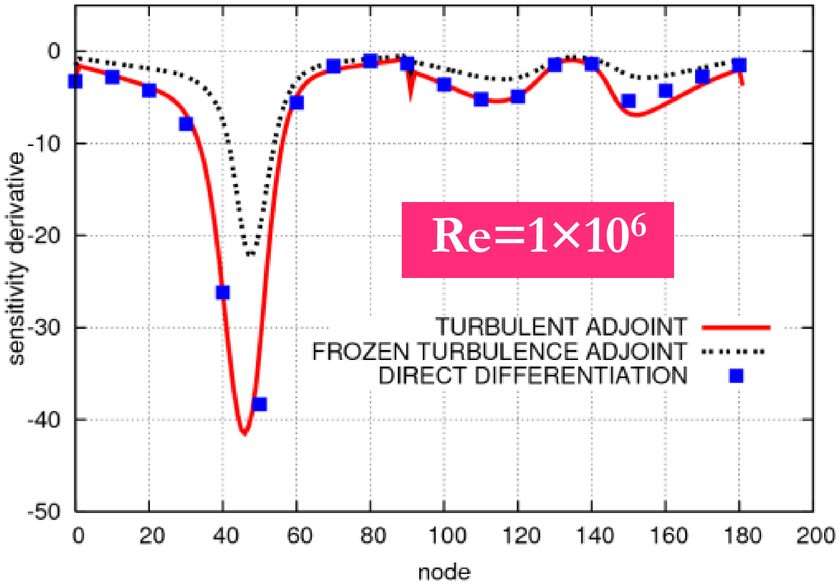
A.S. ZYMARIS, D.I. PAPADIMITRIOU, K.C. GIANNAKOGLOU, C. OTHMER: 'Continuous Adjoint Approach to the Spalart-Allmaras Turbulence Model for Incompressible Flows', Computers & Fluids, 38, pp. 1528-1538, 2009.

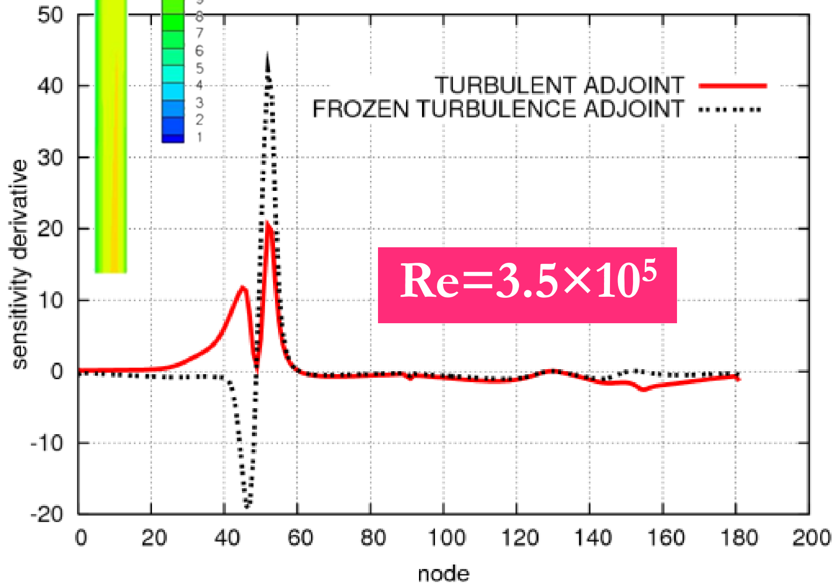
How Important is to Differentiate the Turbulence Model Eqs.?

The computationally expensive Direct Differentiation (DD) method
is used to compute reference sensitivities (to compare with).



$$F = \int_{S_{in}} \rho V_n p_t dS - \int_{S_{out}} \rho V_n p_t dS$$

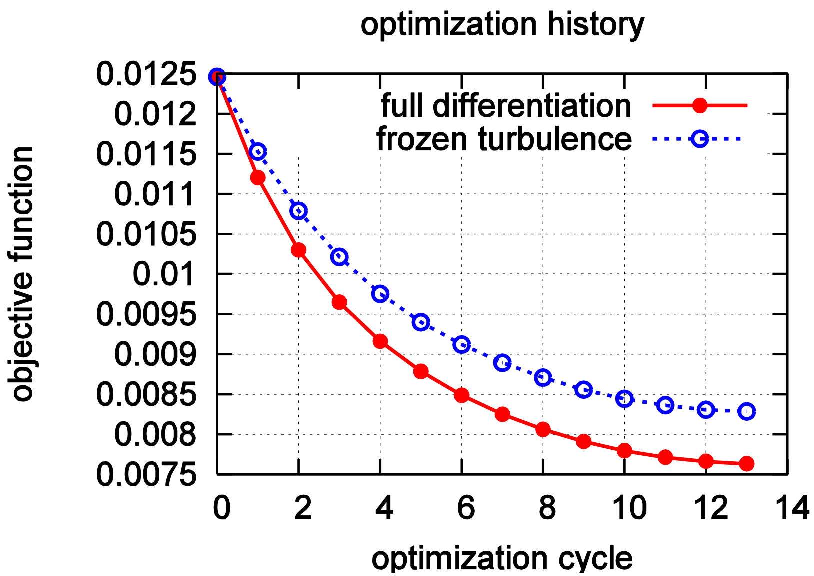




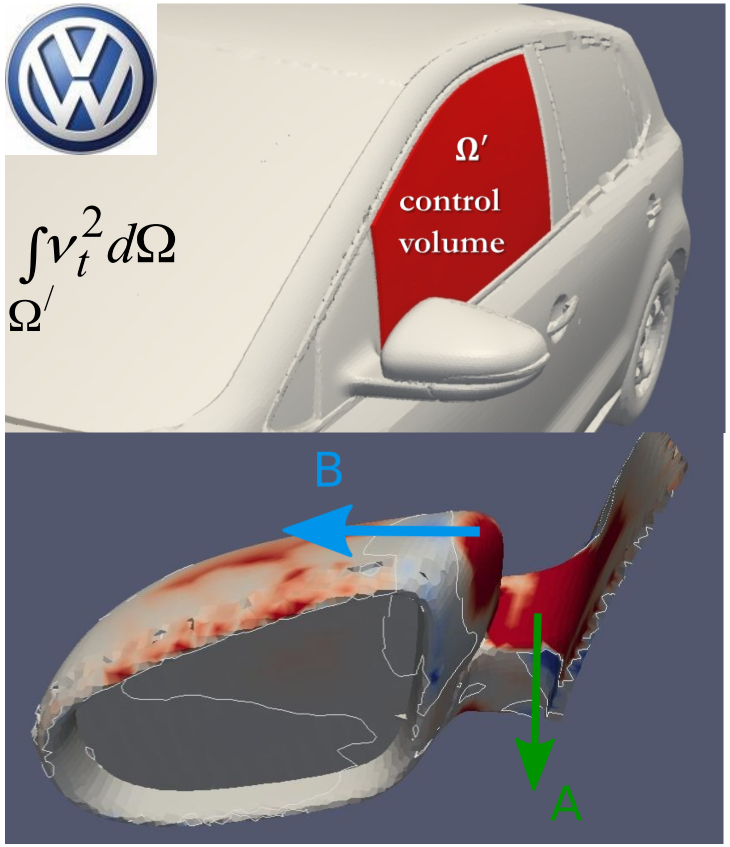
<u>CONCLUSION:</u> Depending on the case & the Reynolds number, the "frozen turbulence assumption" may lead to wrongly signed sensitivity derivatives!

Two Good Reasons for Differentiating the TM Eqs.

$$F = \int_{S_{in}} \rho V_n p_t dS - \int_{S_{out}} \rho V_n p_t dS$$



$$F = \int_{\Omega'} v_t^2 d\Omega$$



RED : inwards displacement

BLUE : outwards displacement

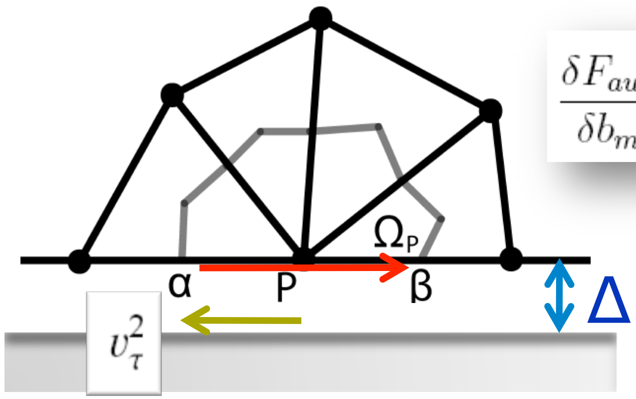
CONCLUSIONS: (a) Some problems are solved faster if the exact gradient is used.
(b) Some problems cannot be solved without differentiating the turbulence model!

Adjoint Wall Functions (k- ε Model)

Differentiation of High-Re Turbulence Models

A New Adjoint Law of the Wall

Demonstrated for the k- ε model. Exists for Spalart-Allmaras & k- ω



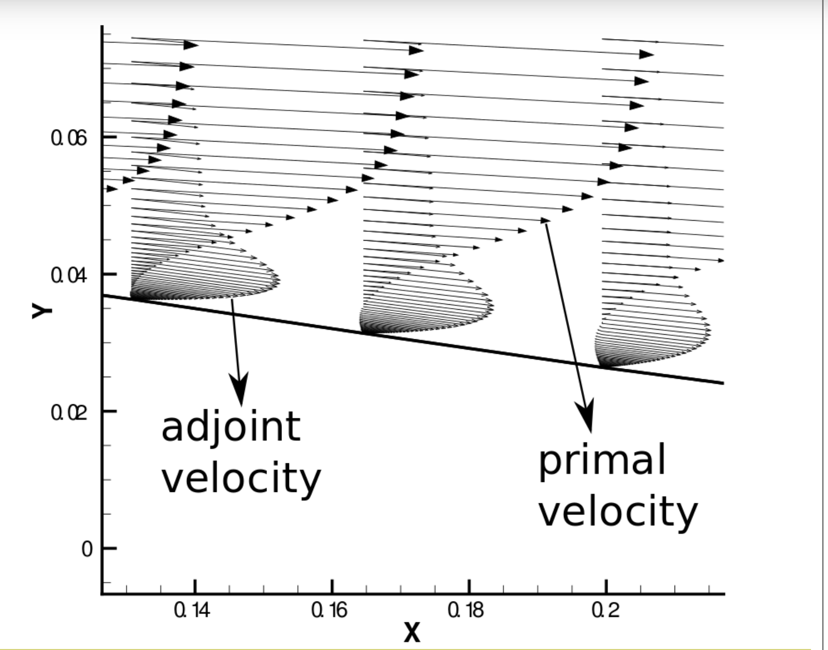
$$\frac{\delta F_{aug}}{\delta b_m} = \frac{\delta F}{\delta b_m} + \int_\Omega u_i \frac{\delta R_i^v}{\delta b_m} d\Omega + \int_\Omega q \frac{\delta R^p}{\delta b_m} d\Omega + \int_\Omega k_a \frac{\delta R^k}{\delta b_m} d\Omega + \int_\Omega \varepsilon_a \frac{\delta R^\varepsilon}{\delta b_m} d\Omega$$

Friction velocity

$$v_\tau^2 = (\nu + \nu_t)\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) n_j t_i$$

Adjoint friction velocity

$$u_\tau^2 = (\nu + \nu_t)\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) n_j t_i$$



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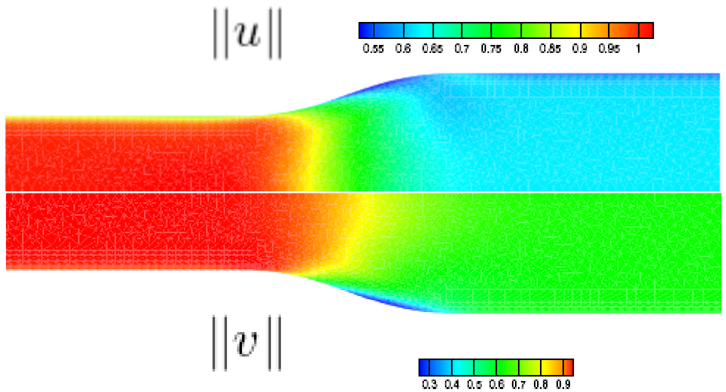
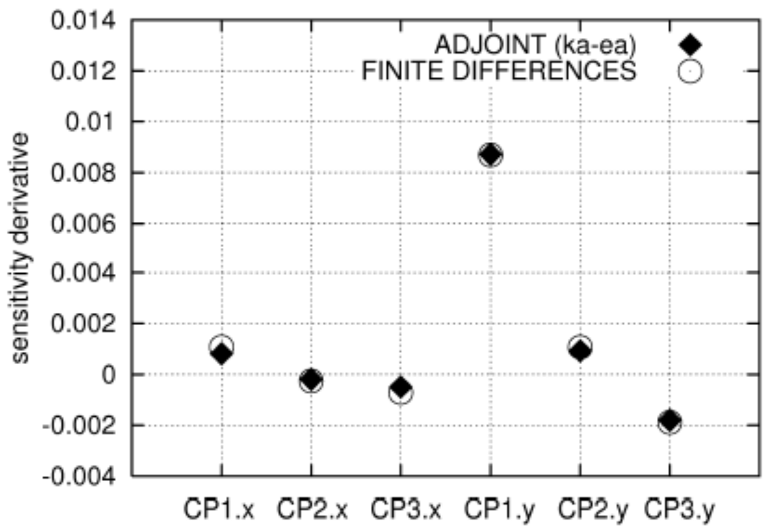
Computation of Sensitivity Derivatives on the starting geometry

Subsonic Flow in an axial diffuser, with incipient separation, Re=1x10^6

Objective function: mass-averaged total pressure losses

Without running the Optimization Loop



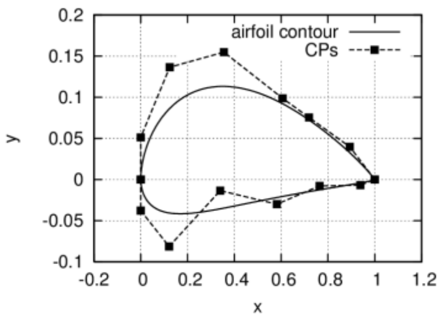


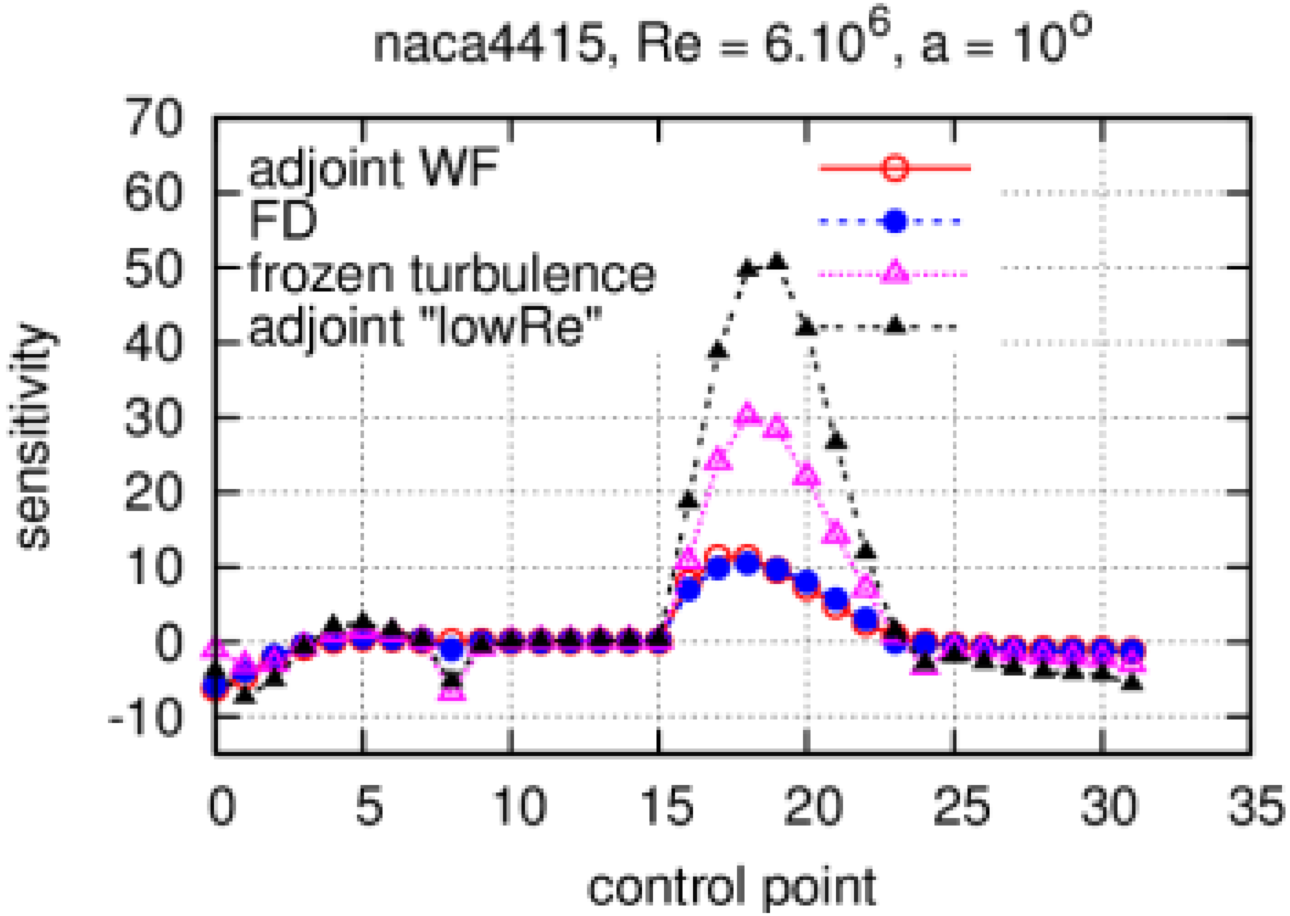
CONCLUSION: Turbulence models based on the wall function technique may/should be differentiated!

Adjoint Wall Functions (Spalart-Allmaras)

Computation of Sensitivity Derivatives on the starting geometry

Subsonic Flow around NACA4415





<u>CONCLUSION:</u> Primal model with Wall Functions? Using the adjoint "low-Re" model yields worst results than the "Frozen Turbulence Assumption"!!!

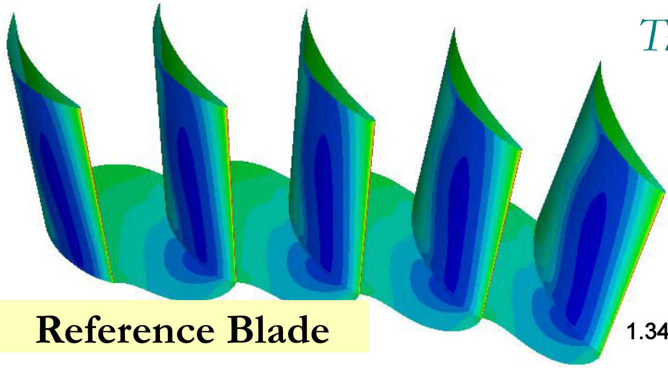
Design-Optimization of two Peripheral Compressor Cascades

Target: Minimum Viscous Losses

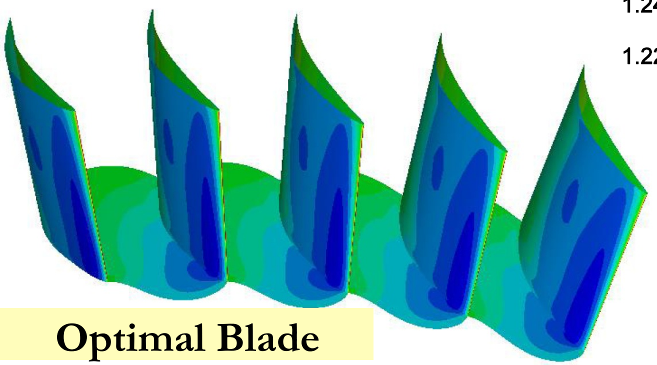
Constraints on the Flow Turning & the Blade Thickness

Turbulence Model: Spalart-Allmaras

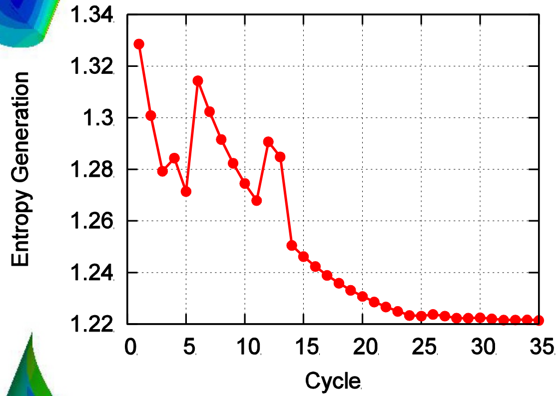
Row 1



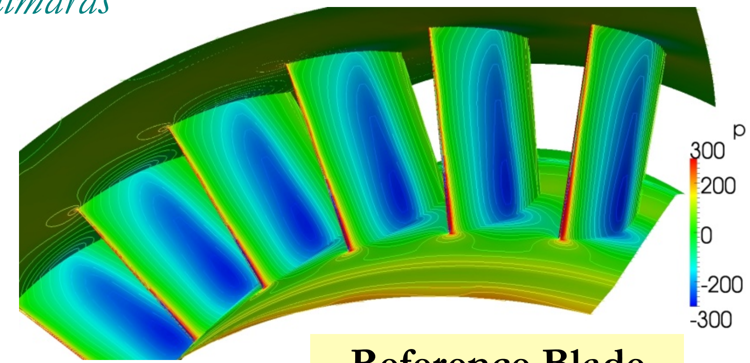
Reference Blade



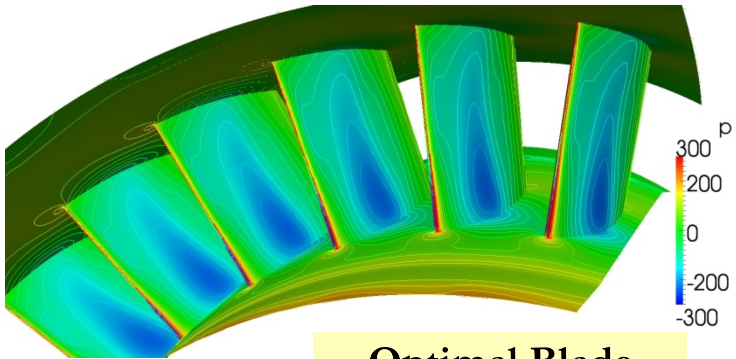
Optimal Blade



Row 2



Reference Blade



Optimal Blade

Differentiation of Distance Δ (in Turbulence Models)

Applied for Turbulence Models involving the Distance from the Wall

Including Wall Functions

Inspired by the AIAA J. paper, March 2012 by Bueno-Orovio, et al.

Differentiate the Hamilton-Jacobi eq., governing the distance Δ

$$\frac{\delta F_{aug}}{\delta b_n} = -\int_{S_{W_p}} \left[(\nu+\nu_t) \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) n_j - q n_i \right] \frac{\partial v_i}{\partial x_m} n_m n_k \frac{\delta x_k}{\delta b_n} dS + \boxed{\int_\Omega \tilde{\nu}\tilde{\nu}_a \mathcal{C}_\Delta \frac{\partial \Delta}{\partial b_n} d\Omega}$$

New State Eq.: $$R^\Delta = \frac{\partial (c_j \Delta)}{\partial x_j} - \Delta \frac{\partial^2 \Delta}{\partial x_j^2} - 1 = 0 \quad , \quad c_j = \partial \Delta / \partial x_j$$

$$F_{aug} = F + \int_\Omega u_i R_i^v d\Omega + \int_\Omega q R^p d\Omega + \int_\Omega \tilde{\nu}_a R^{\tilde{\nu}} d\Omega + \boxed{\int_\Omega \Delta_a R^\Delta d\Omega}$$

New Adjoint Eq. (decoupled): $$R^{\Delta_a} = -2 \frac{\partial}{\partial x_j} \left(\Delta_a \frac{\partial \Delta}{\partial x_j} \right) + \tilde{\nu}\tilde{\nu}_a \mathcal{C}_\Delta = 0$$

New Sensitivity Derivatives:

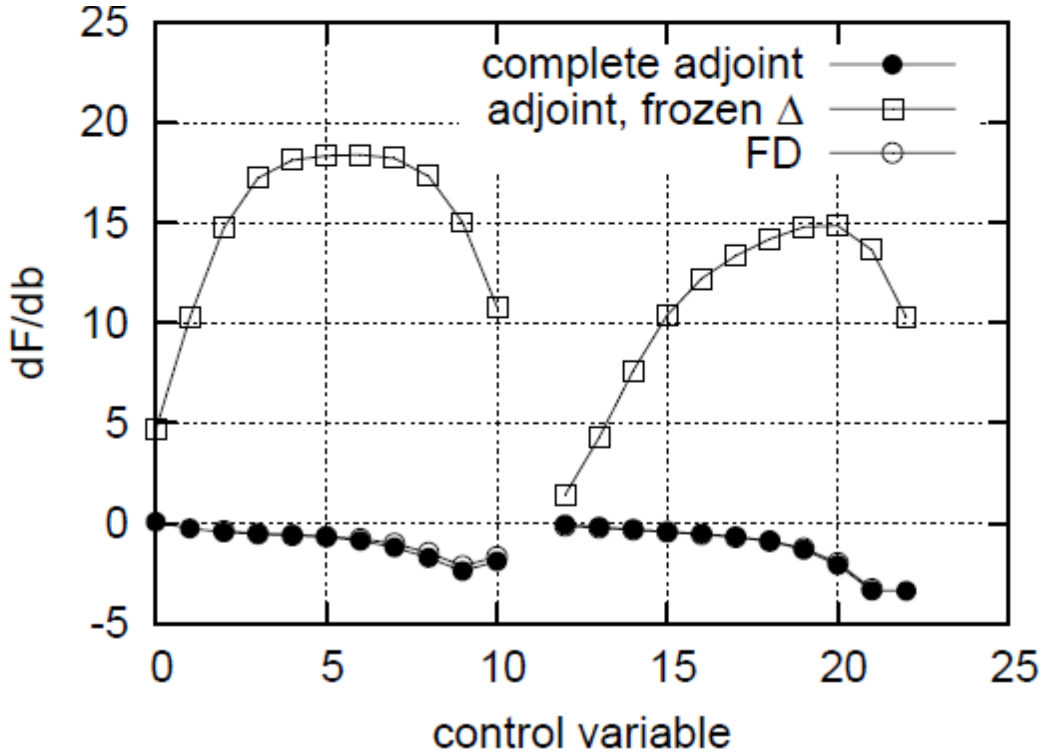
$$\frac{\delta F_{aug}}{\delta b_n} = -\int_{S_{W_p}} \left[(\nu+\nu_t) \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) n_j - q n_i \right] \frac{\partial v_i}{\partial x_m} n_m n_k \frac{\delta x_k}{\delta b_n} dS \underbrace{- \int_{S_{W_p}} 2\Delta_a \frac{\partial \Delta}{\partial x_j} n_j \frac{\partial \Delta}{\partial x_m} n_m n_k \frac{\delta x_k}{\delta b_n} dS}$$

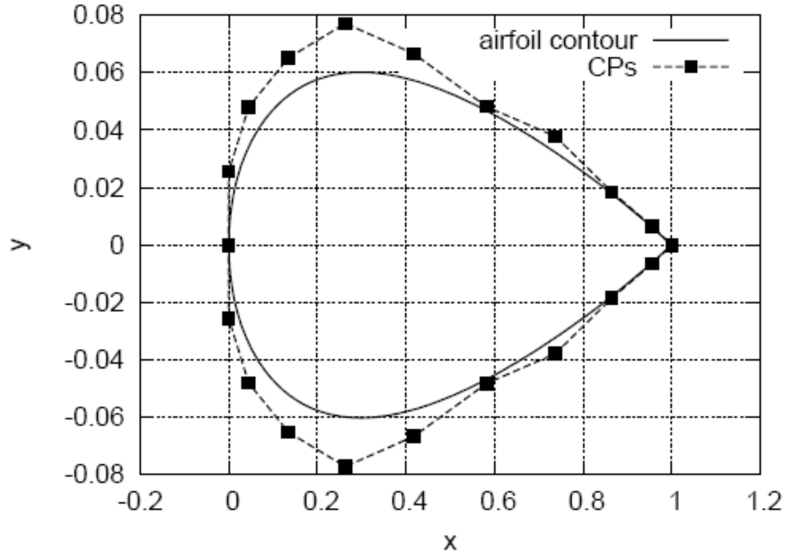
Demo: In some cases, ignoring δ(Δ) might be detrimental

NACA12 Airfoil, Re=6x10^6, $a_{inf}=3^o$

NACA12 F= -Lift, Sensitivities wrt the y of Bezier control points

Spalart-allmaras, low-Re model, Re=6x10^6, $a_{inf}=3^o$





CONCLUSION: In some cases, the "frozen distance assumption" produces wrongly signed sensitivities!

The straightforward way to compute the Hessian

Twice application of the Direct Differentiation Method (DD-DD)

Shown in Discrete. Formulated and programmed also in Continuous Mode

Newton Method:

$$b_i^{n+1} = b_i^n + db_i$$

$$\frac{d^2F}{db_i db_j} db_j = -\frac{dF}{db_i}$$

$$\frac{dF}{db_i} = \frac{\partial F}{\partial b_i} + \frac{\partial F}{\partial U_k}\frac{dU_k}{db_i}$$

k=1,…,N design variables

$$\frac{d^2F}{db_i db_j} = \frac{\partial^2 F}{\partial b_i \partial b_j} + \frac{\partial^2 F}{\partial b_i \partial U_k}\frac{dU_k}{db_j} + \frac{\partial^2 F}{\partial U_k \partial b_j}\frac{dU_k}{db_i} + \frac{\partial^2 F}{\partial U_k \partial U_m}\frac{dU_k}{db_i}\frac{dU_m}{db_j} + \frac{\partial F}{\partial U_k}\frac{d^2 U_k}{db_i db_j}$$

$$\frac{dR_m}{db_i} = \frac{\partial R_m}{\partial b_i} + \frac{\partial R_m}{\partial U_k}\frac{dU_k}{db_i} = 0$$

$$\frac{d^2R_n}{db_i db_j} = \frac{\partial^2 R_n}{\partial b_i \partial b_j} + \frac{\partial^2 R_n}{\partial b_i \partial U_k}\frac{dU_k}{db_j} + \frac{\partial^2 R_n}{\partial U_k \partial b_j}\frac{dU_k}{db_i} + \frac{\partial^2 R_n}{\partial U_k \partial U_m}\frac{dU_k}{db_i}\frac{dU_m}{db_j} + \frac{\partial R_n}{\partial U_k}\frac{d^2 U_k}{db_i db_j} = 0$$

- Cost of the DD-DD approach scales with N^2.

How to compute the Hessian with the lowest CPU cost

DD-AV, equivalent to "tangent mode, then reverse mode"

Shown in Discrete. Formulated and programmed also in Continuous Mode

$$\frac{dR_m}{db_i} = \frac{\partial R_m}{\partial b_i} + \frac{\partial R_m}{\partial U_k}\frac{dU_k}{db_i} = 0 \quad \Rightarrow \quad \frac{dU_k}{db_i} \qquad N$$

System solutions (EFS)

$$\frac{\partial F}{\partial U_k} + \hat{\Psi}_n \frac{\partial R_n}{\partial U_k} = 0 \quad \Rightarrow \quad \hat{\Psi}_n \qquad 1$$

EFS

The Adjoint equation is the same with that solved to compute the Gradient !!!

$$\frac{d^2\hat{F}}{db_i db_j} = \frac{\partial^2 F}{\partial b_i \partial b_j} + \hat{\Psi}_n \frac{\partial^2 R_n}{\partial b_i \partial b_j} + \frac{\partial^2 F}{\partial U_k \partial U_m}\frac{dU_k}{db_i}\frac{dU_m}{db_j} + \hat{\Psi}_n \frac{\partial^2 R_n}{\partial U_k \partial U_m}\frac{dU_k}{db_i}\frac{dU_m}{db_j}$$
$$+ \frac{\partial^2 F}{\partial b_i \partial U_k}\frac{dU_k}{db_j} + \hat{\Psi}_n \frac{\partial^2 R_n}{\partial b_i \partial U_k}\frac{dU_k}{db_j} + \frac{\partial^2 F}{\partial U_k \partial b_j}\frac{dU_k}{db_i} + \hat{\Psi}_n \frac{\partial^2 R_n}{\partial U_k \partial b_j}\frac{dU_k}{db_i}$$
$$+ \left(\frac{\partial F}{\partial U_k} + \hat{\Psi}_n \frac{\partial R_n}{\partial U_k}\right)\frac{d^2 U_k}{db_i db_j}$$

$$\frac{d^2 F^\lambda}{db_i db_j} db_j^\lambda = -\frac{dF^\lambda}{db_i}$$

- **The cost per Newton cycle is N+1+1=N+2 EFS! Scales with N, not N^2.**

CONCLUSION: Much better if N<<. But, what about N>>!

Computation of the Hessian Matrix, via DD-AV

With Continuous Adjoint

See references (on both discrete & continuous approaches)

$$\frac{\delta F_{aug}}{\delta b_j} = \frac{\delta F}{\delta b_j} + \int_\Omega \Psi_n \frac{\partial}{\partial b_j}\left(\frac{\partial f^{inv}_{nk}}{\partial x_k}\right) d\Omega + \int_S \Psi_n \frac{\partial f^{inv}_{nk}}{\partial x_k}\frac{\delta x_l}{\delta b_j} n_l dS + \int_\Omega \frac{\partial \Psi_n}{\partial b_j}\frac{\partial f^{inv}_{nk}}{\partial x_k} d\Omega$$

$$\begin{aligned}
\frac{\delta^2 F_{aug}}{\delta b_i \delta b_j} &= \frac{\delta^2 F}{\delta b_i \delta b_j} + \int_\Omega \Psi_n \frac{\partial^2}{\partial b_i \partial b_j}\left(\frac{\partial f^{inv}_{nk}}{\partial x_k}\right) d\Omega \\
&+ \int_\Omega \frac{\partial^2 \Psi_n}{\partial b_i \partial b_j}\frac{\partial f^{inv}_{nk}}{\partial x_k} d\Omega + \int_\Omega \frac{\partial \Psi_n}{\partial b_i}\frac{\partial^2 f^{inv}_{nk}}{\partial x_k \partial b_j} d\Omega + \int_\Omega \frac{\partial \Psi_n}{\partial b_j}\frac{\partial^2 f^{inv}_{nk}}{\partial x_k \partial b_i} d\Omega \\
&+ \int_S \frac{\partial \Psi_n}{\partial b_i}\frac{\partial f^{inv}_{nk}}{\partial x_k}\frac{\delta x_l}{\delta b_j} n_l dS + \int_S \frac{\partial \Psi_n}{\partial b_j}\frac{\partial f^{inv}_{nk}}{\partial x_k}\frac{\delta x_l}{\delta b_i} n_l dS + \\
&+ \int_S \Psi_n \frac{\partial}{\partial b_i}\left(\frac{\partial f^{inv}_{nk}}{\partial x_k}\right)\frac{\delta x_l}{\delta b_j} n_l dS + \int_S \Psi_n \frac{\partial}{\partial b_j}\left(\frac{\partial f^{inv}_{nk}}{\partial x_k}\right)\frac{\delta x_l}{\delta b_i} n_l dS \\
&+ \int_S \Psi_n \frac{\partial f^{inv}_{nk}}{\partial x_k}\frac{\delta^2 x_l}{\delta b_i \delta b_j} n_l dS + \int_S \frac{\partial \Psi_n}{\partial x_m}\frac{\partial f^{inv}_{nk}}{\partial x_k}\frac{\delta x_m}{\delta b_i}\frac{\delta x_l}{\delta b_j} n_l dS \\
&+ \int_S \Psi_n \frac{\partial^2 f^{inv}_{nk}}{\partial x_k \partial x_l}\frac{\delta x_l}{\delta b_i}\frac{\delta x_m}{\delta b_j} n_m dS + \int_S \Psi_n \frac{\partial f^{inv}_{nk}}{\partial x_k}\frac{\delta x_l}{\delta b_j}\frac{\delta (n_l dS)}{\delta b_i}
\end{aligned}$$

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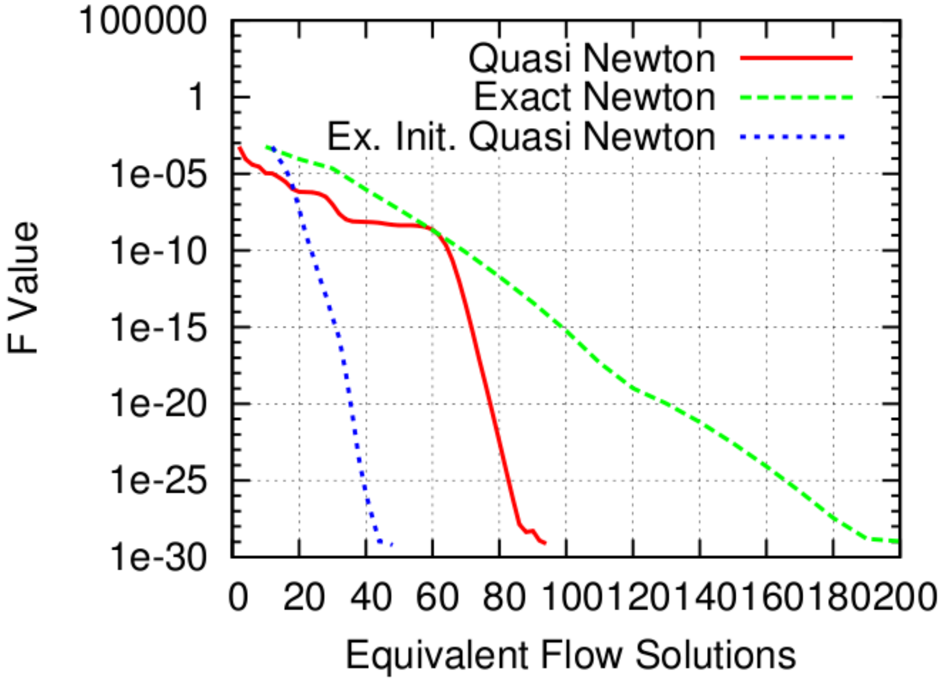
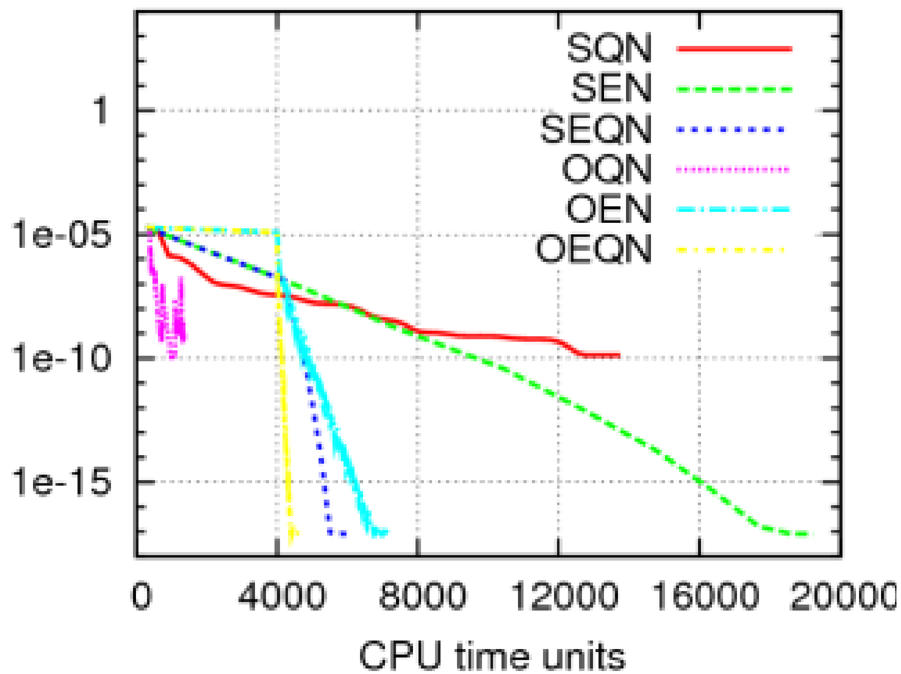
An Improved Approach – Application 1

The Exactly-Initialized-then-Quasi-Newton method & its One-Shot variant

Application: Inverse design of a Compressor blading

Compute the Hessian only in the first cycle, then switch to quasi-Newton method (BFGS)





D.I. PAPADIMITRIOU, K.C. GIANNAKOGLOU: 'One-Shot Shape Optimization Using the Exact Hessian', ECCOMAS CFD 2010, 5th European Conference on CFD, Lisbon, Portugal, June 14-17, 2010.

CONCLUSION: Quite often, it suffices to initialize the solution with the exact Hessian. Work with the one-shot approach!

The AV-DD Truncated Newton Method (with CG)

Handling problems with N>>

Compute Hessian-vector products instead of the Hessian itself

$k \leftarrow 0$

$x \leftarrow \text{init}()$

$r^0 \leftarrow Ax - q;\ p \leftarrow -r^0$

while $r^k \neq 0$ **and** $k \leq M_{CG}$ **do**

$\eta \leftarrow \frac{(r^k)^T r^k}{p^T Ap}$

$x \leftarrow x + \eta p$

$r^{k+1} \leftarrow r^k + \eta Ap$

$\beta \leftarrow \frac{(r^{k+1})^T r^{k+1}}{(r^k)^T r^k}$

$p \leftarrow -r_{k+1} + \beta p$

$k \leftarrow k+1$

end while

Ax=b

$$b_i^{n+1} = b_i^n + db_i$$

$$\frac{d^2F}{db_i db_j} db_j = -\frac{dF}{db_i}$$

$k \leftarrow 0$

$b_j \leftarrow \text{init}()$

while $k \leq k_{max}$ **do**

$U_n \leftarrow$ Flow Equations [1 EFS] ★

$\Psi_n \leftarrow$ Adjoint Equations [1 EFS] ★

$r_j^0 = \frac{dF}{db_j} \leftarrow$ Gradient Expression

$db_j^0 \leftarrow \text{init}(0)$

$p_j \leftarrow -r_j^0$

$m \leftarrow 0$

while $r^m \neq 0$ **and** $m \leq M_{CG}$ **do**

$\frac{dU_n}{db_j} p_j \leftarrow$ DD (Flow Equations) [1 EFS] ★

$\frac{d\Psi_n}{db_j} p_j \leftarrow$ DD (Adjoint Equations) [1 EFS] ★

$w_i = \frac{d^2F}{db_i db_j} p_j \leftarrow$ Hessian Expression

$\eta \leftarrow \frac{r_i^m r_i^m}{p_j w_j}$

$db_j^{m+1} \leftarrow db_j^m + \eta p_j$

$r_j^{m+1} \leftarrow r_j^m + \eta w_j$

$\beta \leftarrow \frac{r_i^{m+1} r_i^{m+1}}{r_j^m r_j^m}$

$p_j \leftarrow -r_j^{m+1} + \beta p_j$

$m \leftarrow m+1$

end while

$b_j \leftarrow b_j + db_j$

$k \leftarrow k+1$

end while

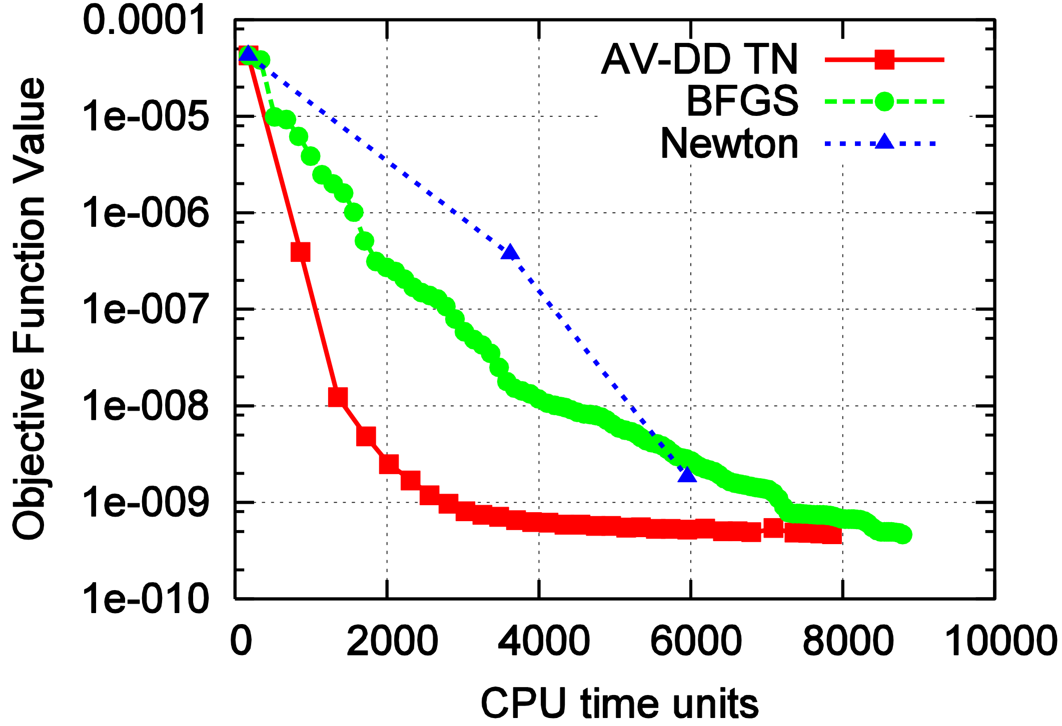
Total Cost= $2+2M_{CG}$ << N

AV-DD Truncated Newton method – Why?

Application: Inverse design of an isolated airfoil, N=42 DOFs

Compute Hessian-vector products instead of the Hessian itself

Comparison of three solution methods



AV-DD Truncated Newton method

Quasi-Newton BFGS

(Exact) Newton

D.I. PAPADIMITRIOU, K.C. GIANNAKOGLOU: 'Aerodynamic design using the truncated Newton algorithm and the continuous adjoint approach', Int. J.for Numerical Methods in Fluids, 68, 6, pp. 724-739, 2012.

CONCLUSION: Truncated Newton method is a viable alternative!

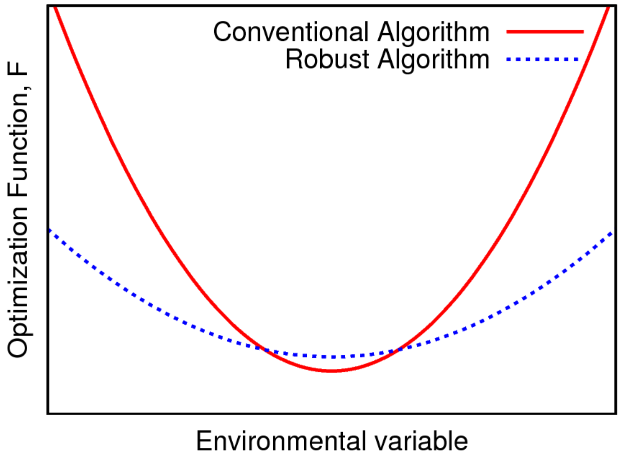
The Second-Order, Second-Moment (SOSM) Approach

For N design (b_i) & M environmental (c_j) variables

Minimize the estimated mean & standard deviation of F

Third-order mixed derivatives must be computed

Proposed method: DD_c-DD_c-Av_b (if M<N)



$$\widehat{F} = \hat{\mu_F} + k\hat{\sigma_F}$$

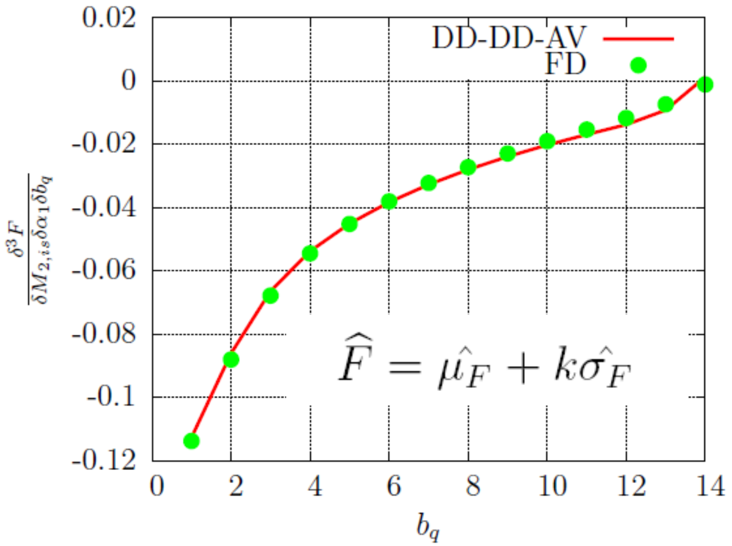
$$\hat{\mu_F} = F_{\mathrm{D}} + \frac{1}{2}\left[\frac{d^2F}{dc_i^2}\right]_{\mathrm{D}}\sigma_i^2$$

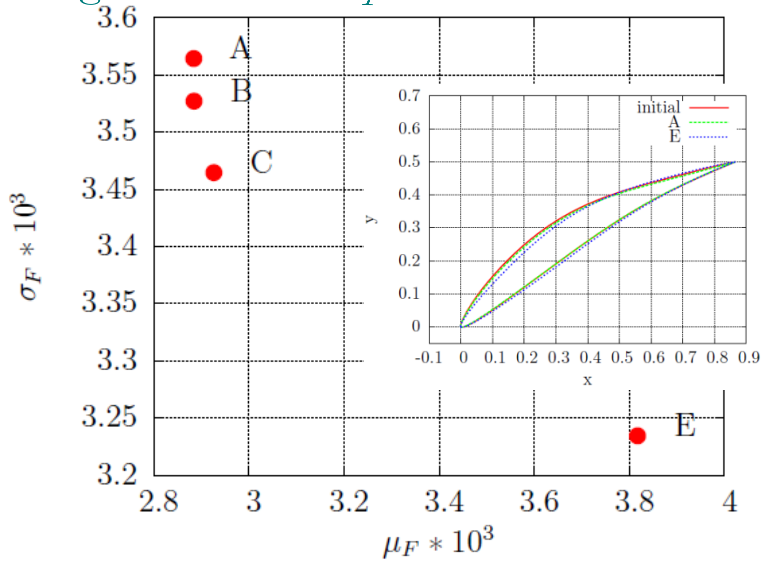
$$\hat{\sigma_F} = \sqrt{\left[\frac{dF}{dc_i}\right]_{\mathrm{D}}^2\sigma_i^2 + \frac{1}{2}\left[\frac{d^2F}{dc_idc_j}\right]_{\mathrm{D}}^2\sigma_i^2\sigma_j^2}$$

$$\frac{d\widehat{F}}{db_l} = \frac{dF}{db_l} + \frac{1}{2}\frac{d^3F}{dc_i^2db_l}\sigma_i^2 + k\frac{2\frac{dF}{dc_i}\frac{d^2F}{dc_idb_l}\sigma_i^2 + \frac{d^2F}{dc_idc_j}\frac{d^3F}{dc_idc_jdb_l}\sigma_i^2\sigma_j^2}{2\sqrt{\left[\frac{dF}{dc_i}\right]^2\sigma_i^2 + \frac{1}{2}\left[\frac{d^2F}{dc_idc_j}\right]^2\sigma_i^2\sigma_j^2}}$$

Robust Design of a Compressor Cascade

Two environmental variables (M=2): Inlet flow angle & exit isentropic Mach number





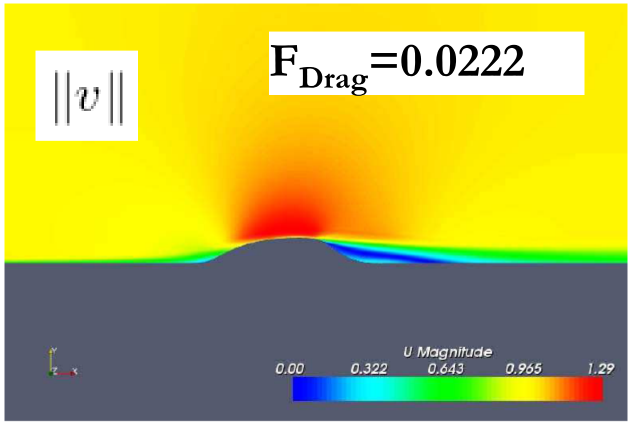
E.M. PAPOUTSIS-KIACHAGIAS, D.I. PAPADIMITRIOU, K.C. GIANNAKOGLOU: 'Robust Design in Aerodynamics using 3rd-Order Sensitivity Analysis based on Discrete Adjoint. Application to Quasi-1D Flows', International Journal for Numerical Methods in Fluids, Vol. 69, No. 3, pp. 691-709, 2012.
E.M. PAPOUTSIS-KIACHAGIAS, D.I. PAPADIMITRIOU, K.C. GIANNAKOGLOU: Discrete and Continuous Adjoint Methods in Aerodynamic Robust Design problems, CFD and Optimization 2011, ECCOMAS Thematic Conference, Antalya, Turkey, May 23-25, 2011.
D.I. PAPADIMITRIOU, K.C. GIANNAKOGLOU: 'Third-Order Sensitivity Analysis for Robust Aerodynamic Design using Continuous Adjoint', International Journal for Numerical Methods in Fluids, Vol. 71, No. 5, pp. 652-670, 2013.

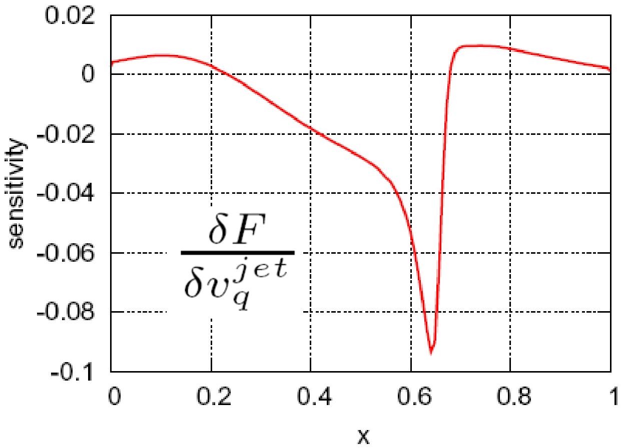
CONCLUSION: Robust Design? Don't go with EAs!

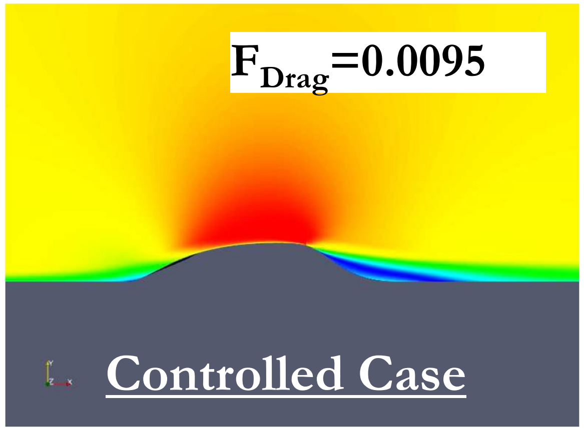
Flow Control Optimization

Optimal flow control using suction/blowing/pulsating jets

Idea: Compute the sensitivity derivatives by solving the flow & adjoint problem once, for normal_jet_velocity=0. Use the computed sensitivity maps to optimally locate the jets and their sign to decide whether suction or blowing is needed. Stop here or iterate to optimize all jet parameters.





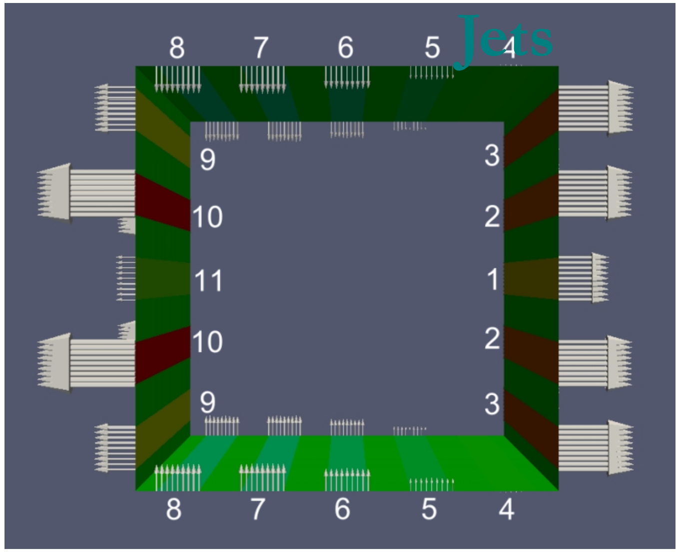


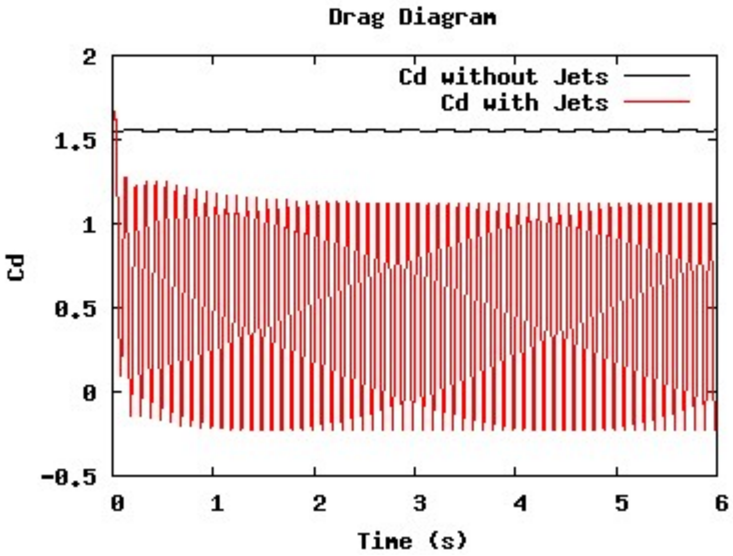
A.S. ZYMARIS, D.I. PAPADIMITRIOU, K.C. GIANNAKOGLOU, C. OTHMER: 'Optimal Location of Suction or Blowing Jets Using the Continuous Adjoint Approach', ECCOMAS CFD 2010, Lisbon, June 14-17, 2010.

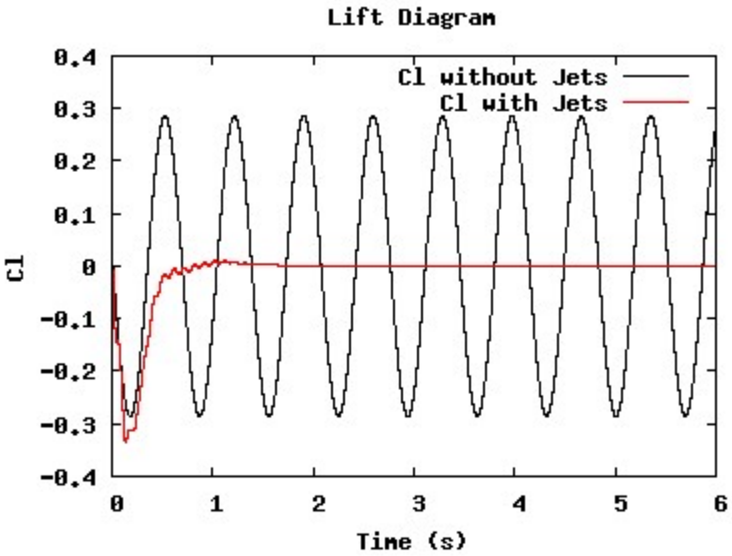
A.S. ZYMARIS, D.I. PAPADIMITRIOU, E.M. PAPOUTSIS-KIACHAGIAS, K.C. GIANNAKOGLOU, C. OTHMER: 'The Continuous Adjoint Method as a Guide for the Design of Flow Control Systems Based on Jets", Engineering Computations, to appear 2013.

Flow around a square cylinder (Re=100) – Control with Pulsating Jets

Slot	Amplitude
1	0.0484
2	0.0707
3	0.0721
4	0.0186
5	-0.0124
6	-0.0218
7	-0.0264
8	-0.0260
9	0.0400
10	0.0948
11	0.0193





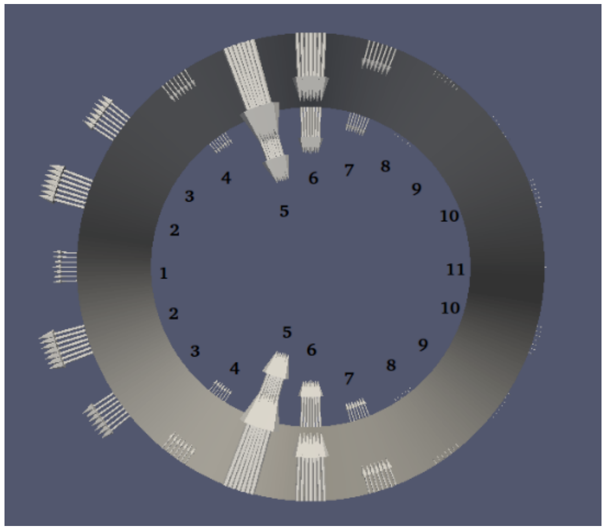


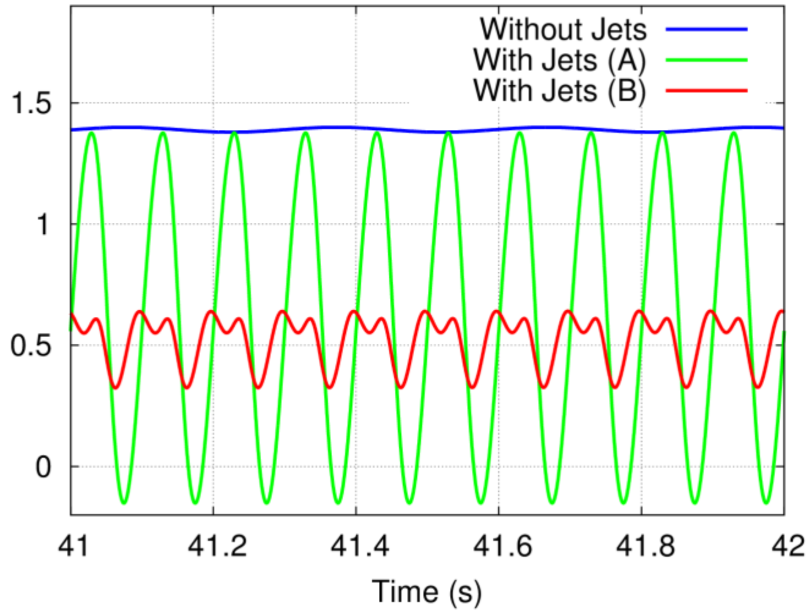
Flow around a circular cylinder (Re=100) – "More" Control

Case A: Design variables=Amplitudes

Case B: Design variables=Amplitudes & phases

	Mean Lift Minimization				Mean Drag Minimization			
	$\overline{c_L}$	$\overline{c_L^2}$	$\overline{c_D}$	$\overline{c_D^2}$	$\overline{c_L}$	$\overline{c_L^2}$	$\overline{c_D}$	$\overline{c_D^2}$
Uncontrolled Case	0.0	0.0303	1.3892	0.9656	0.0	0.0303	1.3892	0.9656
Case A	0.0	0.0	1.0630	0.7892	0.0	0.0	0.6119	0.3382
Case B	0.0	0.0	1.0946	0.6045	0.0	0.0	0.4902	0.1023

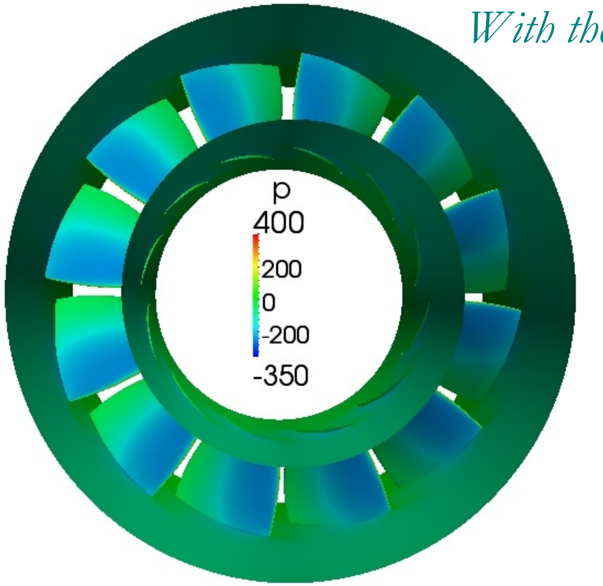


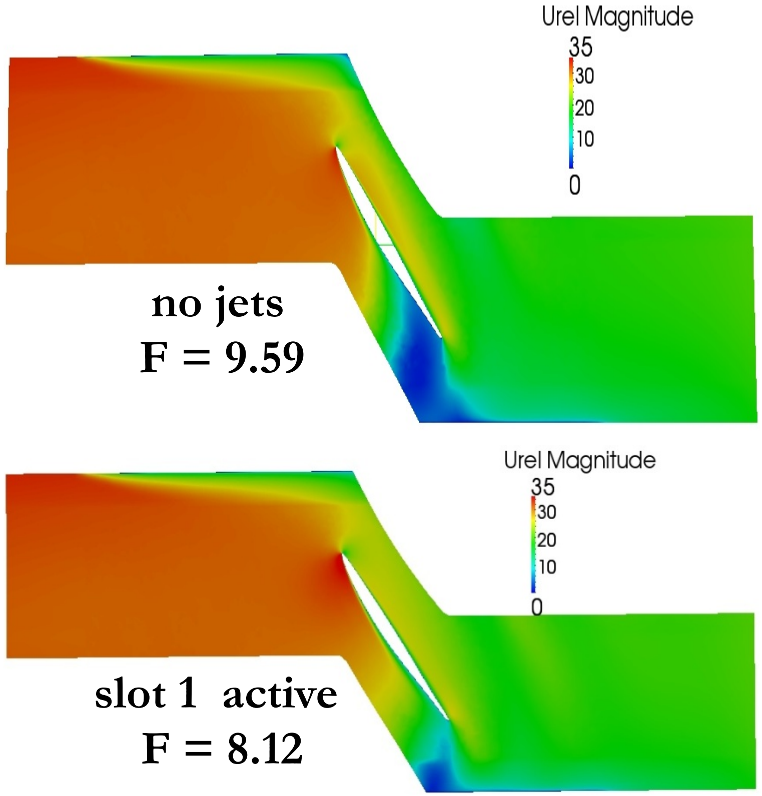


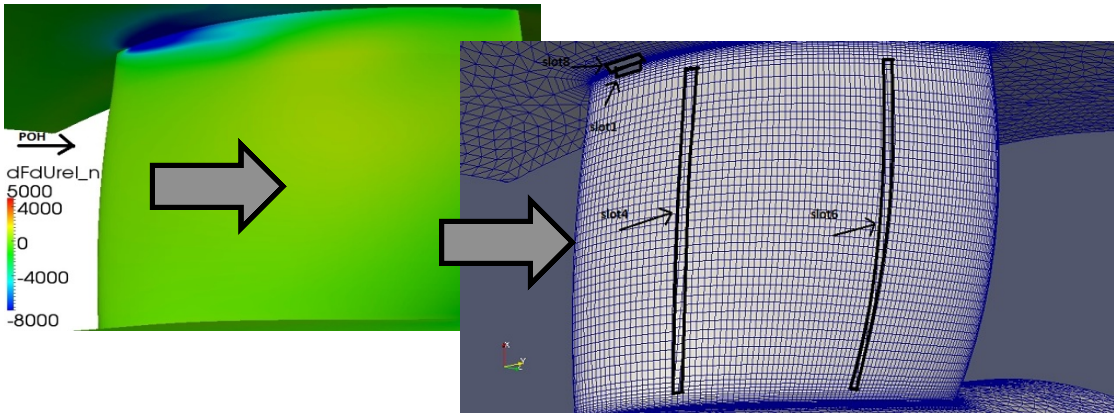
Optimal flow control of a Compressor Cascade

Sensitivity maps were computed & then the jets were placed "manually"

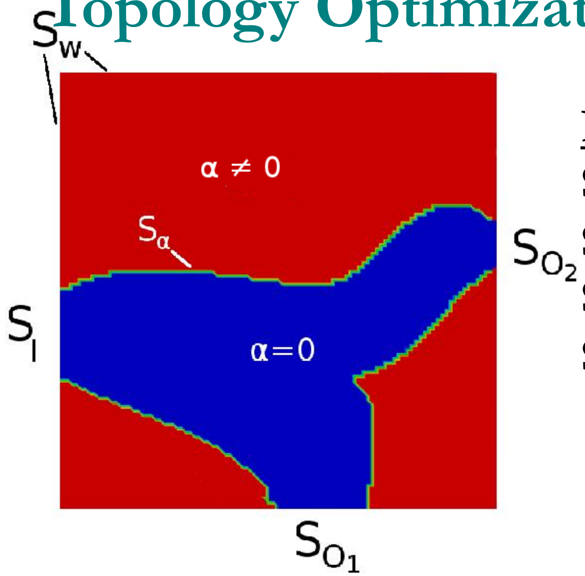
With the continuous adjoint to the k-ω SST model







Topology Optimization: Formulations based on porosity (a)



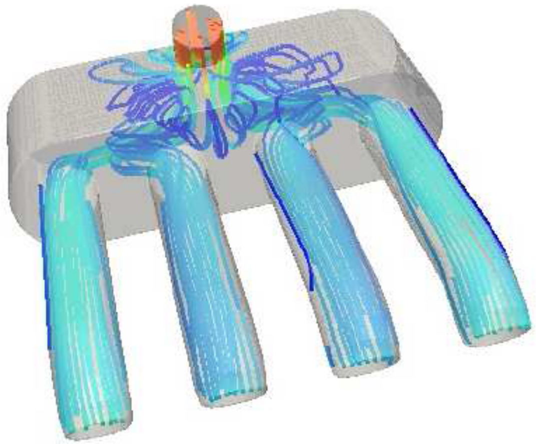
Primal Equations:

Std_Continuity=0

Std_Momenum$_i$+av_i=0

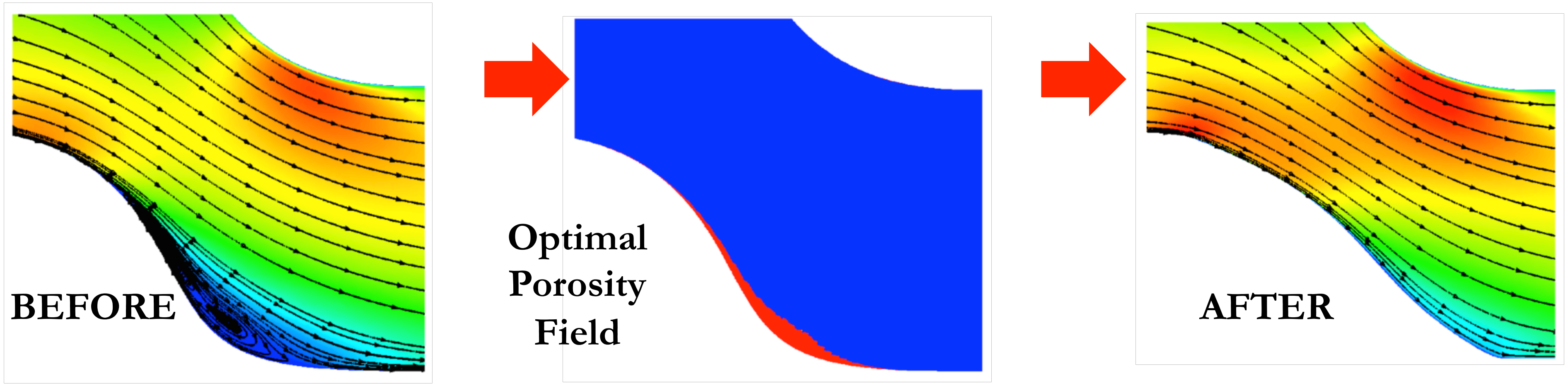
Std_Energy+a(T-T_w)=0

Std_TurbModel(ν_t)+aν_t=0

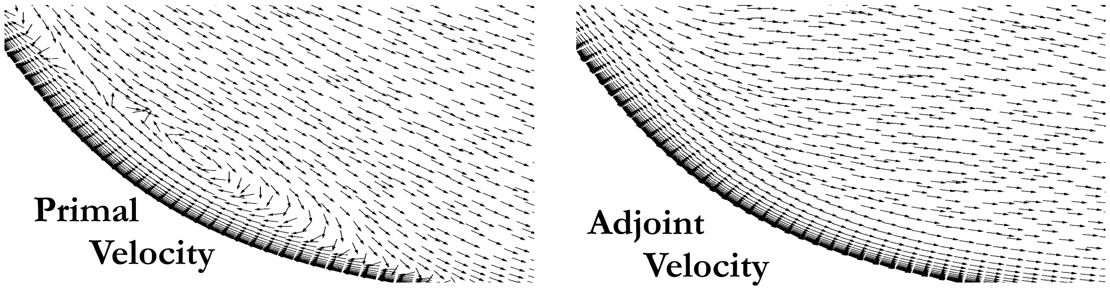


E.A. KONTOLEONTOS, E.M. PAPOUTSIS-KIACHAGIAS, A.S. ZYMARIS, D.I. PAPADIMITRIOU, K.C. GIANNAKOGLOU: 'Adjoint-based constrained topology optimization for viscous flows, including heat transfer, Engineering Optimization, 2012.

Why Continuous Adjoint?



Objective: Min. pt Losses – Continuous Adjoint to [RANS & Spalart-Allmaras].
Recirculation areas disappeared - 15% reduction in total pressure losses.



$$\frac{\delta F_{aug}}{\delta \alpha} = \boxed{v_i u_i} + \tilde{\nu}\tilde{\nu}_a + \int_{\Omega} \tilde{\nu}_a \tilde{\nu} \mathcal{C}_d(\tilde{\nu}, \vec{v}) \frac{\partial d}{\partial \alpha} d\Omega$$

CONCLUSION: Interesting physical interpretation of the adjoint fields!

On-going Research

- Make the continuous adjoint as consistent as the discrete adjoint. Consistent discretization schemes for the adjoint PDEs & their boundary conditions. The *Think-discrete-do-continuous* approach.
- Low(er)-cost solution of robust design problems using adjoint methods and truncated Newton methods.
- Efficient adjoint methods for Pareto optimization. Truncated Newton.
- Approximate adjoints for DES solutions.