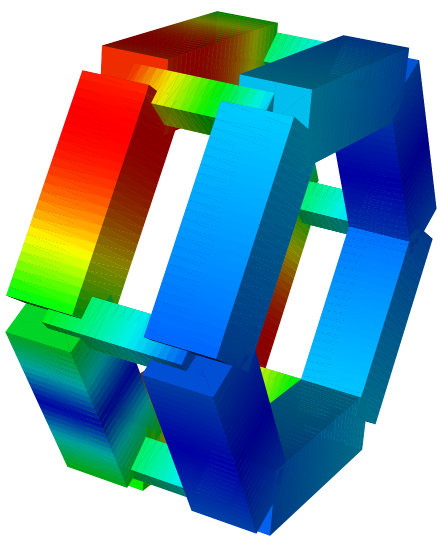
SOUNDTUBE1_5D Users' Guide

Global tool
for thermo-acoustic instabilities analysis
in 1.5 dimensional configurations

september 2002



Cover photo:
3D visualization of a mode with ENSIGHT7. The mode visualized is the first mode ($f = 285.6\,Hz$) of the example 12 in chapter 3. It is obtained by using the soundtube option “mode-shape3d”.

Introduction

Soundtube1_5D constitutes a tool for acoustic analysis in 2 or 3-dimensional network of elementary tubes. In each tube, acoustic waves are seen as longitudinal waves in a 1-dimensional approach. Consequently, these geometries can be called ”1.5-dimensional”. This code has been conceived in order to calculate quickly the eigen modes of combustion chambers viewed as ”1.5-dimensional” configurations. It can also provide the shape of the correspondant eigen modes.

Soundtube1_5D is written in C++. It has been made by Laurent Benoit and Andre Kaufmann at CERFACS in 2002.

The current guide deals with the main characteristics of the code.
First, the theoretical aspect of the code and the choice of acoustic modelization is described. The different evaluation methods of eigen frequencies are also detailed. This part is essential for whoever wants to understand the logic of this code. The knowledge of this part is also necessary to use correctly soundtube1_5d because it contains the definitions of all parameters required in the input file of the code.
Second, the way of filling the input file and executing the code is detailed. This part deals with visualizing the results also.
Third, some basic examples are given. They can be seen as a sort of tutorial and reference for the user.
Finally, the last part contains the description of different auxiliar tools to use easily the code.

Contents

Chapter 1

Principle of soundtube1_5D

1.1 Acoustic modelisation

1.1.1 Geometries considered

As explained in the introduction, the geometries considered are networks of elementary tubes. Each tube can be viewed as a basic component characterized by:

- its length L
- its surface S (constant in all the tube)
- its mean temperature T_0 (constant in all the tube)
- its mean pressure p_0 (constant in all the tube)
- the mean molecular weigth W (constant in all the tube) of the gas inside
- the thermodynamic parameter γ (constant in all the tube)

The boundaries of such a configuration correspond to acoustical reflection coefficents as shown by figure 1.1. The exact definition retains for this reflection coefficient is given here.

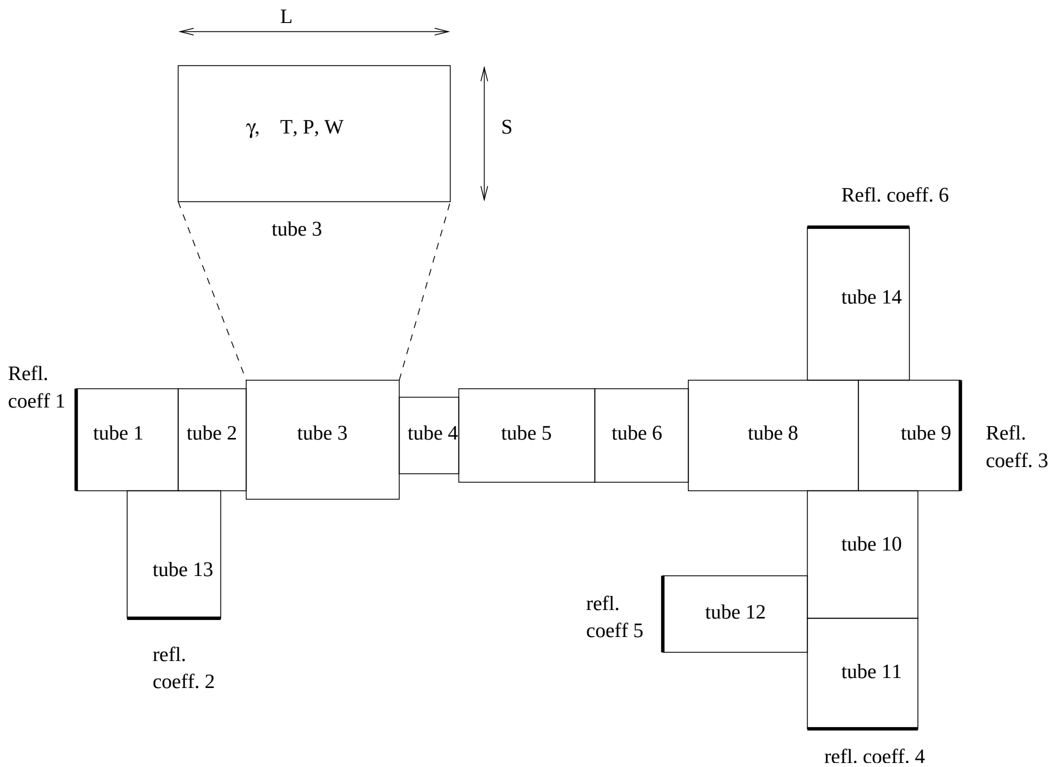


Figure 1.1: General configuration in Soundtube1_5D

SoundTube1_5D is also able to find eigen modes in the case of tubes linked such in figure 1.2. Such an example is available in chapter 3.

Theoretically there is no limit to the number of tubes used; nevertheless, the speed of solving is linked with this number. The way of "discretizing" the real configuration studied, i.e: the choice of the number of tubes and their features, has to be chosen by considering the variation of the section and the thermodynamic fields (mean temperature, mean pressure) of the real problem. The number of tubes connected at an intersection is currently limited to 20 but can be changed without particular problems.

1.1.2 Acoustics without combustion

At first, the situation without flame is considered. The model is based on the following assumption:

- low Mach number flow
- volumic forces neglected
- viscosity neglected
- small acoustic fluctuations (linear acoustics)
- isentropic flow (since no flame are taken into account for the moment)
- low frequencies and consequently longitudinal waves.

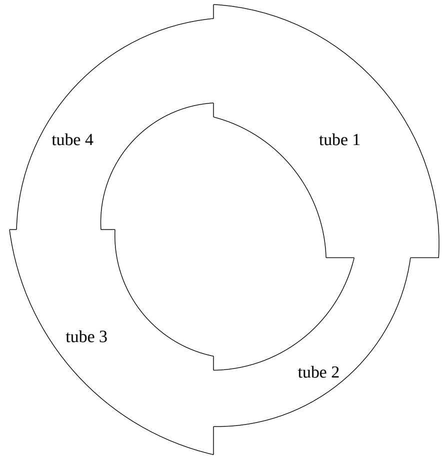


Figure 1.2: Ring configuration in Soundtube1_5D

In these conditions, the linearized equations of mass and momentum can be reduced to the homogeneous wave equation:

$$\nabla^2 p' - \frac{1}{c_0^2}\frac{\partial^2 p'}{\partial t^2} = 0 \tag{1.1}$$

In a given elementary tube, the solving of this equation yields to the following expressions for u' and p' (with complex notation):

$$p' = A^+ e^{ikz - i\omega t} + A^- e^{-ikz - i\omega t} \tag{1.2}$$

$$u' = \frac{A^+}{\rho_0 c_0} e^{ikz - i\omega t} - \frac{A^-}{\rho_0 c_0} e^{-ikz - i\omega t} \tag{1.3}$$

k is the wave vector and, in a $1-D$ flow, without dispersion phenomenon, it verifies:

$$k = \frac{\omega}{c_0} = \frac{2.\pi.f}{c_0} \tag{1.4}$$

As a consequence, for each tube of a given SoundTube1_5D configuration, two corresponding amplitudes A^+ and A^- (the so-called Riemann invariants) are associated. This can be seen in figure 1.3.

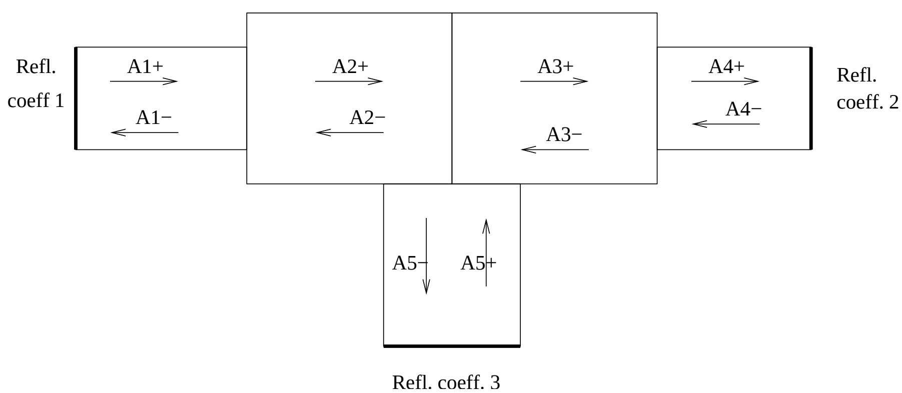


Figure 1.3: General configuration in Soundtube1_5D with amplitudes of each tube

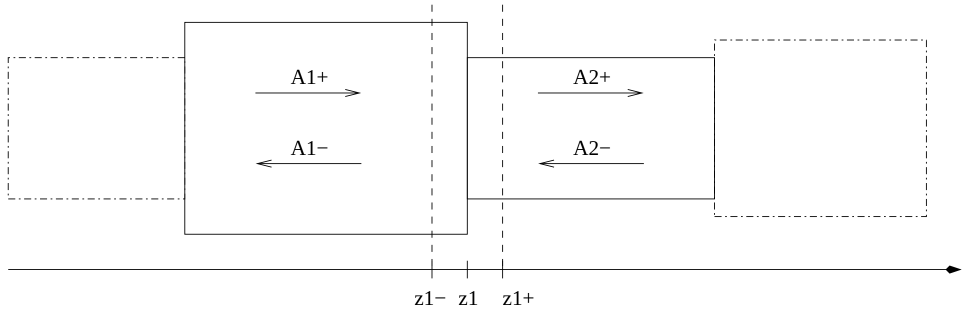


Figure 1.4: Intersection without flame between two tubes

Now, the point is to link the amplitudes of each tubes at an intersection and, as a first attempt, at an intersection with only two tubes (see figure 1.4).

In order to obtain a relation between the amplitudes of the two tubes connected, balance equations of mass and momentum are integrated over the volume delimited by z_1^+ and z_1^-:

$$\frac{\partial \rho}{\partial t} + \frac{1}{S(z)} \frac{\partial(\rho u S(z))}{\partial z} = 0 \tag{1.5}$$

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial z} \tag{1.6}$$

Linearizing these equations under the hypothesis of the model (mean speed flow null) yields to:

$$\frac{\partial u'}{\partial t} + \frac{1}{\rho_0} \frac{\partial p'}{\partial z} = 0 \tag{1.7}$$

$$\frac{1}{\gamma p_0} \frac{\partial p'}{\partial t} + \frac{1}{S} \frac{\partial S u'}{\partial z} = 0 \tag{1.8}$$

By making the length $z_1^+ - z_1^-$ go to zero, the continuity of pressure and speed flux yield to the following "jump-relations":

$$p_1'(z_1) = p_2'(z_1) \tag{1.9}$$

$$S_1 u_1'(z_1) = S_2 u_2'(z_1) \tag{1.10}$$

The detail of the calculus can be found in the chapter 8 of *Theoretical and numerical combustion* of Poinsot and Veynante [1].

Generalization with any number of tubes

The same reasoning can be performed whatever the number of tubes connected at an intersection. However, some additional parameters have to be introduced: in order to satisfy the jump-relation for speed at a node, an orientation of the tube and its sides is needed. As seen on figure 1.5 where four tubes are linked, the beginning of a tube is called $side-$ and its end $side+$. This convention can also be justified by other reasons detailed in the part "definition of eigen frequencies and generalities about their calculus". This orientation is taken into account in jump-relations via the variable $side$ and the function $g(side)$ defined as:

$$side = 0 \Longleftrightarrow side-$$

$$side = 1 \Longleftrightarrow side+$$
$$g(side) = -1 \Longleftrightarrow side-$$
$$g(side) = +1 \Longleftrightarrow side+$$

Finally, for k tubes linked at an intersection (see figure 1.5 where $k = 4$), the jump-relations become:

$$p'_1(z_1) = p'_2(z_1)$$
$$p'_1(z_1) = p'_3(z_1)$$
$$\vdots$$
$$p'_1(z_1) = p'_k(z_1)$$

$$g(side_1).S_1u'_1(z_1) + g(side_2).S_2u'_2(z_1) + g(side_3).S_3u'_3(z_1) + \cdots + g(side_k).S_ku'_k(z_1) = 0$$

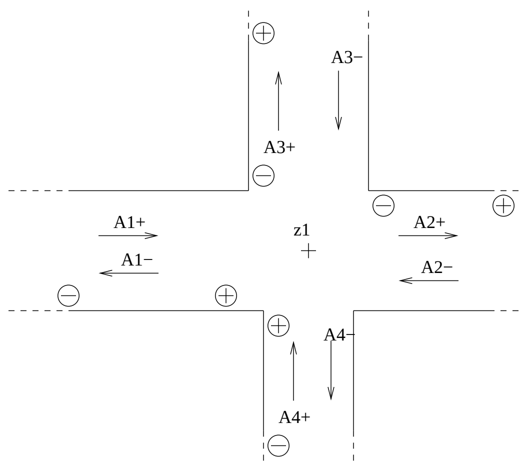


Figure 1.5: Intersection without flame between four tubes

1.1.3 Acoustics with combustion

The acoustic model with combustion uses the same hypothesis as previously **excepted the isentropy of the flow** which can not be valid in this situation. So, the hypothesis with combustion are:

- low Mach number flow
- volumic forces neglected
- viscosity neglected
- small acoustic fluctuations (linear acoustics)
- low frequencies and consequently longitudinal waves.

By adding the enthalpy equation to the linearized equations of mass and momemtum, the inhomogeneous wave equation can be deduced with the unsteady flame rate $\dot{\omega}_T = \dot{\omega}_T^0 + \dot{\omega}_T^{'}$:

$$\nabla.(c_0^2 \nabla p') - \frac{\partial^2 p'}{\partial t^2} = (\gamma - 1)\frac{\partial \dot{\omega}_T^{'}}{\partial t} \tag{1.11}$$

This last equation (eq. 1.11) is not directly solved. Instead of that, another hypothesis is added to simplify the problem: **the flame is considered infinitely thin**. As a result, a flame is always viewed as a reactive intersection separating tubes where the flow is isentropic and where the previous solutions (eq. 1.2 and eq. 1.3) can be applied. Now, solving the acoustic problem with a flame means find the jump-relations at a flame-interface.
The first step consists in obtaining these jump-relations in the case of two tubes linked (see figure 1.6).

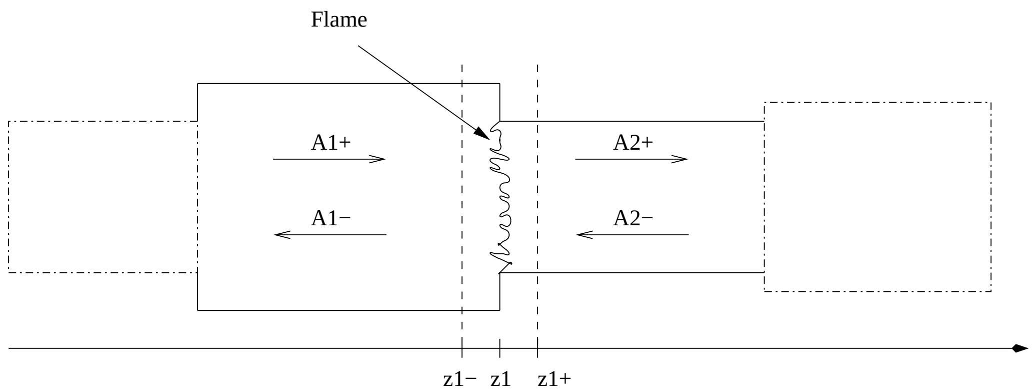


Figure 1.6: Intersection with flame between two tubes

In this case, the linearized equations of mass and momemtum applied to the control volume (between z_1^+ and z_1^-) give the following result:

$$\frac{\partial u'}{\partial t} + \frac{1}{\rho_0}\frac{\partial p'}{\partial z} = 0 \tag{1.12}$$

$$\frac{1}{\gamma p_0}\frac{\partial p'}{\partial t} + \frac{1}{S}\frac{\partial S u'}{\partial z} = \frac{\gamma - 1}{\gamma p_0}\dot{\omega}_T^{'} \tag{1.13}$$

By integrating over the control volume and by tending towards zero the length $z_1^+ - z_1^-$, equations 1.12 and 1.13 yield to:

$$p_1'(z_1) = p_2'(z_1) \tag{1.14}$$

$$S_1 u_1'(z_1) + \frac{\gamma - 1}{\gamma p_0}\Omega' = S_2 u_2'(z_1) \tag{1.15}$$

$\Omega' = lim_{z_1^-, z_1^+ \to z_1} \int_{z_1^-}^{z_1^+} S \dot{\omega}_T^{'} dz$ represents the global fluctuating heat release. This term has to be modelized, i.e: linked with u' and p', in order to close the system. This is the role of the $n - \tau$ model.

$n - \tau$ **model**

This model uses two parameters, n and a time lag τ, to link heat release with a reference acoustic speed u'_{ref}. The result is as follows:

$$\frac{\gamma-1}{\rho_1 c_1^2}\Omega' = S_{ref}.n.u'_{ref}(t-\tau) = S_{ref}.n.e^{i\omega\tau}u'_{ref} \tag{1.16}$$

The reference speed u'_{ref} corresponds to the acoustic speed at a given side of a given tube in the configuration. This choice is made by the user. The parameters n and τ can be functions of the (real) frequency. As explained in chapter 2, $n(f)$ and $\tau(f)$ can be any polynomial or rational function of the frequency.

Influence of the fuel injection line

In addition to the heat release, the influence of fuel injection is taken into account as follows:

$$\frac{\gamma-1}{\rho_1 c_1^2}\Omega' = \underbrace{S_{ref}.n.e^{i\omega\tau}u'_{ref}}_{n-\tau\, model} + \underbrace{S_{ref}.n_p.e^{i\omega\tau_p}p'_{ref}}_{fuel\, line} \tag{1.17}$$

Like n and τ, n_p and τ_p can be any polynomial or rational function of the frequency.

Generalization with any number of tubes

By performing a similar derivation with a number k of tubes (figure 1.7 illustrates this case with $k=4$) and by using the same convention as in the case without combustion, some more general jump-relations can be obtained:

$$p'_1(z_1) = p'_2(z_1)$$

$$\vdots$$

$$p'_1(z_1) = p'_k(z_1)$$

$$g(side_1).S_1u'_1(z_1) + \cdots + g(side_k).S_ku'_k(z_1) + g(side_{ref}).S_{ref}n.e^{i\omega\tau}u'_{ref} + S_{ref}n_p.e^{i\omega\tau_p}p'_{ref} = 0$$

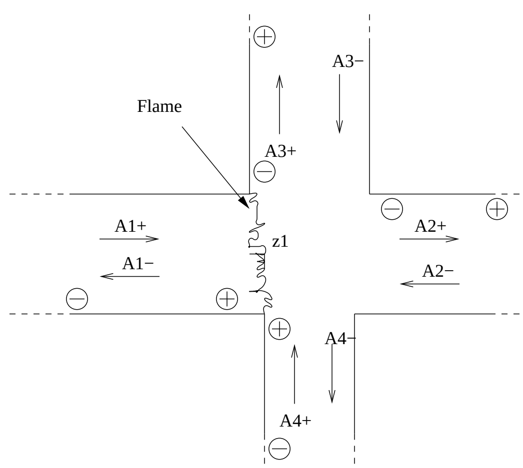


Figure 1.7: Intersection with flame between four tubes

Acoustic thick flame

Until now, flames are seen as infinitely thin in soundtube1_5d, but sometimes this view is too limited. A more realistic conception is to consider the flame as a spatial distribution of heat release. It means that an acoustic thick flame can be modelled as a succession of thin flames with their own n and τ and with a same speed reference u'_{ref}. This situation is illustrated by the figure 1.8.

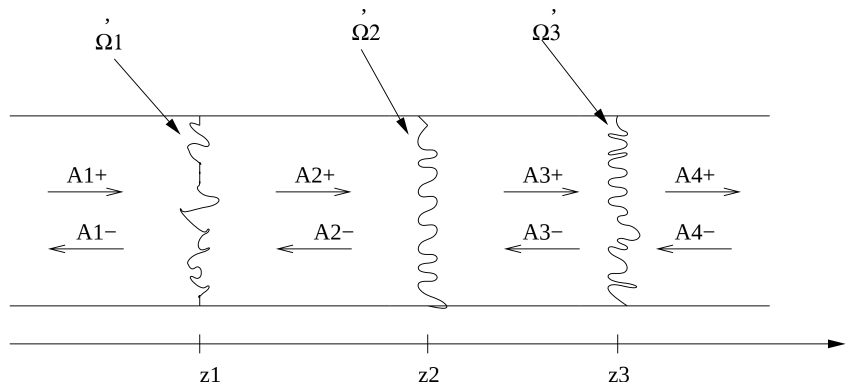


Figure 1.8: Acoustic thick flame

The speed reference u'_{ref} is for instance $u'(z_1)$ and consequently S_{ref} is S_1. In this case, the global fluctuating heat release of the thick flame corresponds to the sum of the fluctuating heat releases of each zone of the flame: Ω'_1, Ω'_2, Ω'_3.

$$\frac{\gamma-1}{\rho c^2}\Omega'_1 = S_{ref}.n_1.e^{i\omega\tau_1}u'_{ref}$$

$$\frac{\gamma-1}{\rho c^2}\Omega'_2 = S_{ref}.n_2.e^{i\omega\tau_2}u'_{ref}$$

$$\frac{\gamma-1}{\rho c^2}\Omega'_3 = S_{ref}.n_3.e^{i\omega\tau_3}u'_{ref}$$

1.1.4 Reflection coefficient

This part is very important because the definition of the reflection coefficient governs the values of the eigen frequencies. Consider a tube where z is the local abscissa (i.e: z measures the distance from the begining of the tube). Thus, the reflection coefficient R is defined as:

$$R = \frac{A^+}{A^-}e^{2ikz} \tag{1.18}$$

Two cases are possible:

- If the reflection coefficient is at the end of the tube (so, $z = L$ with $''L''$, the length of the tube) as in figure 1.9, the value of R is:

$$R = \frac{A^+}{A^-}e^{2ikL}$$

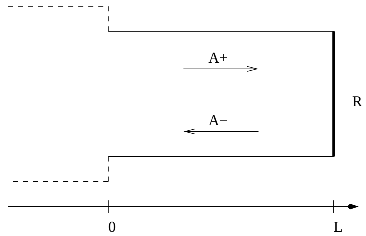


Figure 1.9: A tube ended by a reflecting boundary

- On the contrary, if the reflection coefficient is at the begining of the tube ($z = 0$) as in figure 1.10, then the value of R is:

$$R = \frac{A^+}{A^-}$$

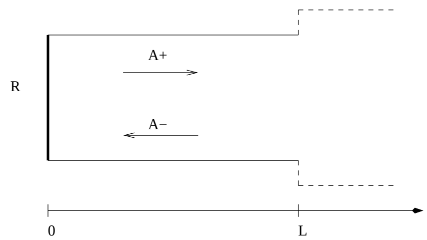


Figure 1.10: A tube beginning by a reflecting boundary

With the definition chosen, the elementary cases $R = 1$ and $R = -1$ can be easily interpreted:

- $R = 1 \Longleftrightarrow u'(z) = 0$
- $R = -1 \Longleftrightarrow p'(z) = 0$

Influence of the frequency

Like n and τ, the reflection coefficient can be a function of the real frequency. In soundtube1_5d, R can be any polynomial or rational function of the frequency. This feature will be explained in the chapter 2.

1.2 Evaluation of acoustic eigen modes

1.2.1 Definition of eigen frequencies and generalities about their calculus

For a given configuration, an eigen frequency is a frequency which verifies all the boundary conditions (reflection coefficient value) and all the jump-relations of all the intersections. Therefore, the way of evaluating an eigen frequency is relied on the way of describing the configuration.

A relevant point is **the need of an orientation for each tube**. Indeed, in the definition of a reflection coefficient, it is required to know which side is defined as the begining of the tube.

Moreover, it is also necessary to know which side of a tube is connected with which intersection in order to satisfy the jump-relations. Thus, the variable *side* and the function $g(side)$ precedently introduced (in acoustics without combustion) are again used.

Reflection condition verified by eigen frequencies:

The relation deduced from the definition of a reflection coefficient becomes (in the case where the tube number q is linked with the reflection coefficient number j):

$$A_q^+.e^{2.i.k_q.side.L_q} - R_j.A_q^- = 0 \tag{1.19}$$

Jump-relation for pressure verified by eigen frequencies:

This jump-relation (see here) can be developped similarly to the reflection condition. At an intersection with m tubes connected and eventually with a flame attached to tube 1, the jump-relation for acoustic pressure is:

$$\forall j \in [|1,m|],\ p_1'(z_1) = p_j'(z_1)$$

It becomes:

$$A_1^+.e^{i.k_1.side_1.L_1} + A_1^-.e^{-i.k_1.side_1.L_1} - A_j^+.e^{i.k_j.side_j.L_j} - A_j^-.e^{-i.k_j.side_j.L_j} = 0 \tag{1.20}$$

Jump-relation for speed verified by eigen frequencies: For acoustic speed, the following relation is reminded:

$$\sum_{j=1}^{m} g(side_j).S_j u_j'(z_1) + g(side_{ref})S_{ref}.n.e^{i\omega\tau}u_{ref}' + S_{ref}n_p.e^{i\omega\tau_p}p_{ref}' = 0$$

It yields:

$$A_{ref}^+\alpha_{ref}^+.e^{i.k_{ref}.side_{ref}.L_{ref}} + A_{ref}^-\alpha_{ref}^-.e^{-i.k_{ref}.side_{ref}.L_{ref}} + \sum_{j=1}^{m}(\alpha_j^+A_j^+.e^{i.k_j.side_j.L_j} + \alpha_j^-A_j^-.e^{-i.k_j.side_j.L_j}) = 0 \tag{1.21}$$

with

$$\alpha_{ref}^+ = S_{ref}.(n_p.e^{i\omega\tau_p} + g(side_{ref}).\frac{n.e^{i\omega\tau}}{\rho_{ref}.c_{ref}})$$

$$\alpha_{ref}^- = S_{ref}.(n_p.e^{i\omega\tau_p} - g(side_{ref}).\frac{n.e^{i\omega\tau}}{\rho_{ref}.c_{ref}})$$

$$\alpha_j^+ = g(side_j).\frac{S_j}{\rho_j.c_j}$$

$$\alpha_j^- = -g(side_j).\frac{S_j}{\rho_j.c_j}$$

The case of an intersection without flame can be easily deduced from relation 1.21 by imposing $n = 0$ and $n_p = 0$.

Stability of an eigen mode:

The value of an eigen frequency is complex. Because of the solution form retained (equations 1.2 and 1.3), **an eigen frequency is stable if its imaginary part is negative**. If $f = f_r + i.f_i$ is an eigen frequency then:

- $f_i > 0 \Longrightarrow$ **unstable** eigen mode
- $f_i < 0 \Longrightarrow$ **stable** eigen mode

1.2.2 Modal method

This method is a strict application of the definition of an eigen frequency. The whole relations verified by eigen frequencies (reflection conditions and jump-relations) can be writing into a matrix $\mathbf{M}(\omega)$ such that (in a case with n tubes):

$$\mathbf{M}(\omega).\begin{pmatrix} A_1^+ \\ A_1^- \\ \vdots \\ A_n^+ \\ A_n^- \end{pmatrix} = 0$$

Eigen frequencies $f_0 = \frac{\omega_0}{2\pi}$ are such that:

$$det(\mathbf{M}(\omega_{\mathbf{0}})) = 0 \tag{1.22}$$

The modal method seeks, in an given complex domain, complex frequencies which verify equation 1.22. The characteristic function $\mathcal{F}(\omega)$ of the configuration is defined as:

$$\mathcal{F}(\omega) = det(\mathbf{M}(\omega))$$

To find the complex zeros of $\mathcal{F}$ in a given complex domain of contour C , the following result is reminded:

$$\frac{1}{2i\pi}\oint_C \frac{\mathcal{F}'(\omega)}{\mathcal{F}(\omega)} d\omega = N - P \tag{1.23}$$

where N is the number of zeros inside C and P is the number of poles inside C.

The strategy to find eigen modes consists in dividing the initial contour C into smaller ones until each eigen frequencies are enclosed. At this point, eigen frequencies are numerically found by minimizing $|\mathcal{F}(\omega)|$ on the small resulting contours.

Result of the method

This method gives the numerical values of the eigen frequencies that is to say their real part f_r and their imaginary part (growth rate) f_i. They are sorted following the value of their frequency (i.e: their real part).

Limitation of the modal method

This method is not well adapted to 1.5D cases because the resulting determinant $\mathcal{F}(\omega)$ varies too rapidly with the frequency. Indeed, in the formula 1.23, the continuity of $arg(\mathcal{F}(\omega))$ inside a subdivision of the contour C is required (result of complex analysis). when $arg(\mathcal{F}(\omega))$ varies rapidly more subdivisions are necessary. Otherwise, some eigen frequencies can be missed. Consequently, the computing time induced by the high number of subdivisions is too important.

In 1.5D cases, this method gives reliable results only for small configurations (with 5 or 6 tubes maximum).

1.2.3 Forcing method

This method consists in measuring the response of the configuration at a monochromatic excitation. The resonant frequencies, that is to say the frequencies for which the response is maximal, are considered as the eigen frequencies of the configuration. The two key points of the forcing method are (1) which device is used to force and (2) which acoustic variable quantifies the response of the excited configuration.

Forcing device

The forcing device is a side-mounted loudspeaker modelled as a source of volume flow in one of the tube of the configuration. The situation is similar to the figure 1.11.

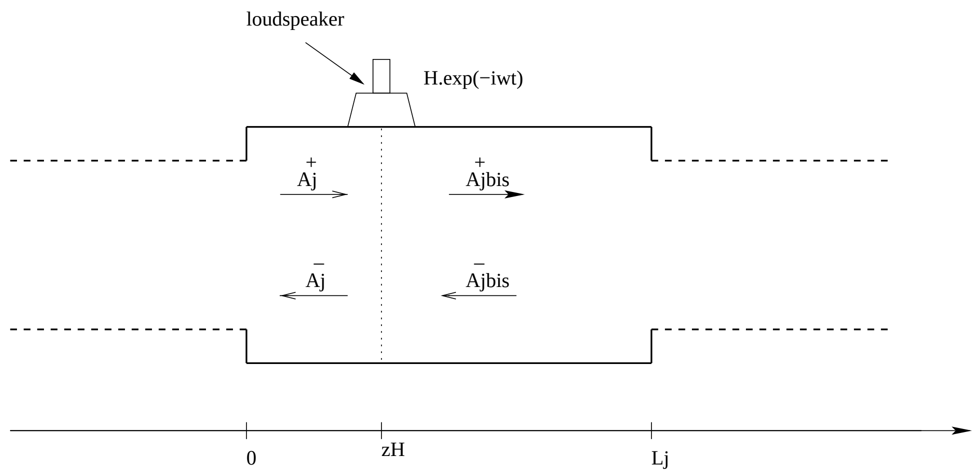


Figure 1.11: Forcing device: lateral loudspeaker in a given tube of the configuration at z_H in the local abscissa.

In this case, the jump-relation for pressure is:

$$A_j^+ e^{ik_j z_H} + A_j^- e^{-ik_j z_H} = A_{jbis}^+ + A_{jbis}^- \tag{1.24}$$

And the jump-relation for speed is as follows:

$$\frac{S_j}{\rho_j c_j} A_j^+ e^{ik_j z_H} - \frac{S_j}{\rho_j c_j} A_j^- e^{-ik_j z_H} = \frac{S_j}{\rho_j c_j} A_{jbis}^+ - \frac{S_j}{\rho_j c_j} A_{jbis}^- + H \tag{1.25}$$

Concerning the acoustic variable which measures the response of the configuration to the forcing, two variables can be selected: the total acoustic energy in the whole domain or the acoustic flux at the loudspeaker (at $z = z_H$ in the example of figure 1.11.

EnergyForcing method

The total acoustic energy E is the acoustic energy of all the tubes averaged over one period (the period corresponding to the forced frequency). As defined in the chapter 8 of *Theoretical and numerical combustion* of Poinsot and Veynante [1] and chapter 8 of *Physique theorique* [2] of Landau and Lifchitz, e is the density of acoustic energy:

$$e = \frac{1}{2}\rho_0 u'^2 + \frac{1}{2}\frac{p'^2}{\rho_0 c_0^2}$$

E is the integral of e over volume and time:

$$E = \frac{1}{T}\frac{1}{V}\int_V \int_T e dt dV$$

At resonance (when the forcing frequency almost reaches the real part of an eigen frequency), E is maximal. Therefore, a sweep in forced frequencies can detect eigen frequencies as peaks of total acoustic energy E . The drawback with total acoustic energy is the impossibility to determine the growth rate of the eigen frequency. Since the sweep concerns only the real frequencies, only the real part of the eigen frequencies can be evaluated with this method. In addition, the eigen frequencies with high imaginary parts are detected with difficulty. Indeed,in such cases, the resonance peak is located very far from the real axis so its height seen from the real axis is very low. As a consequence, the difficulty to detect such a peak is more important.

FluxForcing method

The acoustic flux of the loudspeaker can evaluate the real part of an eigen frequency and give information about its growth rate. The acoustic flux f is defined as:

$$f = p'.u'$$

By calling $\Delta u' = \frac{H}{S}$ the speed induced by the loudspeaker and z_H its local position, the acoustic flux of the loudspeaker f_{LS} corresponds to:

$$f_{LS} = p'(z_H).\Delta u' = p'(z_H).\frac{H}{S}$$

Because of the sinusoidal excitation imposed, F_{LS} -the acoustic flux of loudspeaker averaged on a period of excitation- is calculated instead of the instantaneous flux f_{LS}.

$$F_{LS} = \frac{1}{T}\int_T f_{LS} dt$$

In cases where z_H does not correspond to a pressure node, F_{LS} is maximum at the resonance since $p'(z_H)$ is. Moreover, the sign of F_{LS} indicates if the mode is stable or unstable. It is possible to explain this particularity by using the balance equation of the total acoustic energy:

$$\frac{1}{T}\frac{1}{V}\int_V \int_T \frac{\partial e}{\partial t} dt dV + F_{LS} + \underbrace{\frac{1}{T}\int_T \int_{Surf.} \mathbf{f_{bound.}}.\mathbf{n} dS dt}_{flux\,at\,the\,boundaries} = \underbrace{\frac{1}{T}\int_V \int_T s dt dV}_{source\,term:\,flame}$$

When forcing is applied, a steady harmonic regime is reached where:

$$\frac{1}{T}\frac{1}{V}\int_V \int_T \frac{\partial e}{\partial t} dt dV = 0$$

The stability of the mode depends if the losses at boundary can balance the source term. Thus, the sign of F_{LS} is a measure of the stability of the mode:

$$F_{LS} = \frac{1}{T}\int_V \int_T s dt dV - \frac{1}{T}\int_T \int_{Surf.} \mathbf{f_{bound.}}.\mathbf{n} dS dt$$

With the modelization chosen, the sign of F_{LS} varies as follows:

- $F_{LS} > 0 \Longleftrightarrow$ **unstable mode**: to keep the oscillations at a fixed amplitude, the loudspeaker must evacuate energy ($F_{LS} > 0$).

- $F_{LS} < 0 \Longleftrightarrow$ **stable mode** : to maintain the oscillations, the loudspeaker must inject energy ($F_{LS} < 0$).

Thus, the sign of the imaginary part of an eigen frequency can be obtained by this method but not its value. Likewise "energyforcing", this method detects with difficulty the eigen frequencies with high imaginary parts.

EnergyForcing_dom method

The aim of this method is to evaluate the imaginary part of the eigen frequencies with an "energyforcing". The idea is to find the peaks of energy in the whole complex domain instead of searching them only on the real axis (as in the precedent forcing methods). So, the forced frequencies are now complex (their imaginary part f_i is not necessary zero). However, a global sweep in the whole complex domain can not be performed for computing time reasons. As a consequence, the sweep must concern only frequencies of particulary axis of the complex plan. The strategy retained is the following:

- First, a forcing on the real axis is performed as in the ordinary energyforcing method. The peaks of acoustic energy indicate the real parts of the eigen frequencies.

- Second, the imaginary part of each eigen frequency is evaluated by a forcing on the imaginary axis. The abscissa of this axis is equal to the real part of the eigen frequency detected precedently (figure 1.12).

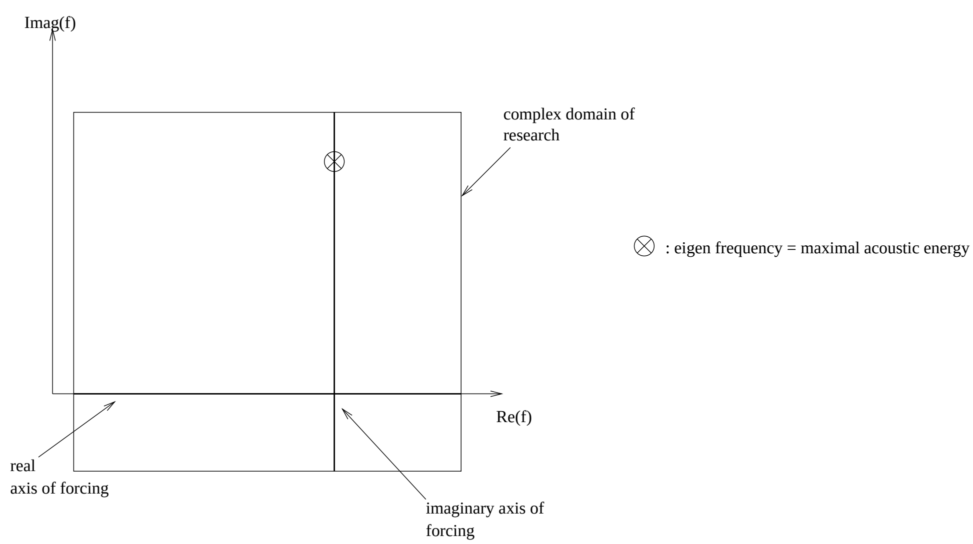


Figure 1.12: complex domain of research with the axis of forcing

To be sure to capture eigen frequencies with large imaginary part, the initial domain is divided into zones where the precedent process is applied. This is illustrated by figure 1.13. Experience has shown that the ideal zone's width is typically $20\,Hz$ for burners similar to gas turbines. Thus, eigen frequencies with high imaginary parts can be detected.

Limitation of the forcing methods

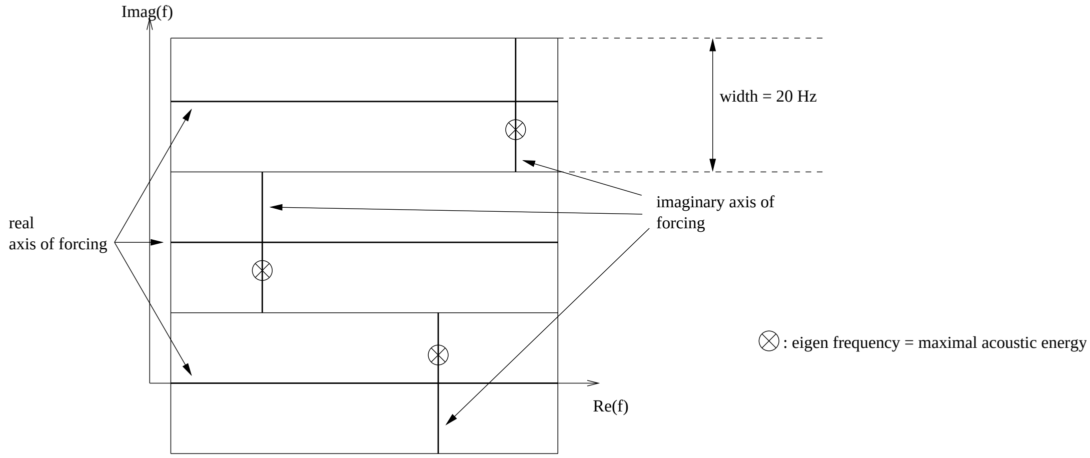


Figure 1.13: division of the complex domain into 3 zones of research

- The "Energyforcing" and "Fluxforcing" methods give graphically the real part of the eigen frequencies. The frequency at which a resonance peak is located is the real part of the eigen frequency. The main drawback of these methods is that they can not give a numerical value of the imaginary part. The height of a peak is not directly linked with the growth rate (imaginary part) of the mode. However, the modes which are very stable or unstable (i.e: with $|Im(f_0)| < 20Hz$) can not appear clearly because of their low resonance peak. Moreover, this peak's height is strongly linked with the position of the loudspeaker. Indeed, if the location of the loudspeaker corresponds to a node of a given mode, the resonance peak of this mode will be lower than the other modes' peak. The principal advantage of these methods is that they give very quickly their results. They are the quickest of all the methods.

- For "energyforcing_dom" method, the results are given numerically at screen as the real part and the imaginary parts of the eigen frequencies. It is much slower than the "Energyforcing" and "Fluxforcing" methods and like them, results of " energyforcing_dom" method can be dependent on the loudspeaker's location.

1.2.4 Hybrid method

This method is named hybrid because its characteristics are borrowed from forced method and modal (or impulsional) method.
This method uses a forcing at an extremity of the configuration (i.e. at the place of the first reflecting boundary R_1). This forcing imposes the amplitude A^+ to a chosen value (typically A^+ =1). So, it is not the same forcing as previously since with a loudspeaker it is the value of an acoustic speed which is imposed.

Principle

A wave A^+ of frequency fixed f is emitted at the entry of the configuration. After travelling in the whole device, this wave comes back as the reflected wave A^- (figure 1.14).

A^- constitutes the response of the device to the monochromatic excitation A^+. It is possible to

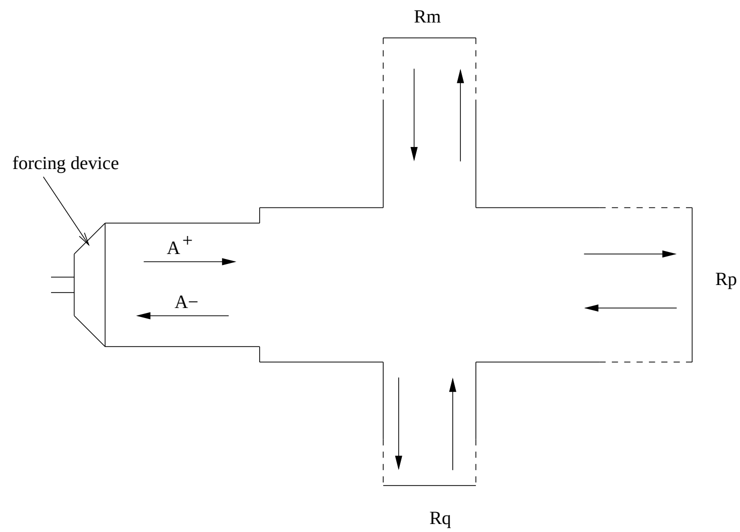


Figure 1.14: Configuration considered in the hybrid method

build an equivalent impedance or even a equivalent reflection coefficient $1/G(\omega)$ such as:

$$A^- = A^+ G(\omega)$$

$G(\omega)$ contains all the jump relations and reflection conditions.

From A^-, it is also possible to build a “virtual wave” A^+_{bis} that would be the wave reflected at the entry R_1. This situation is illustrated by the figure 1.15.

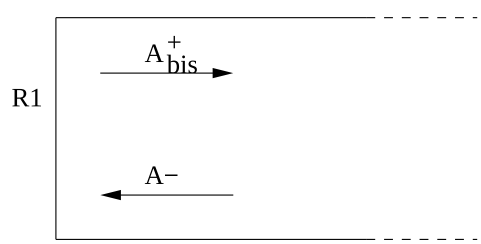


Figure 1.15: Reflection at the entry of the configuration

If the frequency (f) is an eigen frequency (f_{eigen}) then the wave emitted (A^+) and the wave recovered (A^+_{bis}) are in phase. So the relation verified by an eigen frequency (f_{eigen}) is:

$$arg(A^+_{bis}) = arg(A^+)$$

which can be also written:

$$arg(R_1 G(\omega)) = 0 \tag{1.26}$$

since the forced frequency f is real, the relation 1.26 gives the real part of the eigen frequency.

Information about the growth rate can be deduced from the comparison of the amplitudes $|A^+_{bis}|$ and $|A^+|$. Indeed, if the mode is stable, the modulus of the wave $|A^+_{bis}|$ is smaller than the modulus of the wave emitted $|A^+|$. The behaviour is reversed when the mode is unstable. In addition, **for 1D configuration only**, it is possible to evaluate the growth from $|A^+_{bis}|$ and $|A^+|$.

In this situation, it is possible to introduce a time t^* which corresponds to the time between the first emission of the wave and its "first comeback" at the entry. Thus, the imaginary part f_i of the eigen frequency is deduced from the following relation:

$$\frac{|A^+_{bis}|}{|A^+|} = e^{2\pi f_i t^*}$$

$$f_i = \frac{1}{2\pi t^*} ln|R_1.G(\omega_{eigen})| \tag{1.27}$$

with ω_{eigen} the frequency which verifies the relation 1.26.

Limitation of the hybrid method

The hybrid method is well adapted for the 1D cases. In the 1.5D cases, the real part of the eigen frequencies can be found but not the imaginary part. The value of the imaginary part given by the hybrid method in the 1.5D cases is not usable. Only its sign is reliable in such cases.

1.2.5 Mode's shape

It is possible to trace the spatial shape of a mode when its eigen frequency is known. To reach this goal, the amplitude A^+ and A^- in all the tubes must be known since the acoustic pressure $p'(x)$ and acoustic speed $u'(x)$ are evaluated by the formula:

$$p' = A^+ e^{ikz-i\omega t} + A^- e^{-ikz-i\omega t}$$

$$u' = \frac{A^+}{\rho c} e^{ikz-i\omega t} - \frac{A^-}{\rho c} e^{-ikz-i\omega t}$$

with the eigen frequency f such that:

$$k = \frac{\omega}{c} = \frac{2\pi f}{c}$$

The relevant quantities to visualize a mode are

- $Re(u'(x))$
- $Re(p'(x))$
- $arg(u'(x))$
- $arg(p'(x))$

Even if the wave studied is stationary it is clear that for some values of t, the value of $Re(u'(x)$ and $Re(p'(x))$ can be null for all abscissa x in a given tube. In order to avoid this situation it is possible to chosen the value of t in the input file. A step of spatial discretization of the mode is also necessary. These characteristics will be described in chapter 2 and chapter 4.

Calculus of the amplitudes A^+ and A^- in all the tubes

The amplitude A_1^+ of the first tube is arbitrarily set to 1 and the frequency is chosen by the user:

$$A_1^+ = 1 \tag{1.28}$$

All the other amplitudes are deduced from jump-relations and reflection relations. Since A_1^+ is fixed to 1, the number of unknowns is $2n-1$ in a system of n tubes. The number of equations (jump-relations and reflection conditions) is still $2n$. So, an equation can be suppressed in the system. It is the jump-relation for speed of the "intersection 1". Indeed, this equation is common to every configuration.

Thus, the amplitudes are deduced from the resolution of a system of size $2n-1$. This system is composed of, on the one hand, the relations involving A_1^+, and on the other hand, of the other jump-relations and reflection condition as described in the corresponding paragraph of this chapter. The system resulting is of the following form:

$$\mathbf{M}(\omega).\begin{pmatrix} A_1^- \\ A_2^+ \\ A_2^- \\ \vdots \\ A_n^+ \\ A_n^- \end{pmatrix} = X(\omega)$$

where **M** is an $2n-1 \times 2n-1$ matrix and X is a $2n-1$ vector. The coefficients of X are null excepted these stemming from the relations involving A_1^+.

The solving of this system gives all the amplitudes and mode shape can be plotted.

1D mode visualization

Even for a 1_5D configuration, a 1D visualization is possible. The user can select a succession of tubes in which he wants to visualize the mode shape. It is relevant only if the tubes selected are linked. In this case, this succession of tubes constitutes a sort of "path". The features of this path are:

- The number of tubes
- For each tube, the side considered as the origin of the tube according to the direction induced by the order of succession of the tubes.

To illustrate more precisely these points, let's consider the configuration of the figure 1.16.

In this case, a possible "path" is:

- 1 , -
- 3 , -
- 4 , -
- 7 , +

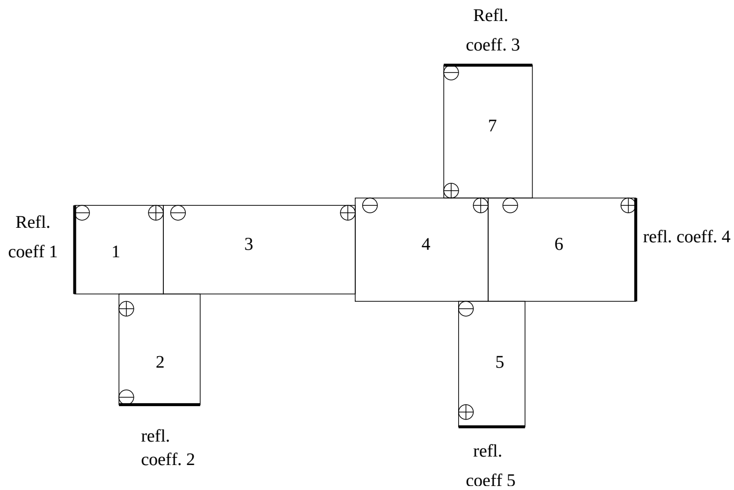


Figure 1.16: General configuration in which eigen modes are plotted

This path is consistent with the configuration of the figure 1.16. If, for instance, (3 ,-) would have been replaced by (3 , +) it would mean that the face "+" of "tube1" was linked with the face "+" of "tube3" which is not the case. It would have been a mistake. So keep in mind that the order of succession of the tube induces a direction. The faces of the tubes selected must be consistent with this direction. The description and the way of using the 1D mode visualization will be discussed in chapter 2 and chapter 4.

3D mode visualization

A 3D visualization of the mode shape is possible by using the software "ensight". This visualization required additional specifications in the input file. In particulary, for each tube, the coordinates X,Y,Z of the centers of the side $-$ and side $+$ must be specified by the user. Once the step of spatial discretization is fixed by the user, soundtube1_5d creates plot3d format files: a mesh file and a solution file. **These files are written in ASCII**. Then, a visualization software able to read plot3d format file can create the configuration's tubes with the spatial discretization step (for the mode) by reading the mesh file. The visualization software makes a mode appear by applying on the mesh the corresponding values of $u'(x)$ or $p'(x)$ contained in the solution file. The selection of the plot3d files' names or the selection between $Re(p'(x)$, $Re(u'(x)$, $arg(p'(x))$, $arg(u'(x))$, $|p'(x)|$ to represent the mode are detailed in chapter 2 and chapter 4.

Chapter 2

Using soundtube1_5D

2.1 Description of the input file

This section deals with the features of the input file. Input arguments are explained one by one, in the order of apparition in the input file.

2.1.1 Edit the general characteristics of the configuration chosen

The first arguments deal with the general features of the configuration in the section "[Geometry]".

```
[Geometry]

TubeNumber=           2
ReflectionNumber=     2
IntersecNumber=       1
```

- "[Geometry]" is the corresponding name of the section.
- "TubeNumber" is the total number of tubes used in the configuration.
- "ReflectionNumber" is the total number of reflective boundaries in the configuration.
- "IntersecNumber" is the total number of intersection in the configuration.

2.1.2 Edit the characteristics of the reflection conditions

After that, the user has to specified the features of each reflection condition. A reflection coefficient is linked to a given side of a given tube of the configuration. Moreover, a reflection coefficient R is always considered as a rational function of the real frequency f:

$$R = \frac{R\_num0 + R\_num1 * f + R\_num2 * f^2 + ...R\_numk * f^k}{R\_denom0 + R\_denom1 * f + ...R\_denomj * f^j}$$

These features are specified in the inputfile as follows:

```
[Reflection2]
TubeReflect=     2 , +
Refl_numorder_denomorder= 0 , 0
R_num0=    -1.0 , 0.0
R_denom0=  1.0 , 0.0
```

- "TubeReflect" indicates the number of the tube concerned with this reflection condition and also specifies which side of the tube is concerned (side + or side -). See part 1.1.2 for information about the orientation of a tube. The first reflection coefficient is particular so the remark2 at the end of this part has to be kept in mind.
- "Refl_numorder_denomorder" indicates the order of the numerator and the denominator. These two numbers are integer. They are separated by a comma, so the remark1 at the end of this part is reminded.
- "R_num0"indicates the value of the first numerator coefficient (zero-th order term). This value is complex (see remark1); that is why there are two numerals: a real part , an imaginary part.
- "R_denom0" indicates the value of the first denominator coefficient (zero-th order term). This value is complex (see remark1).

REMARK 1:

When two arguments , or two elements of an argument, in the input file are separated by a comma (like in "TubeReflect" and "Reflectvalue"), **do not forget the blanks on both sides of the comma !** Otherwise arguments are not correctly taken into account by soundtube1_5d.
For example, for the complex value of "R_num0" it yields:
RIGHT: 1.0 , 0.23
WRONG: 1.0,0.23
WRONG: 1.0, 0.23
WRONG: 1.0 ,0.23

REMARK 2:

The reflection condition's number and the tube's number concerned are independent except for the first reflection condition. The first reflection condition concerns ALWAYS the side"-" of "tube1". So the begining "reflection1" is always as follow:

[Reflection1]
TubeReflect= 1 , -
Refl_numorder_denomorder= *integer , integer*
R_num0= *real , real*
R_denom0= *real , real*
etc...

2.1.3 Edit the characteristics of the tubes

Once all the reflection conditions are filled, the next step is editing the features of the configuration tubes. Again, each tube is numbered and described one by one. For a given tube, its characteristics stored are:

```
[Tube1]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.25
surface=          3E-2
coordxyz_minus=   0.0 , 0.0 , 0.0
coordxyz_plus=    0.25 , 0.0 , 0.0
```

- "temperature" is the mean temperature in the tube (in K degrees)
- "pressure" is the mean pressure in the tube (in Pa)
- "weight" is the mean molecular weight of the gas in the tube (in kg/mol)
- "gamma" is the thermodynamic parameter $\gamma = Cp/Cv$
- "length" is the length of the tube (in m)
- "surface" is the surface of a section of the tube (in m^2)
- "coordxyz_minus" is the X,Y,Z coordinates of the center of $side-$. X,Y,Z are real numbers separetated by a comma (see remark1). "coordxyz_minus" is taken into account by soundtube1_5d only when the option "modeshape3d" is selected (see part 2.1.5). That is to say, "coordxyz_minus" only concerns the 3D visualization of the mode shape (see chapter4).
- "coordxyz_plus" is the X,Y,Z coordinates of the center of $side+$. Similarly to "coordxyz_minus", "coordxyz_plus" plays a role only with "modeshape3d" option.

2.1.4 Edit the characteristics of the intersections

Once all tubes are characterized, the intersections are described. Like reflection conditions and tubes, each intersection is numbered and detailed one by one. An intersection is defined by the values of its parameters n,τ,n_p,τ_p as it was explained in chapter1. Similarly to the reflection coefficient, the parameters n,τ,n_p,τ_p are always considered as rationnal function of the real frequency. For instance, the value of the parameter n is evaluated from the following formula:

$$n = \frac{n\_num0 + n\_num1 * f + n\_num2 * f^2 + ...n\_numk * f^k}{n\_denom0 + n\_denom1 * f + ...n\_denomj * f^j}$$

An intersection is defined by the tubes which it links. Finally, an intersection is also defined by its flame reference tube. Indeed, in the $n-\tau$ model a reference speed is needed. The speed at the point indicated by the "flame reference tube" argument is the reference speed.

These features are specified in the input file by the following way:

```
[Intersection1]
N_numorder_denomorder=    0 , 0
n_num0=                   0.0 , 0.0
n_denom0=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 1.0E-5 , 0.0
tau_denom0=               1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                  0.0 , 0.0
np_denom0=                1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=                1.0E-5 , 0.0
taup_denom0=              1.0 , 0.0

connectTubeNum=        2

connectTube1=          1 , +
connectTube2=          2 , -
flame_ref_tube=        1 , +
```

- "N_numorder_denomorder" indicates the order of the numerator and the denominator for parameter *n*. These two numbers are integer. They are separated by a comma (see remark1).
- "n_num0" indicates the value of the first numerator coefficient (zero-th order term) of parameter *n*. This value is complex (see remark1).
- "n_denom0" indicates the value of the first denominator coefficient (zero-th order term) of of parameter *n*. This value is complex (see remark1).
- "connectTubeNum" corresponds to the total number of tubes linked at this intersection.
- "flame_ref_tube" indicates a tube and a side that is to say a point of the configuration. The speed at this point is the reference speed for the flame of the intersection.

REMARK 3:

Whatever the configuration, there must be at least one intersection (see example of the simple tube in chapter 3: tutorial.

REMARK 4:

The first intersection is particular. Indeed, for this intersection,"connectTube1" must be ALWAYS "Tube1" of the configuration with side "-", that is to say:
connectTube1= 1 , -

2.1.5 Edit the characteristics of the detection methods

Once characteristics of the configuration are edited in the input file, user has to determine, in the section “[Simulation]” of input file, the method to search eigen frequencies. The first stage is to specify in the subsection “option”, the method chosen:

- “modal”: to select modal method.
- “fluxforcing”: to select forcing method with acoustic flux of loud-speaker evaluated.
- “energyforcing”: to select forcing method with total acoustic energy.
- “energyforcing_dom”: to select forcing method with total acoustic energy. In this case the forcing is performed in the whole complex domain and the result is numerical instead of graphical.
- “hybrid”: to select hybrid method.
- “modeshape”: to select the 1D visualization of the mode shape.
- “modeshape3d”: to select the 3D visualization of the mode shape.

The principle of these methods is reminded in chapter 1 of this help guide. The option for mode shape visualization are also detailed in the chapter4. Thus, the begining of the section “[Simulation]” looks like in the following example where the modal is selected:

```
[Simulation]

option=     modal
```

The following arguments concern only the modal option and the “energyforcing_dom” option. Therefore, if another option is selected, these arguments are not taken into account by the code.

```
LowerLeft_domain=       1.0 , -100.0
UpperRight_domain=      2000.0 , 100.0
```

- “LowerLeft_domain” corresponds to the lower left point of the domain of eigen frequencies research. It is a complex value so keep in mind the remark1 about this kind of input file argument.
- “UpperRight_domain” corresponds to the upper right point of the domain of eigen frequencies research. It is a complex value so keep in mind the remark1 about this kind of input file argument.

The figure 2.1 illustrates this point.

The following arguments concern only the options fluxforcing and energyforcing methods:

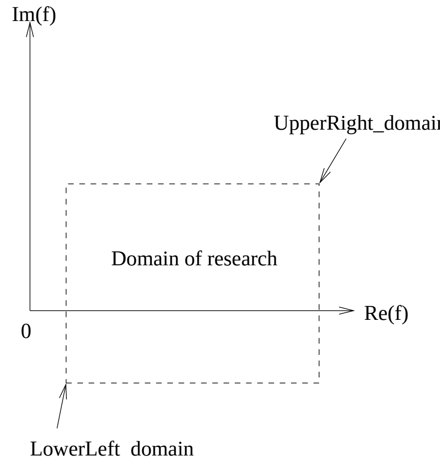


Figure 2.1: Complex domain for the research of eigen frequencies

```
LS_tube=                2
LS_relative_position=   0.15
LS_amplitude=           1.0
LS_phase=               0.0
```

- "LS_tube" is the number of the tube where the side-mounted loudspeaker is located.
- "LS_relative_position" is the position of the loudspeaker in the "LS_tube". More precisely, it is the value (in m) of the local abscissa of the loudspeaker in the tube.
- "LS_amplitude" corresponds to the amplitude of the loudspeaker (in m^3/s). Loudspeaker is a source of flow. Generally this value is set to 1.0.
- "LS_phase" corresponds to the loudspeaker's phase (in *rad*).

The following arguments concern fluxforcing and energyforcing methods but also and hybrid method:

```
LS_beginfreq=     1.0
LS_endfreq=       2000.0
LS_resol_forcing= 2.0
```

- "LS_beginfreq" corresponds to the value of the frequency (in Hz) at which the forcing starts.
- "LS_endfreq" corresponds to the value of the frequency (in Hz) at which the forcing stops.
- "LS_resol_forcing" corresponds to the resolution (in Hz) of the forcing. In the "energyforcing_dom", the ideal resolution (i.e: the resolution which provides the better results the more quickly) is $2\,Hz$. For the other forcing methods and hybrid method, this resolution is directly linked with the precision of the result. In these methods a resolution of $1\,Hz$ is a good compromise between rapidity and precision.

REMARK 5:

Real numbers in the input file should be written with a decimal, because their format is ”double” in the code. So, instead of writting 1, user should write 1.0 .

2.1.6 Edit the characteristics of the mode shape visualization

The principle of the calculus of the eigen mode is reminded in chapter1. The way of visualizing the mode shape is detailed in chapter4. In the input file, the different parameters necessary to these methods (modeshape and modeshape3d) are at the end of the section “[Simulation]” as in the following example:

```
modeshape_freq=        448.0 , 0.0
modeshape_resol=       0.01
modeshape_time=        0.017
modeshape_TubeNum=     2
modeshape_Tube1=       1 , -
modeshape_Tube2=       2 , -
```

- “modeshape_freq” specifies the complex frequency (in Hz) of the mode visualized. Remember the remark1 concerning arguments separated by comma.
- “modeshape_resol” corresponds to the resolution (in m). It can be seen as a “spatial discretization”. Since calculus are 1D in each tube, this discretization stands for the discretization along the direction of the tube.
- “modeshape_time” is the time at which mode shape is visualized.

The following arguments concern only the modeshape option (not modeshape3d method):

- “modeshape_TubeNum” is the total number of the tube in which the modes are visualized. All this tubes constitutes a sort of “path”and the modes are visualized along this path.
- “modeshape_Tube i” is a parameter which specified the i^{th} tube of the path, that is to say its number and its side. Remember the remark1 concerning arguments separated by comma.

2.1.7 Edit the characteristics of the output files

Even if hybrid, modal and energyforcing_dom methods give their results at screen , the results are also stored in output files. For energyforcing and fluxforcing methods, this storage is the only output. For modeshape and modeshape3d methods, the values of the $p'(x)$ and $u'(x)$ are stored in an output file. The names of the corresponding files are specified in the section ”Output” of the input file as in the following example:

```
[Output]
```

```
Resultfile=               in-out/new1tube.res
geometryfile=             in-out/new1tube_geom.dat
fluxResultfile=           in-out/new1tube_FluxForcing.dat
totenergyResultfile=      in-out/new1tube_totEForcing.dat
modeshapefile=            in-out/modes/new1tube.mode
xmgrcmdfile=              dir_moulinette/xmgrmodecmd.dat
meshplot3d_file=          in-out/modes/tubemesh3D.g
solplot3d_file=           in-out/modes/tubesol3D.q
```

All the paths specified for all these files are local paths (from soundtube1_5d's location of execution).

- "Resultfile" is the file where the results of modal, energyforcing_dom and hybrid methods are stored.
- "geometryfile" is the file where the characteristics of the configuration are stored.
- "fluxResultfile" is the file where the results of fluxforcing method are stored.
- "totenergyResultfile" is the file where the results of energyforcing method are stored.
- "modeshapefile" is the file where the results of modeshape method are stored.
- "xmgrcmdfile" is a file necessary to the tool of 1D visualization of the mode shape
- "meshplot3d_file" is the plot3d format file which contains the virtual mesh used with modeshape3d method.
- "solplot3d_file" is the plot3d format file which contains the variables visualized with modeshape3d method.

2.2 Execution of soundtube1_5d and visualization of results

2.2.1 Determination of eigen frequencies by using modal method

WARNING:

This method is not well adapted to all 1_5D configurations. This method has to be use on the simple 1_5D cases (with maximum 5 or 6 tubes). Otherwise this method can fail or not give all the eigen frequencies.

Let's begin with the using of the modal method:

- After editing the characteristics of the configuration in the input file, the user has to select the "modal" option in the argument option of the section "[simulation]" of the input file:

```
[Simulation]

option=     modal
```

- After that, user has to determine the domain of research of eigen frequencies (see figure 2.1), that is to say the values of LowerLeft_domain and UpperRight_domain in the input file.
- Once these stages are achieved, user just has to execute soundtube1_5D by the following way:

  soundtube1_5d.exe *input file name*

Visualization of the results:

- Eigen frequencies appear at screen. They are also stored in "Resultfile" of input file. Eigen frequencies are given with their real part Re(f) and imaginary part (growth rate) Im(f). They are sorted by increasing real part order.

2.2.2 Determination of eigen frequencies by using energyforcing method and fluxforcing method

- After editing the characteristics of the configuration in the input file, user has to select the "fluxforcing" option or "energyforcing" option in the argument "option" of the section "[Simulation]" of the input file:

```
[Simulation]

option=     fluxforcing
```

- After that, user has to specify the value of "LS_tube", "LS_relative_position", "LS_amplitude", "LS_phase", the characteristics of the loudspeaker. User has also to determine the begining forced frequency ("LS_beginfreq"), the last forced frequency ("LS_endfreq") and the resolution ("LS_resol_forcing").
- Once these stages are achieved, user just have to execute soundtube1_5d by the following way:

  soundtube1_5d.exe *input file name*

Visualization of the results:

- Results are stored in "fluxResultfile" or "totenergyResultfile", specified in the input file. They can be easily visualized by using the "macros" "xmgrFluxForcing" or "xmgrEnergyForcing" (see chapter4). By executing "xmgrFluxForcing" or "xmgrEnergyForcing", the corresponding result appears at screen:

  xmgrFluxForcing *input file name*

Details about "xmgrFluxforcing" are available in chapter 4 dealing with the "macros".

REMARK 6:

The argument "LS_beginfreq" must have a non zero value, ALWAYS!

2.2.3 Determination of eigen frequencies by using energyforcing_dom method

- After editing the characteristics of the configuration in the input file, user has to select the ”energyforcing_dom” option in the argument “option” of the section “[Simulation]” of the input file:

```
[Simulation]

option=     energyforcing_dom
```

- After that, user has to specify the value of “LS_tube”, “LS_relative_position”, “LS_amplitude”, “LS_phase”, “LS_resol_forcing” the characteristics of the loud-speaker. The complex domain of research is specified in the option “LowerLeft_domain” and “UpperRight_domain”.
- Once these stage are achieved, user just have to execute soundtube1_5d by the following way:

  soundtube1_5d.exe *input file name*

Visualization of the results:

- Eigen frequencies appear at screen. They are also stored in “Resultfile” of input file. Eigen frequencies are given with their real part Re(f) and imaginary part (growth rate) Im(f). They are sorted by increasing real part order.

2.2.4 Determination of eigen frequencies by using hybrid method

- After editing the characteristics of the configuration in the input file, user has to select the ”hybrid” option in the argument “option” of the section “[Simulation]” of the input file:

```
[Simulation]

option=     hybrid
```

- After that, user has to specify the begining forced frequency (“LS_beginfreq”, see the remark6), the last forced frequency (“LS_endfreq”) and the resolution (“LS_resol_forcing”).
- Once these stages are achieved, user just have to execute soundtube1_5d by the following way:

  soundtube1_5d.exe *input file name*

Visualization of results:

- As in the case of the ”modal” option, results appear at screen. Results are also stored in “Resultfile” specified in the input file.

2.2.5 Determination of eigen modes shape with “modeshape” method

- After editing the characteristics of the configuration in the input file, user has to select the ”modeshape” option in the argument “option” of the section “[Simulation]” of the input file:

```
[Simulation]

option=     modeshape
```

- After that, user has to specify the eigen frequency chosen in the argument “modeshape_freq”. User has also to specify the parameters “modeshape_resol”, “modeshape_time”, “modeshape_TubeNum” and the all the “modeshape_Tube”.
- Once these stages are achieved, user just has to execute soundtube1_5d by the following way:

  soundtube1_5d.exe *input file name*

Visualization of the results:

- Results are stored in “modeshapefile”, specified in the input file. By executing the macro “xmgrmodeshape”, the mode shape of the frequency retained can be visualized in terms of acoustic pressure and acoustic speed:

  xmgrmodeshape *input file name*

Details about “xmgrmodeshape” are available in chapter 4 dealing with the “macros”.

2.2.6 Determination of eigen modes shape with “modeshape3d” method

- After editing the characteristics of the configuration in the input file, user has to select the ”modeshape” option in the argument “option” of the section “[Simulation]” of the input file:

```
[Simulation]

option=     modeshape3d
```

- After that, user has to specify the eigen frequency chosen in the argument “modeshape_freq”. User has also to specify the parameters “modeshape_resol”, “modeshape_time”.
- Once these stages are achieved, user just has to execute soundtube11_5d by the following way:

  soundtube1_5d.exe *input file name*

Visualization of the results:

- Results are stored in plot3d format files “meshplot3d_files” and “solplot3d_file”, specified in the input file. The 3d visualization can be done with ENSIGHT by reading these two files.

Details about “modeshape3d” are available in chapter 4.

Chapter 3

Basic examples of soundtube1_5d

3.1 Simple Tube

3.1.1 General features and analytical results

It corresponds to the most basic example since only one tube is considered with reflecting boundaries (see figure 3.1).

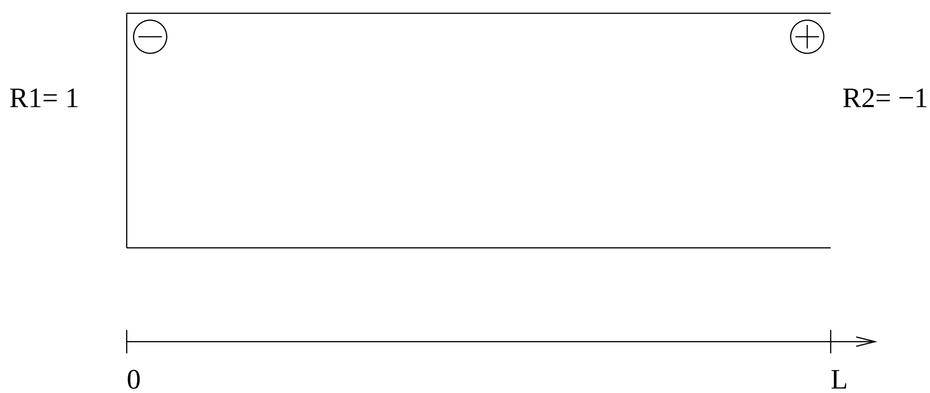


Figure 3.1: Tube without flame

Mean temperature, mean pressure, length and section area are respectively named T, P, L and S.

Eigen frequencies values of the configuration

According to these hypothesis and the definition of a reflection coefficient, the reflection relations are in the current case:

- 1^{st} reflection condition:
$$R_1 = 1 \Longrightarrow A_1^+ = A_1^- \quad (3.1)$$

- 2^{nd} reflection condition:
$$R_2 = -1 \Longrightarrow A_2^+ e^{i.2.k.L} = -A_2^- \quad (3.2)$$

In this trivial example, no jump-relations are needed. Combining the relations 3.1 and 3.2 leads to:

$$R_1.e^{i.2.k.L} = R_2$$

The real and imaginary parts (growth rate) of eigen frequencies can be easily deduced. For the m^{th} mode, the results are the following:

$$f_r = \frac{c}{2\pi}.k_r = \frac{c}{4\pi L}.(arg(R_2/R_1) + 2.m.\pi) \tag{3.3}$$

$$f_i = \frac{c}{2\pi}.k_i = \frac{c}{4\pi L}.ln|\frac{R_1}{R_2}| \tag{3.4}$$

In this case, the fundamental mode ($m = 1$) is called “quarter wave mode” since the tube length L corresponds to the quarter of its wave length. Likewise, the second mode ($m = 2$) is called “three quarter wave mode”.

Features of the configuration

To compare analytics with soundtube1_5d results, the following features are chosen:

- $T = 2000\,K$
- $P = 1.013\,bars$
- $S = 0.03\,m^2$
- $W = 0.02897\,kg/mol$
- $\gamma = 1.4$
- $L = 0.5\,m$

3.1.2 soundtube1_5D input file

```
[Geometry]

TubeNumber=          2
ReflectionNumber=    2
IntersecNumber=      1


[Reflection1]
TubeReflect=                 1 , -
Refl_numorder_denomorder= 0 , 0
R_num0=                      1.0 , 0.0
R_denom0=                    1.0 , 0.0

[Reflection2]
TubeReflect=      2 , +
Refl_numorder_denomorder= 0 , 0
```

```
R_num0=                  -1.0 , 0.0
R_denom0=                1.0 , 0.0

[Tube1]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.25
surface=          3E-2
coordxyz_minus=   0.0 , 0.0 , 0.0
coordxyz_plus=    0.25 , 0.0 , 0.0

[Tube2]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.25
surface=          3E-2
coordxyz_minus=   0.25 , 0.0 , 0.0
coordxyz_plus=    0.50 , 0.0 , 0.0

[Intersection1]
N_numorder_denomorder=    0 , 0
n_num0=                  0.0 , 0.0
n_denom0=                1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                1.0E-5 , 0.0
tau_denom0=              1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                 0.0 , 0.0
np_denom0=               1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=               1.0E-5 , 0.0
taup_denom0=             1.0 , 0.0

connectTubeNum=        2

connectTube1=          1 , +
connectTube2=          2 , -
flame_ref_tube=        1 , +

[Simulation]

option=    modal
```

```
LowerLeft_domain=      1.0 , -20.0
UpperRight_domain=     2000.0 , 100.0

LS_tube=               1
LS_relative_position=  0.12
LS_amplitude=          1.0
LS_phase=              0.0
LS_beginfreq=          1.0
LS_endfreq=            2000.0
LS_resol_forcing=      2.0

modeshape_freq=        1344.656 , 0.0
modeshape_resol=       0.01
modeshape_time=        0.019
modeshape_TubeNum=     2
modeshape_Tube1=       1 , -
modeshape_Tube2=       2 , -

[Output]

Resultfile=              in-out/new1tube.res
geometryfile=            in-out/new1tube_geom.dat
fluxResultfile=          in-out/new1tube_FluxForcing.dat
totenergyResultfile=     in-out/new1tube_totEForcing.dat
modeshapefile=           in-out/modes/new1tube.mode
xmgrcmdfile=             dir_moulinette/xmgrmodecmd.dat
meshplot3d_file=         in-out/modes/tubemesh3D.g
solplot3d_file=          in-out/modes/tubesol3D.q
```

3.1.3 soundtube1_5D results and comparison with analytical results

Modal method

mode1	Re(f):4.48218696620299e+02	Im(f):-4.48704699933630e-14
mode2	Re(f):1.34465608986090e+03	Im(f):-3.27711610188750e-14

Hybrid method

mode1_5D[1]:	Re(f)=4.482187e+02	Im(f)=0.000000e+00
mode1_5D[2]:	Re(f)=1.344656e+03	Im(f)=0.000000e+00

Energyforcing_dom method

mode 1:	Re(f)=4.482187e+02	Im(f)=1.301649e-14
mode 2:	Re(f)=1.344656e+03	Im(f)=-5.951374e-15

Energyforcing method

The result of the “energyforcing”option is illustrated by the figure 3.2.

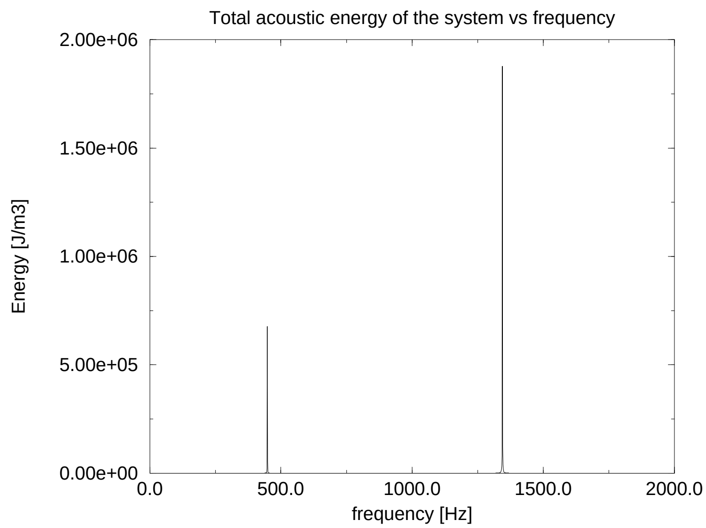


Figure 3.2: acoustic energy evaluated with “energyforcing” in the case of the simple tube without combustion

Analytical results

In this situation, the relations 3.3 and 3.4 give the following results:
mode 1: Re(f)=f_r=448.2187 Im(f)=f_i=0
mode 2: Re(f)=f_r=1344.656 Im(f)=f_i=0

3.2 Simple Tube with a variable reflection coefficient

3.2.1 General features and analytical results

Again, the situation of a simple tube is considered. However, the output reflection coefficient (R_2) is variable in frequency as illustrated by the figure 3.3.

Mean temperature, mean pressure, length and section area are respectively named T, P, L and S.

Eigen frequencies values of the configuration

The precedent results (in the case of the simple tube) are reminded:

- 1^{st} reflection condition:
$$R_1 = 1 \Longrightarrow A_1^+ = A_1^- \tag{3.5}$$
- 2^{nd} reflection condition:
$$R_2 = R_2(f) \Longrightarrow A_2^+ e^{i.2.k.L} = R_2(f).A_2^- \tag{3.6}$$

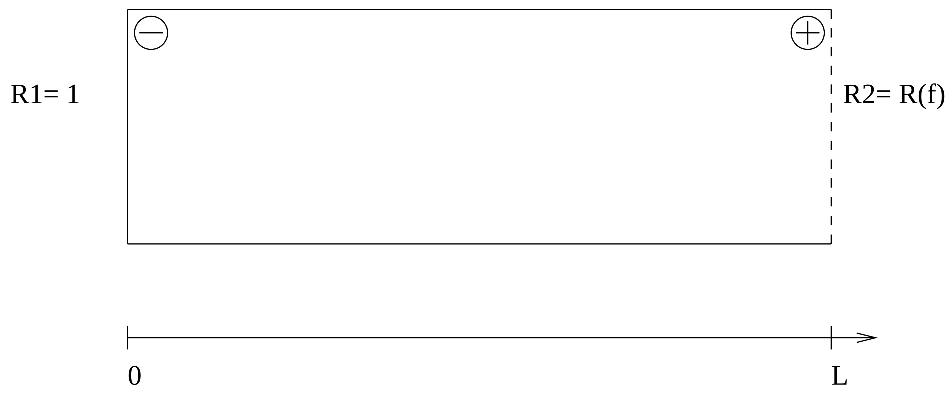


Figure 3.3: Simple tube with a variable output reflection coefficient

Hence:

$$R_1.e^{i.2.k.L} = R_2$$

The real and imaginary parts (growth rate) of eigen frequencies can be easily deduced:

$$f_r = \frac{c}{2\pi}.k_r = \frac{c}{4\pi L}.(arg(R_2(f)/R_1) + 2.n.\pi) \tag{3.7}$$

$$f_i = \frac{c}{2\pi}.k_i = \frac{c}{4\pi L}.ln|\frac{R_1(f)}{R_2}| \tag{3.8}$$

The function $R_2(f)$ is chosen to vary continuously and "smoothly" with only the real part of f. From $1\,Hz$ to $2000\,Hz$, $R_2(f)$ varies from -1 to -0.84. The exact formula chosen is as follows:

$$R_2(f) = \frac{f_r^2}{\alpha} - 1 \tag{3.9}$$

with $\alpha = 25 \times 10^6$. So, in the range $[1\,Hz, 2000\,Hz]$, $R_2(f)$ is a negative number as it can be seen in the figure 3.4.

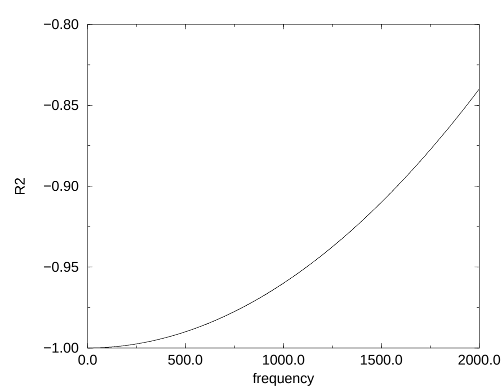


Figure 3.4: Evolution of the Reflection coefficient with frequency

The eigen frequencies can be now evaluated. The m^{th} mode of the configuration is:

$$f_r = \frac{c}{4L}.(1 + 2.m) \tag{3.10}$$

$$f_i = \frac{-c}{4\pi L}.ln(1 - \frac{c^2(1+2.m)^2}{16\alpha L^2}) \quad (3.11)$$

In this case, the real part of the eigen frequency is not modified in comparison with the situation of the simple tube. On the other hand, the growth rate (or imaginary part of the eigen frequency) is not zero anymore .

Features of the configuration

To compare analytics with soundtube1_5d results, the following features are chosen:

- $T = 2000\,K$
- $P = 1.013\,bars$
- $S = 0.03\,m^2$
- $W = 0.02897\,kg/mol$
- $\gamma = 1.4$
- $L = 0.5\,m$

3.2.2 soundtube1_5D input file

```
[Geometry]

TubeNumber=           2
ReflectionNumber=     2
IntersecNumber=       1


[Reflection1]
TubeReflect=                1 , -
Refl_numorder_denomorder= 0 , 0
R_num0=                     1.0 , 0.0
R_denom0=                   1.0 , 0.0

[Reflection2]
TubeReflect=                2 , +
Refl_numorder_denomorder= 2 , 0
R_num0=                     -1.0 , 0.0
R_num1=                     0.0 , 0.0
R_num2=                     4E-8 , 0.0
R_denom0=                   1.0 , 0.0

[Tube1]
temperature=       2000.0
pressure=          1.013E5
weight=            0.02897
```

```
gamma=            1.4
length=           0.25
surface=          3E-2
coordxyz_minus=   0.0 , 0.0 , 0.0
coordxyz_plus=    0.25 , 0.0 , 0.0

[Tube2]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.25
surface=          3E-2
coordxyz_minus=   0.25 , 0.0 , 0.0
coordxyz_plus=    0.50 , 0.0 , 0.0

[Intersection1]
N_numorder_denomorder=    0 , 0
n_num0=                  0.0 , 0.0
n_denom0=                1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                1.0E-5 , 0.0
tau_denom0=              1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                 0.0 , 0.0
np_denom0=               1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=               1.0E-5 , 0.0
taup_denom0=             1.0 , 0.0

connectTubeNum=        2

connectTube1=          1 , +
connectTube2=          2 , -
flame_ref_tube=        1 , +

[Simulation]

option=     modal

LowerLeft_domain=      1.0 , -100.0
UpperRight_domain=     2000.0 , 100.0

LS_tube=               1
LS_relative_position=  0.12
LS_amplitude=          1.0
```

```
LS_phase=             0.0
LS_beginfreq=         1.0
LS_endfreq=           2000.0
LS_resol_forcing=     2.0

modeshape_freq=       1344.656 , 0.0
modeshape_resol=      0.001
modeshape_time=       0.019
modeshape_TubeNum=    2
modeshape_Tube1=      1 , -
modeshape_Tube2=      2 , -

[Output]

Resultfile=             in-out/ex1R1R2.res
geometryfile=           in-out/ex1R1R2.dat
fluxResultfile=         in-out/ex1R1R2FluxForcing.dat
totenergyResultfile=    in-out/ex1R1R2totEForcing.dat
modeshapefile=          in-out/modes/ex1R1R2.mode
xmgrcmdfile=            dir_moulinette/xmgrmodecmd.dat
meshplot3d_file=        in-out/modes/ex1R1R2mesh3.g
solplot3d_file=         in-out/modes/ex1R1R2sol3D.q
```

3.2.3 soundtube1_5D results and comparison with analytical results

Modal method

mode1	Re(f):4.48218696620299e+02	Im(f):1.15114727598094e+00
mode2	Re(f):1.34465608986090e+03	Im(f):1.07108118926279e+01

Hybrid method

mode[1]:	Re(f)=4.482187e+02	Im(f)=1.151147e+00
mode[2]:	Re(f)=1.344656e+03	Im(f)=1.071081e+01

Energyforcing_dom method

mode 1:	Re(f)=4.482187e+02	Im(f)=1.151147e+00
mode 2:	Re(f)=1.344655e+03	Im(f)=1.070862e+01

Fluxforcing method

The result of the “fluxforcing” method is illustrated by the figure 3.5.

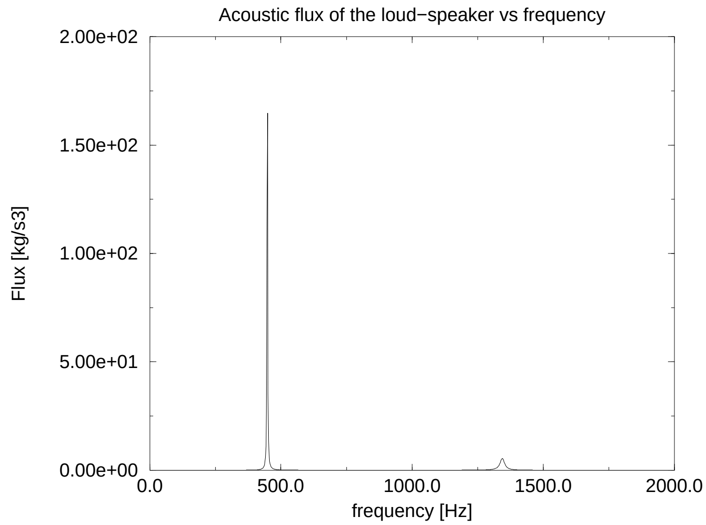


Figure 3.5: acoustic flux at the loudspeaker evaluated with “fluxforcing” in the case of a tube with a variable reflection coefficient

Analytical results

In this situation, the relations 3.10 and 3.11 give the following results:

mode 1: Re(f)=f_r=448.2187 Im(f)=f_i=1.151147
mode 2: Re(f)=f_r=1344.656 Im(f)=f_i=10.71081

3.3 Tube with flame

3.3.1 General features and analytical results

The configuration of the figure 3.6 is considered:

To find more easily an analytical solution, the following simplifications are applied:

- Same mean temperature in the two tubes:

$$T_1 = T_2 = T$$

- Same section in the two tubes:

$$S_1 = S_2 = S$$

- Same mean pressure in the two tubes:

$$P_1 = P_2 = P$$

- Same length for the two tubes:

$$l_1 = l_2 = L/2$$

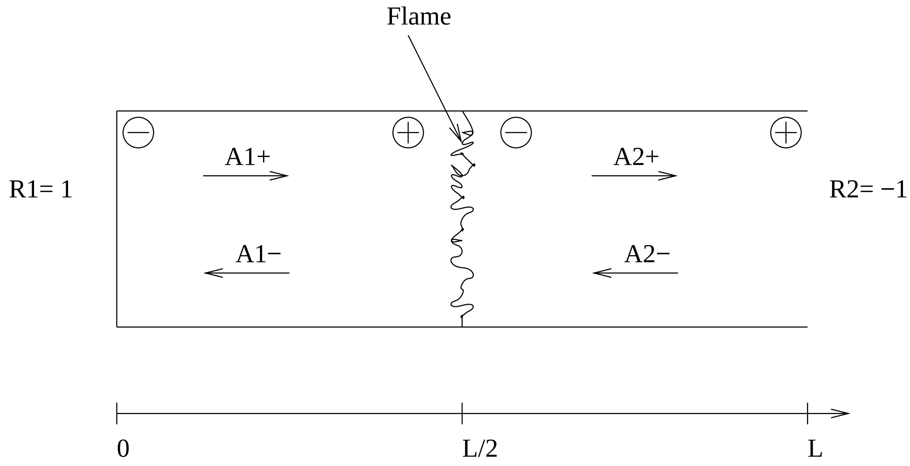


Figure 3.6: Tube with a flame, without heat release and without a fuel line

Reflection relations

According to these hypothesis and the definition of a reflection coefficient, the reflection relations are in the current case:

- 1st reflection condition:

$$R_1 = 1 \Longrightarrow A_1^+ = A_1^- \tag{3.12}$$

- 2nd reflection condition:

$$R_2 = -1 \Longrightarrow A_2^+ e^{i.k.L} = -A_2^- \tag{3.13}$$

Jump-relations for pressure

This relation in the current case is:

$$p_1'(L/2) = p_2'(L/2) \Longrightarrow A_1^+.e^{i.k.L/2} + A_1^-.e^{-i.k.L/2} = A_2^+ + A_2^- \tag{3.14}$$

Jump-relation for speed

The general jump-relation for speed is applied in the current case. It is reminded that the orientation of each tube is essential in this relation.

$$S.u_1'(L/2) + S.u_1'(L/2).n.e^{i\omega\tau} - S.u_2'(L/2) = 0$$

By introducing the "Riemann invariants", it yields:

$$(A_1^+.e^{i.k.L/2} - A_1^-.e^{-i.k.L/2}).(1 + ne^{i\omega\tau}) - A_2^+ + A_2^- = 0 \tag{3.15}$$

Characteristic function and results

By combining the relations 3.12, 3.13, 3.14 and 3.15 it yields:

$$cos(k.L) = ne^{i.\omega.\tau} sin^2(k.L/2) \tag{3.16}$$

Here, another hypothesis is needed to close the calculus: since no heat release is taken into account, the value of n is very small ($n << 1$). As a consequence, the value f of eigen frequencies with combustion is close to their value without combustion f_0 (example 1 of this help guide). So, The Taylor expansion of 3.16 can be applied with $k = k_0 + \delta k$ and $|\delta k| << |k_0|$.

For the first mode (quarter wave mode, $k_0 L = \frac{\pi}{2}$):

$$\Re(\delta f) = -\frac{n.c}{4\pi L} cos(\omega_0 \tau) \tag{3.17}$$

$$\Im(\delta f) = -\frac{n.c}{4\pi L} sin(\omega_0 \tau) \tag{3.18}$$

Similarly, for the second mode (three-quarter wave mode, $k_0 L = \frac{3\pi}{2}$):

$$\Re(\delta f) = \frac{n.c}{4\pi L} cos(\omega_0 \tau) \tag{3.19}$$

$$\Im(\delta f) = \frac{n.c}{4\pi L} sin(\omega_0 \tau) \tag{3.20}$$

Features of the configuration

To compare analytics with soundtube1_5d results, the following features are chosen:

- $T = 2000\,K$
- $P = 1.013\,bars$
- $S = 7 \times 10^{-4}\,m^2$
- $W = 0.02897\,kg/mol$
- $\gamma = 1.4$
- $L = 0.5\,m$
- $n = 0.01$
- $\tau = 1.5 \times 10^{-4}\,s$

3.3.2 soundtube1_5D input file

```
[Geometry]

TubeNumber=           2
ReflectionNumber=     2
IntersecNumber=       1

[Reflection1]
TubeReflect=     1 , -
Refl_numorder_denomorder= 0 , 0
R_num0=                   1.0 , 0.0
```

```
R_denom0=                 1.0 , 0.0

[Reflection2]
TubeReflect=     2 , +
Refl_numorder_denomorder= 0 , 0
R_num0=                   -1.0 , 0.0
R_denom0=                 1.0 , 0.0

[Tube1]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.25
surface=          3E-2
coordxyz_minus=   0.0 , 0.0 , 0.0
coordxyz_plus=    0.25 , 0.0 , 0.0

[Tube2]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.25
surface=          3E-2
coordxyz_minus=   0.25 , 0.0 , 0.0
coordxyz_plus=    0.50 , 0.0 , 0.0

[Intersection1]
N_numorder_denomorder=    0 , 0
n_num0=                   0.01 , 0.0
n_denom0=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 1.5E-4 , 0.0
tau_denom0=               1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                  0.0 , 0.0
np_denom0=                1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=                0.0 , 0.0
taup_denom0=              1.0 , 0.0


connectTubeNum=        2

connectTube1=          1 , +
connectTube2=          2 , -
```

```
flame_ref_tube=        1 , +

[Simulation]

option=        modeshape3d

LowerLeft_domain=      1.0 , -20.0
UpperRight_domain=     2000.0 , 100.0

LS_tube=               2
LS_relative_position=  0.1
LS_amplitude=          1.0
LS_phase=              0.0
LS_beginfreq=          1.0
LS_endfreq=            2000.0
LS_resol_forcing=      2.0

modeshape_freq=        446.9 , -0.58
modeshape_resol=       0.001
modeshape_time=        0.019
modeshape_TubeNum=     2
modeshape_Tube1=       1 , -
modeshape_Tube2=       2 , -

[Output]

Resultfile=              in-out/tubef.res
geometryfile=            in-out/tubef_geom.dat
fluxResultfile=          in-out/tubef_FluxForcing.dat
totenergyResultfile=     in-out/tubef_totEForcing.dat
modeshapefile=           in-out/modes/tubef.mode
xmgrcmdfile=             xmgrmodecmd.dat
meshplot3d_file=         in-out/modes/tubefmesh3D.g
solplot3d_file=          in-out/modes/tubefsol3D.q
```

3.3.3 soundtube1_5D results and comparison with analytical results

Modal method

mode1	Re(f):4.46920708024079e+02	Im(f):-5.78369560587885e-01
mode2	Re(f):1.34508718499304e+03	Im(f):1.35587820432314e+00

hybrid method

mode1_5D[1]:	Re(f)=4.469214e+02	Im(f)=-5.780593e-01
mode1_5D[2]:	Re(f)=1.345088e+03	Im(f)=1.357622e+00

Energyforcing_dom method

mode 1:	Re(f)=4.469207e+02	Im(f)=-5.783696e-01
mode 2:	Re(f)=1.345087e+03	Im(f)=1.355878e+00

Fluxforcing method

The result of the “fluxforcing” option is available in figure 3.7.

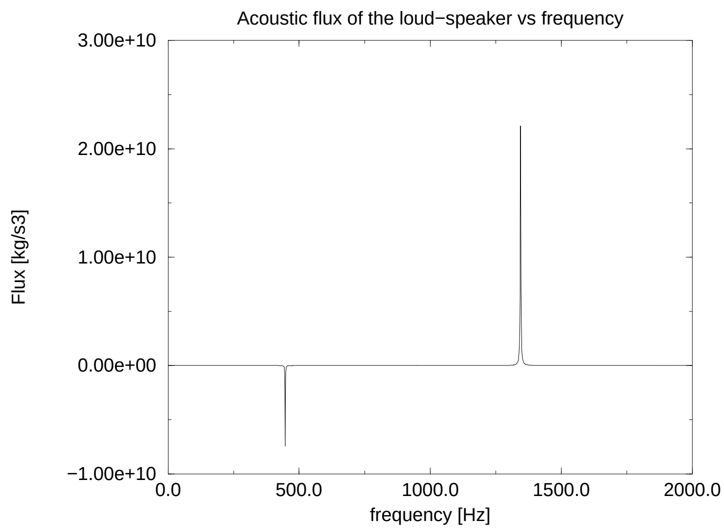


Figure 3.7: acoustic flux of the loudspeaker evaluated with “fluxforcing”. Case of the simple tube with a flame, without heat release and without fuel line

Analytical results

The modes without combustion are the modes of the example 1, that is to say:
$f_{01} = 448.2187\,Hz$ and Im(f_{01})=0
$f_{02} = 1344.656\,Hz$ and Im(f_{02})=0
In this situation, the relations 3.17, 3.18, 3.19, 3.20 give the following results:

- mode 1:
  $\Re(\delta f)$ =-1.3013 so Re(f)=446.9174 Hz
  $\Im(\delta f)$ =-0.5849 so Im(f)=-0.5849 Hz

- mode 2:
  $\Re(\delta f)$=0.4264 so Re(f)=1345.082 Hz
  $\Im(\delta f)$=1.3615 so Im(f)=1.3615 Hz

3.4 Tube with a flame and with a fuel line

3.4.1 General features and analytical results

Again, the configuration of the figure 3.6 is considered. However, a fuel line is added.

To find more easily an analytical solution, the following simplifications are applied:

- Same mean temperature in the two tubes:

$$T_1 = T_2 = T$$

- Same section in the two tubes:

$$S_1 = S_2 = S$$

- Same mean pressure in the two tubes:

$$P_1 = P_2 = P$$

- Same length for the two tubes:

$$l_1 = l_2 = L/2$$

Reflection relations

According to these hypothesis and the definition of a reflection coefficient, the reflection relations are in the current case:

- 1^{st} reflection condition:

$$R_1 = 1 \Longrightarrow A_1^+ = A_1^- \tag{3.21}$$

- 2^{nd} reflection condition:

$$R_2 = -1 \Longrightarrow A_2^+ e^{i.k.L} = -A_2^- \tag{3.22}$$

Jump-relations for pressure

This relation in the current case is:

$$p_1'(L/2) = p_2'(L/2) \Longrightarrow A_1^+.e^{i.k.L/2} + A_1^-.e^{-i.k.L/2} = A_2^+ + A_2^- \tag{3.23}$$

Jump-relation for speed

The general jump-relation for speed is applied in the current case. It is reminded that the orientation of each tube is essential in this relation.

$$S.u_1'(L/2) + S.u_1'(L/2).n.e^{i\omega\tau} + S.p_1'(L/2).n_p.e^{i\omega\tau_p} - S.u_2'(L/2) = 0$$

By introducing the “Riemann invariants”, it yields:

$$A_1^+.e^{i.k.L/2}[(1+ne^{i\omega\tau}) + \lambda n_p e^{i\omega\tau_p}] - A_1^-.e^{-i.k.L/2}.[(1+ne^{i\omega\tau}) - \lambda n_p e^{i\omega\tau_p}] - A_2^+ + A_2^- = 0 \tag{3.24}$$

with $\lambda = \rho c$.

Characteristic function and results

By combining the relations 3.21, 3.22, 3.23 and 3.24 it yields:

$$cos(k.L) = ne^{i.\omega.\tau} sin^2(k.L/2) - \frac{i.\lambda.n_p.e^{i.\omega.\tau_p}}{2} sin(k.L) \tag{3.25}$$

Again, n and n_p are very small. Moreover, they are chosen such that $n = \lambda n_p$ and $\tau = \tau_p$. With these new simplifications and by performing Taylor expansion of relation 3.25 around eigen mode without combustion, it yields:

For the first mode (quarter wave mode, $k_0 L = \frac{\pi}{2}$):

$$\Re(\delta f) = -\frac{n.c}{2\sqrt{2}\pi L} sin(\omega_0 \tau + \frac{\pi}{4}) \tag{3.26}$$

$$\Im(\delta f) = \frac{n.c}{2\sqrt{2}\pi L} cos(\omega_0 \tau + \frac{\pi}{4}) \tag{3.27}$$

Similarly, for the second mode (three-quarter wave mode, $k_0 L = \frac{3\pi}{2}$):

$$\Re(\delta f) = \frac{n.c}{2\sqrt{2}\pi L} cos(\omega_0 \tau + \frac{\pi}{4}) \tag{3.28}$$

$$\Im(\delta f) = \frac{n.c}{2\sqrt{2}\pi L} sin(\omega_0 \tau + \frac{\pi}{4}) \tag{3.29}$$

Features of the configuration

To compare analytics with soundtube1_5d results, the following features are chosen:

- $T = 2000\,K$
- $P = 1.013\,bars$
- $S = 7 \times 10^{-4}\,m^2$
- $W = 0.02897\,kg/mol$
- $\gamma = 1.4$
- $L = 0.5\,m$
- $n = 0.01$
- $\rho c n_p = 0.01$
- $\tau = \tau_p = 2 \times 10^{-4}\,s$

3.4.2 soundtube1_5D input file

```
[Geometry]

TubeNumber=          2
ReflectionNumber=    2
IntersecNumber=      1


[Reflection1]
TubeReflect=     1 , -
Refl_numorder_denomorder= 0 , 0
R_num0=                   1.0 , 0.0
R_denom0=                 1.0 , 0.0

[Reflection2]
TubeReflect=     2 , +
Refl_numorder_denomorder= 0 , 0
R_num0=                   -1.0 , 0.0
R_denom0=                 1.0 , 0.0

[Tube1]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.25
surface=          3E-2
coordxyz_minus=   0.0 , 0.0 , 0.0
coordxyz_plus=    0.25 , 0.0 , 0.0

[Tube2]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.25
surface=          3E-2
coordxyz_minus=   0.25 , 0.0 , 0.0
coordxyz_plus=    0.50 , 0.0 , 0.0

[Intersection1]
N_numorder_denomorder=    0 , 0
n_num0=                   0.01 , 0.0
n_denom0=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 2.0E-4 , 0.0
tau_denom0=               1.0 , 0.0
```

```
Np_numorder_denomorder=   0 , 0
np_num0=                 0.01 , 0.0
np_denom0=               1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=               2.0E-4 , 0.0
taup_denom0=             1.0 , 0.0

connectTubeNum=       2

connectTube1=         1 , +
connectTube2=         2 , -
flame_ref_tube=       1 , +

[Simulation]

option=    modeshape3d

LowerLeft_domain=     1.0 , -100.0
UpperRight_domain=    2000.0 , 100.0

LS_tube=              2
LS_relative_position= 0.18
LS_amplitude=         1.0
LS_phase=             0.0
LS_beginfreq=         1.0
LS_endfreq=           2000.0
LS_resol_forcing=     2.0

modeshape_freq=       446.26 , 0.45
modeshape_resol=      0.001
modeshape_time=       0.019
modeshape_TubeNum=    2
modeshape_Tube1=      1 , -
modeshape_Tube2=      2 , -

[Output]

Resultfile=             in-out/tubefl_np.res
geometryfile=           in-out/tubefl_np_geom.dat
fluxResultfile=         in-out/tubefl_np_FluxForcing.dat
totenergyResultfile=    in-out/tubefl_np_totEForcing.dat
modeshapefile=          in-out/modes/tube_np_fl.mode
xmgrcmdfile=            xmgrmodecmd.dat
meshplot3d_file=        in-out/modes/tubefl_np_mesh3D.g
solplot3d_file=         in-out/modes/tubefl_np_sol3D.q
```

3.4.3 soundtube1_5D results and comparison with analytical results

Modal method

mode1: Re(f):4.46262388175255e+02 Im(f):4.52213678373532e-01
mode2: Re(f):1.34308065872080e+03 Im(f):1.25705011813905e+00

Hybrid method

mode1_5D[1]: Re(f)=4.462632e+02 Im(f)=4.536658e-01
mode1_5D[2]: Re(f)=1.343079e+03 Im(f)=1.252841e+00

Energyforcing_dom method

mode 1: Re(f)=4.462623e+02 Im(f)=4.523952e-01
mode 2: Re(f)=1.343080e+03 Im(f)=1.257027e+00

Fluxforcing method

The result of the "fluxforcing" option is illustrated by figure 3.8.

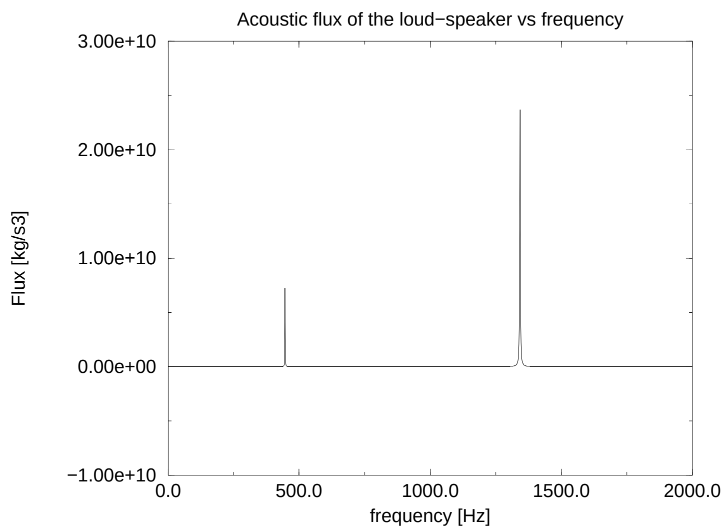


Figure 3.8: Acoustic flux of the loudspeaker evaluated with "fluxforcing". Case of the simple tube with a flame, without heat release but with a fuel line

Analytical results

The modes without combustion are the modes of the example 1, that is to say:
$f_{01} = 448.2187\,Hz$ and Im(f_{01})=0
$f_{02} = 1344.656\,Hz$ and Im(f_{02})=0
In this situation, the relations 3.26, 3.27, 3.28, 3.29 give the following results:

- mode 1:
  $\Re(\delta f)$ =-1.9681 so Re(f)=446.2506 Hz

$\Im(\delta f)$ =0.4446 so Im(f)=0.4446 Hz

- mode 2:
  $\Re(\delta f)$=-1.5860 so Re(f)=1343.07 Hz
  $\Im(\delta f)$=1.2473 so Im(f)=1.2473 Hz

3.5 Tube with a flame and with a heat release

3.5.1 General features and analytical results

Again, the configuration of the figure 3.6 is considered. However, a heat release exists now, that is to say a difference of temperature between the two tubes separated by flame.

To find more easily an analytical solution, the following simplifications are used:

- Different mean temperature in the two tubes:

$$4.T_1 = T_2$$

- Same section in the two tubes with:

$$S_1 = S_2 = S$$

- Same mean pressure in the two tubes:

$$P_1 = P_2 = P$$

- Same length for the two tubes:

$$l_1 = l_2 = L/2$$

In these conditions, it can be checked that:

$$\Gamma = \frac{S_1 \rho_1 c_1}{S_2 \rho_2 c_2} = \frac{\rho_1 c_1}{\rho_2 c_2} = \frac{1}{2}$$

and:

$$k = 2 \times k_2 = k_1 = \frac{2\pi f}{c_1}$$

Reflection relations

According to these hypothesis and the definition of a reflection coefficient, the reflection relations are in the current case:

- 1^{st} reflection condition:

$$R_1 = 1 \Longrightarrow A_1^+ = A_1^- \tag{3.30}$$

- 2^{nd} reflection condition:

$$R_2 = -1 \Longrightarrow A_2^+ e^{i.k.L/2} = -A_2^- \tag{3.31}$$

Jump-relations for pressure

This relation in the current case is:

$$p_1'(L/2) = p_2'(L/2) \Longrightarrow A_1^+.e^{i.k.L/2} + A_1^-.e^{-i.k.L/2} = A_2^+ + A_2^- \tag{3.32}$$

Jump-relation for speed

The general jump-relation for speed is applied in the current case. It is reminded that the orientation of each tube is essential in this relation.

$$S.u_1'(L/2) + S.u_1'(L/2).n.e^{i\omega\tau} - S.u_2'(L/2) = 0$$

By introducing the “Riemann invariants”, it yields:

$$\Gamma.(1 + ne^{i\omega\tau}).(A_1^+.e^{i.k.L/2} - A_1^-.e^{-i.k.L/2}) - A_2^+ + A_2^- = 0 \tag{3.33}$$

with $\lambda = \rho c$.

Characteristic function and results

By combining the relations 3.30, 3.31, 3.32 and 3.33 it yields:

$$cos(\frac{k.L}{4})[ne^{i.\omega.\tau}sin^2(\frac{k.L}{4}) - 3cos^2(\frac{k.L}{4}) + 2] = 0 \tag{3.34}$$

Again, n is chosen very small, so eigen modes with combustion are closed to eigen modes without combustion. But, in this case, the configuration without combustion is not example 1. Eigen modes without combustion can be obtained by applying $n = 0$ in relation 3.34. Two families of eigen modes without combustion appear (k_0 is the corresponding wave vector of the mode):

- the first family verifies:

$$cos(\frac{k_0 L}{4}) = 0 \tag{3.35}$$

- the second family verifies:

$$cos^2(\frac{k_0 L}{4}) - \frac{2}{3} = 0 \tag{3.36}$$

With a flame, it can be deduced from the relation 3.34 that only the modes of the second family are influenced by combustion.

- First family mode with combustion:
  If f_0 corrresponds to a second family eigen frequency without combustion, with $n << 1$, the value f with combustion is obtained by performing Taylor expansion of relation 3.34 around f_0:

$$f = f_0 + \delta f$$

$$\Re(\delta f) = \frac{-n.c_1}{9\pi L}.\frac{cos(\omega_0\tau)}{sin(\frac{\omega_0 L}{4c_1}).cos(\frac{\omega_0 L}{4c_1})} \tag{3.37}$$

$$\Im(\delta f) = \frac{-n.c_1}{9\pi L}.\frac{sin(\omega_0\tau)}{sin(\frac{\omega_0 L}{4c_1}).cos(\frac{\omega_0 L}{4c_1})} \tag{3.38}$$

- Second family mode with combustion:
  Combustion induces no modification for these modes: imaginary part of these eigen frequencies is null and their real part is such that:

$$cos(\frac{kL}{4}) = 0$$

Features of the configuration

To compare analytics with soundtube1_5d results, the following features are chosen:

- $T_1 = 300\,K$
- $P = 1.013\,bars$
- $S = 9 \times 10^{-4}\,m^2$
- $W = 0.02897\,kg/mol$
- $\gamma = 1.4$
- $L = 0.5\,m$
- $n = 0.01$
- $\tau = 2.7 \times 10^{-3}\,s$

3.5.2 soundtube1_5D input file

```
[Geometry]

TubeNumber=           2
ReflectionNumber=     2
IntersecNumber=       1


[Reflection1]
TubeReflect=                1 , -
Refl_numorder_denomorder=   0 , 0
R_num0=                     1.0 , 0.0
R_denom0=                   1.0 , 0.0

[Reflection2]
TubeReflect=                2 , +
Refl_numorder_denomorder=   0 , 0
```

```
R_num0=                   -1.0 , 0.0
R_denom0=                 1.0 , 0.0

[Tube1]
temperature=      300.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.25
surface=          3E-2
coordxyz_minus=   0.0 , 0.0 , 0.0
coordxyz_plus=    0.25 , 0.0 , 0.0

[Tube2]
temperature=      1200.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.25
surface=          3E-2
coordxyz_minus=   0.25 , 0.0 , 0.0
coordxyz_plus=    0.50 , 0.0 , 0.0

[Intersection1]
N_numorder_denomorder=    0 , 0
n_num0=                   0.01 , 0.0
n_denom0=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 2.7E-3 , 0.0
tau_denom0=               1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                  0.0 , 0.0
np_denom0=                1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=                2.0E-4 , 0.0
taup_denom0=              1.0 , 0.0

connectTubeNum=        2

connectTube1=          1 , +
connectTube2=          2 , -
flame_ref_tube=        1 , +

[Simulation]

option=       fluxforcing
```

```
LowerLeft_domain=      1.0 , -100.0
UpperRight_domain=     2000.0 , 100.0

LS_tube=               1
LS_relative_position=  0.1
LS_amplitude=          1.0
LS_phase=              0.0
LS_beginfreq=          1.0
LS_endfreq=            2000.0
LS_resol_forcing=      2.0

modeshape_freq=        272.1 , 0.51
modeshape_resol=       0.001
modeshape_time=        0.019
modeshape_TubeNum=     2
modeshape_Tube1=       1 , -
modeshape_Tube2=       2 , -

[Output]

Resultfile=              in-out/tubeheatr.res
geometryfile=            in-out/tubeheatr_geom.dat
fluxResultfile=          in-out/tubeheatr_FluxForcing.dat
totenergyResultfile=     in-out/tubeheatr_totEForcing.dat
modeshapefile=           in-out/modes/tubeheatr.mode
xmgrcmdfile=             xmgrmodecmd.dat
meshplot3d_file=         in-out/modes/tubeheatrmesh3D.g
solplot3d_file=          in-out/modes/tubeheatrsol3D.q

```

3.5.3 soundtube1_5D results and comparison with analytical results

Modal method

mode1	Re(f):2.72123504999098e+02	Im(f):5.14346028450136e-01
mode2	Re(f):6.94377418987686e+02	Im(f):-2.11721210224378e-14
mode3	Re(f):1.11719557301655e+03	Im(f):5.33210062670802e-02
mode4	Re(f):1.66135111516725e+03	Im(f):-4.72270490805639e-02

Hybrid method

mode1_5D[1]:	Re(f)=2.721248e+02	Im(f)=4.611370e-01
mode1_5D[2]:	Re(f)=6.943774e+02	Im(f)=-6.457897e-08
mode1_5D[3]:	Re(f)=1.117196e+03	Im(f)=4.752126e-02
mode1_5D[4]:	Re(f)=1.661351e+03	Im(f)=-4.187975e-02

Enegryforcing_dom method

mode 1:	Re(f)=2.721235e+02	Im(f)=5.143460e-01
mode 2:	Re(f)=6.943774e+02	Im(f)=3.502698e-15
mode 3:	Re(f)=1.117196e+03	Im(f)=5.332101e-02
mode 4:	Re(f)=1.661351e+03	Im(f)=-4.722705e-02

Fluxforcing method

The result of the “fluxforcing” option is given by figure 3.9.

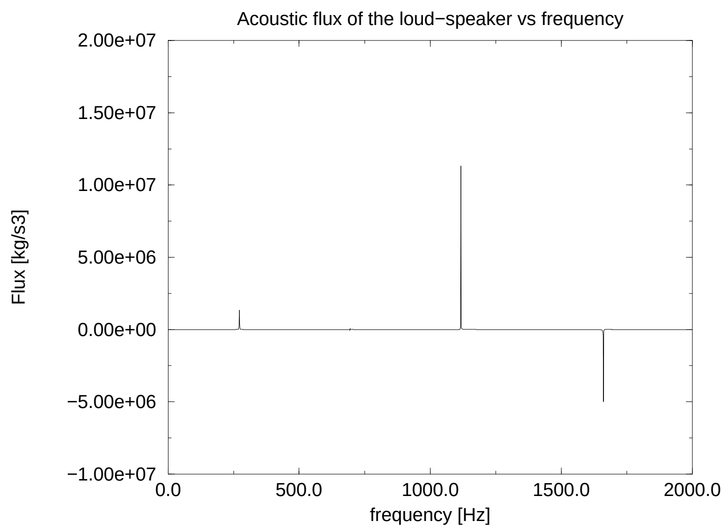


Figure 3.9: Acoustic flux of the loudspeaker evaluated with “fluxforcing”. Case of the simple tube with a flame, with heat release but without fuel line

By zooming, the second mode appears more clearly. An asymptot can be seen on Figure 3.10 at the second eigen frequency and not a peak. This is due to the null value of the imaginary part of its eigen frequency.

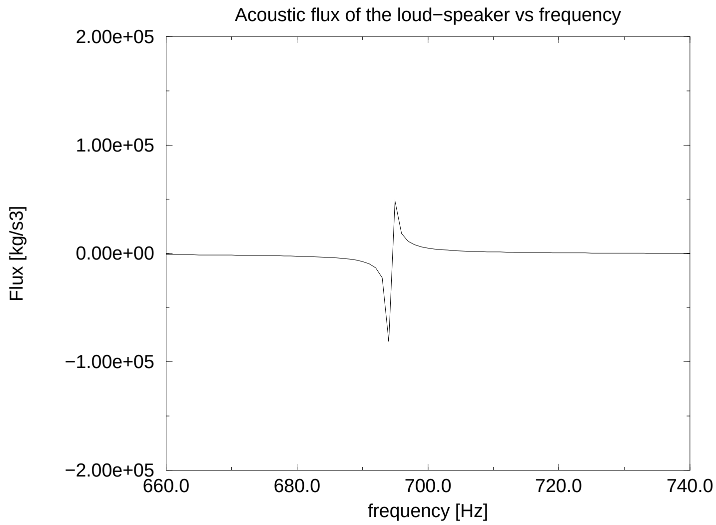


Figure 3.10: Zoom on the second modes at $f = 694\,Hz$. Case of the simple tube with a flame, with heat release but without fuel line

Analytical results

In this example, the modes without combustion are:
$f_{01} = 272.0755\,Hz$ and Im(f_{01})=0
$f_{02} = 694.3774\,Hz$ and Im(f_{02})=0
$f_{03} = 1116.679\,Hz$ and Im(f_{03})=0
$f_{03} = 1660.830\,Hz$ and Im(f_{03})=0
In this situation, the relations 3.35, 3.37, 3.38, give the following results:

- mode 1:
  $\Re(\delta f)$ =0.0503 so Re(f)=272.1258 Hz
  $\Im(\delta f)$ =0.5185 so Im(f)=0.5185 Hz

- mode 2:
  $\Re(\delta f)$ =0.0 so Re(f)=694.3774 Hz
  $\Im(\delta f)$ =0.0 so Im(f)=0.0 Hz

- mode 3:
  $\Re(\delta f)$ =0.5186 so Re(f)=1117.198 Hz
  $\Im(\delta f)$ =0.0491 so Im(f)=0.0491 Hz

- mode 4:
  $\Re(\delta f)$ =0.5184 so Re(f)=1661.348 Hz
  $\Im(\delta f)$ =-0.0515 so Im(f)=-0.0515 Hz

3.6 Tube with a flame varying with the frequency

3.6.1 General features and analytical results

The configuration with a flame, without a fuel line and without heat release is again considered (figure 3.6) . The particularity of this situation is the frequency dependence of the flame through its coefficient $n = n(f)$. But, τ is still constant.

To find more easily an analytical solution, the following simplifications are applied:

- Same mean temperature in the two tubes:

$$T_1 = T_2 = T$$

- Same section in the two tubes:

$$S_1 = S_2 = S$$

- Same mean pressure in the two tubes:

$$P_1 = P_2 = P$$

- Same length for the two tubes:

$$l_1 = l_2 = L/2$$

Reflection relations

The relations found in the case of the simple tube with a flame are reminded here:

- 1^{st} reflection condition:

$$R_1 = 1 \Longrightarrow A_1^+ = A_1^-$$

- 2^{nd} reflection condition:

$$R_2 = -1 \Longrightarrow A_2^+ e^{i.k.L} = -A_2^-$$

Jump-relations for pressure

Again, the relation 3.14 is reminded:

$$A_1^+.e^{i.k.L/2} + A_1^-.e^{-i.k.L/2} = A_2^+ + A_2^-$$

Jump-relation for speed

As previously, the relation 3.15 is reminded. Here, $n = n(f)$:

$$(A_1^+.e^{i.k.L/2} - A_1^-.e^{-i.k.L/2}).(1 + n(f).e^{i\omega\tau}) - A_2^+ + A_2^- = 0$$

Characteristic function and results

By combining the relations 3.12, 3.13, 3.14 and 3.15 it yields:

$$cos(k.L) = n(f).e^{i.\omega.\tau} sin^2(k.L/2) \tag{3.39}$$

The function $n = n(f)$ varies only with the real part of the frequency. The exact formula chosen is:

$$n(f) = \frac{1}{10 + f_r} \tag{3.40}$$

The evolution of n with the frequency can be seen on the figure 3.11.

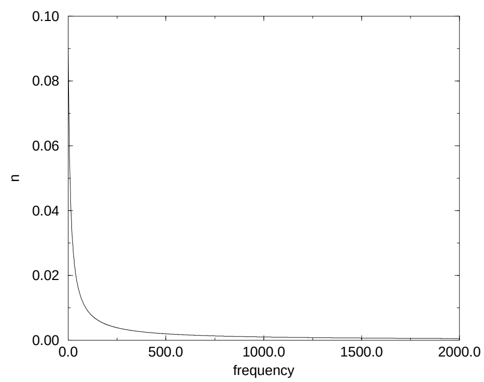


Figure 3.11: Evolution of the parameter "n" with frequency

In a range from $1\,Hz$ to $2000\,Hz$, n can be considered sufficiently small to apply Taylor expansions around the state without combustion (f_0).

By using 3.40 in the equation 3.39, the shift δk induced by the flame is:

$$\delta k = \frac{-sin^2(k_0.L/2)}{a + i.b}$$

With

$$a = L.cos(\omega_0\tau)sin(k_0.L).(10 + \omega_0/2\pi) + (L/2).sin(k_0.L)$$

$$b = c.\tau.sin^2(k_0.L/2) - L.sin(\omega_0\tau)sin(k_0.L)(10 + \omega_0/2\pi)$$

So, for the first mode (quarter wave mode, $k_0L = \frac{\pi}{2}$):

$$\Re(\delta f) = -\frac{c}{4\pi L}\frac{1/2 + cos(\omega_0\tau).(10 + \omega_0/2\pi)}{[(1/2 + cos(\omega_0\tau).(10 + \omega_0/2\pi))^2 + (\frac{c\tau}{2L} - sin(\omega_0\tau).(10 + \omega_0/2\pi))^2]} \tag{3.41}$$

$$\Im(\delta f) = \frac{c}{(4\pi L)}\frac{\frac{c\tau}{2L} - sin(\omega_0\tau).(10 + \omega_0/2\pi)}{[(1/2 + cos(\omega_0\tau).(10 + \omega_0/2\pi))^2 + (\frac{c\tau}{2L} - sin(\omega_0\tau).(10 + \omega_0/2\pi))^2]} \tag{3.42}$$

Similarly, for the second mode (three-quarter wave mode, $k_0L = \frac{3\pi}{2}$):

$$\Re(\delta f) = \frac{c}{4\pi L}\frac{1/2 + cos(\omega_0\tau).(10 + \omega_0/2\pi)}{[(1/2 + cos(\omega_0\tau).(10 + \omega_0/2\pi))^2 + (\frac{c\tau}{2L} + sin(\omega_0\tau).(10 + \omega_0/2\pi))^2]} \tag{3.43}$$

$$\Im(\delta f) = \frac{c}{(4\pi L)} \frac{\frac{c\tau}{2L} + sin(\omega_0\tau).(10+\omega_0/2\pi)}{[(1/2+cos(\omega_0\tau).(10+\omega_0/2\pi))^2 + (\frac{c\tau}{2L} + sin(\omega_0\tau).(10+\omega_0/2\pi))^2]} \quad (3.44)$$

Features of the configuration

To compare analytics with soundtube1_5d results, the following features are chosen:

- $T = 2000\,K$
- $P = 1.013\,bars$
- $S = 7 \times 10^{-4}\,m^2$
- $W = 0.02897\,kg/mol$
- $\gamma = 1.4$
- $L = 0.5\,m$
- $\tau = 1.5 \times 10^{-4}\,s$

3.6.2 soundtube1_5D input file

```
[Geometry]

TubeNumber=          2
ReflectionNumber=    2
IntersecNumber=      1


[Reflection1]
TubeReflect=                1 , -
Refl_numorder_denomorder= 0 , 0
R_num0=                     1.0 , 0.0
R_denom0=                   1.0 , 0.0

[Reflection2]
TubeReflect=                2 , +
Refl_numorder_denomorder= 0 , 0
R_num0=                     -1.0 , 0.0
R_denom0=                   1.0 , 0.0

[Tube1]
temperature=        2000.0
pressure=           1.013E5
weight=             0.02897
gamma=              1.4
length=             0.25
surface=            3E-4
coordxyz_minus=     0.0 , 0.0 , 0.0
```

```
coordxyz_plus=    0.25 , 0.0 , 0.0

[Tube2]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.25
surface=          3E-4
coordxyz_minus=   0.25 , 0.0 , 0.0
coordxyz_plus=    0.50 , 0.0 , 0.0

[Intersection1]
N_numorder_denomorder=    0 , 1
n_num0=                   1.0 , 0.0
n_denom0=                 10.0 , 0.0
n_denom1=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 1.0E-5 , 0.0
tau_denom0=               1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                  0.0 , 0.0
np_denom0=                1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=                1.0E-5 , 0.0
taup_denom0=              1.0 , 0.0

connectTubeNum=        2

connectTube1=          1 , +
connectTube2=          2 , -
flame_ref_tube=        1 , +

[Simulation]

option=     modeshape3d

LowerLeft_domain=      1.0 , -100.0
UpperRight_domain=     2000.0 , 100.0

LS_tube=               1
LS_relative_position=  0.15
LS_amplitude=          1.0
LS_phase=              0.0
LS_beginfreq=          1.0
LS_endfreq=            2000.0
```

```
LS_resol_forcing=      2.0

modeshape_freq=        447.9 , -8.7E-3
modeshape_resol=       0.001
modeshape_time=        0.019
modeshape_TubeNum=     2
modeshape_Tube1=       1 , -
modeshape_Tube2=       2 , -

[Output]

Resultfile=              in-out/nvary.res
geometryfile=            in-out/nvary.dat
fluxResultfile=          in-out/nvary.dat
totenergyResultfile=     in-out/nvary.dat
modeshapefile=           in-out/modes/nvary.mode
xmgrcmdfile=             xmgrmodecmd.dat
meshplot3d_file=         in-out/modes/nvarymesh3d.g
solplot3d_file=          in-out/modes/nvarysol3D.q
```

3.6.3 soundtube1_5D results and comparison with analytical results

Modal method

mode1	Re(f):4.47907584321279e+02	Im(f):-8.74835039764314e-03
mode2	Re(f):1.34476098769844e+03	Im(f):8.88108297065530e-03

hybrid method

mode1_5D[1]:	Re(f)=4.479076e+02	Im(f)=-8.748346e-03
mode1_5D[2]:	Re(f)=1.344761e+03	Im(f)=8.881088e-03

Energyforcing_dom method

mode 1:	Re(f)=4.479076e+02	Im(f)=-8.748350e-03
mode 2:	Re(f)=1.344761e+03	Im(f)=8.881083e-03

Forcing method

The result of the “fluxforcing” method is illustrated by the figure 3.12.

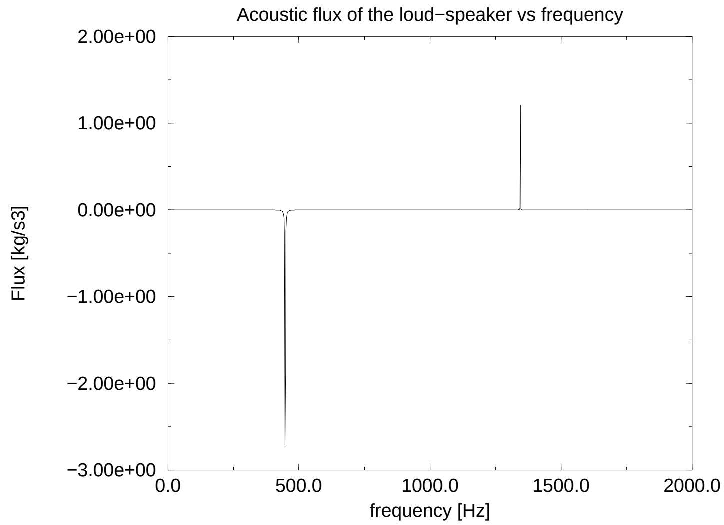


Figure 3.12: Acoustic flux of the loudspeaker evaluated with "fluxforcing". Case of the simple tube with a frequency dependent flame ($n = n(f)$).

Analytical results

The modes without combustion are the modes of the example 1, that is to say:
$f_{01} = 448.2187\,Hz$ and Im(f_{01})=0
$f_{02} = 1344.656\,Hz$ and Im(f_{02})=0
In this situation, the relations 3.41, 3.42, 3.43, 3.44 give the following results:

- mode 1:
  $\Re(\delta f)$ =-0.3109 so Re(f)=447.908 Hz
  $\Im(\delta f)$ =-0.00874 so Im(f)=-0.00874 Hz

- mode 2:
  $\Re(\delta f)$=0.1049 so Re(f)=1344.761 Hz
  $\Im(\delta f)$=0.00888 so Im(f)=0.00888 Hz

3.7 Tube with a thick flame

3.7.1 General features and analytical results

The configuration of the figure 3.13 is considered. The two flames of the device have the same reference speed (in the $n-\tau$ model) that's why they can be seen as a unique flame. This reference speed is the speed at the end of the tube 1, i.e: the speed at the side "+" of the tube 1. More precisely, this case corresponds to a distribution of the heat release in different regions. Each region has its own values for n and τ. Between two regions, acoustics without combustion is applied.

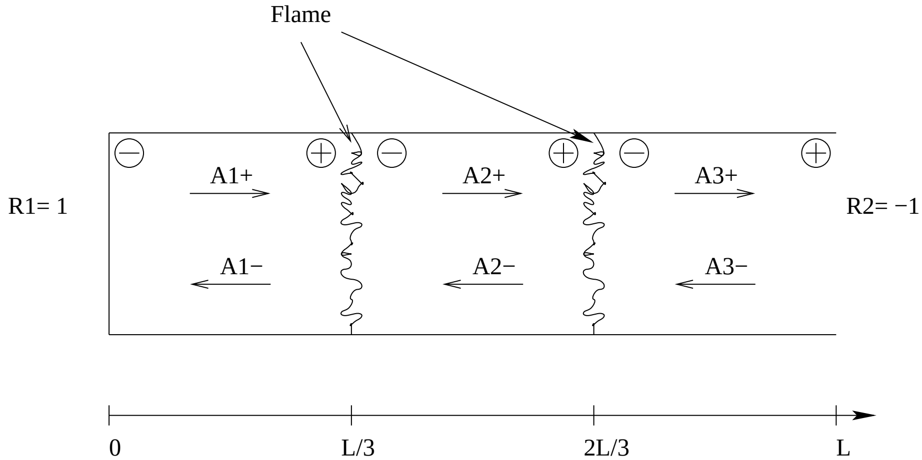


Figure 3.13: Tube with a thick flame

To find more easily an analytical solution, the following simplifications are applied:

- Same mean temperature in the two tubes:

$$T_1 = T_2 = T_3 = T$$

- Same section in the two tubes:

$$S_1 = S_2 = S_3 = S$$

- Same mean pressure in the two tubes:

$$P_1 = P_2 = P_3 = P$$

- Same length for the three tubes:

$$l_1 = l_2 = l_3 = l = L/3$$

Reflection relations

According to these hypothesis and the definition of a reflection coefficient, the reflection relations are in the current case:

- 1^{st} reflection condition:

$$R_1 = 1 \Longrightarrow A_1^+ = A_1^- \tag{3.45}$$

- 2^{nd} reflection condition:

$$R_2 = -1 \Longrightarrow A_3^+ e^{i.2.k.l} = -A_3^- \tag{3.46}$$

Jump-relations for pressure

- At the first intersection, the jump-relation for pressure is:

$$p_1'(L/3) = p_2'(L/3) \Longrightarrow A_1^+.e^{i.k.l} + A_1^-.e^{-i.k.l} = A_2^+ + A_2^- \tag{3.47}$$

- At the second intersection, the jump-relation for pressure is:

$$p'_2(2L/3) = p'_3(2L/3) \Longrightarrow A_2^+.e^{i.k.l} + A_2^-.e^{-i.k.l} = A_3^+ + A_3^- \tag{3.48}$$

Jump-relation for speed

The general jump-relation for speed is applied in the current case. It is reminded that the orientation of each tube is essential in this relation. In this case the two flames have the same reference speed: the acoustic speed at the end of the first tube (at side "+").

- At the first intersection, the jump-relation for speed is:

$$S.u'_1(L/3) + S.u'_1(L/3).n_1.e^{i\omega\tau_1} - S.u'_2(L/3) = 0$$

  Hence:

$$(A_1^+.e^{i.k.l} - A_1^-.e^{-i.k.l}).(1 + n_1.e^{i\omega\tau_1}) - A_2^+ + A_2^- = 0 \tag{3.49}$$

- At the second intersection, the speed reference is also $u'_1(L/3)$. So, the jump-relation for speed is:

$$S.u'_2(2L/3) + S.u'_1(L/3).n_2.e^{i\omega\tau_2} - S.u'_3(2L/3) = 0$$

  Hence:

$$A_2^+.e^{i.k.l} - A_2^-.e^{-i.k.l} + (A_1^+.e^{i.k.l} - A_1^-.e^{-i.k.l}).n_2.e^{i\omega\tau_2} - A_3^+ + A_3^- = 0 \tag{3.50}$$

Characteristic function and results

By combining the relations 3.45, 3.46, 3.47, 3.48, 3.49 and 3.50 it yields:

$$cos(3.k.l) - 2.n_1 e^{i.\omega.\tau_1}.sin^2(k.l).cos(k.l) - n_2 e^{i.\omega.\tau_2}.sin^2(k.l) = 0 \tag{3.51}$$

To finish the calculus, the hypothesis of very small heat release (i.e: $n << 1$) is necessary. As a consequence, the value f of eigen frequencies with combustion is close to their value without combustion f_0 (example 1 of this help guide). So, The Taylor expansion of 3.51 can be applied with $k = k_0 + \delta k$ and $|\delta k| << |k_0|$. Finally, it yields:

$$\delta k = \frac{-sin^2(k_0 l)}{L.sin(k_0 L)}[2n_1 cos(k_0 l).(cos(\omega_0\tau_1) + i.sin(\omega_0\tau_1)) + n_2.(cos(\omega_0\tau_2) + i.sin(\omega_0\tau_2))]$$

The shift in frequency, for the real and the imaginary part, can be expressed as follows:

$$\Re(\delta f) = \frac{-c}{2\pi}\frac{sin^2(k_0 l)}{L.sin(k_0 L)}[2n_1 cos(k_0 l).cos(\omega_0\tau_1) + n_2.cos(\omega_0\tau_2)] \tag{3.52}$$

$$\Im(\delta f) = \frac{-c}{2\pi}\frac{sin^2(k_0 l)}{L.sin(k_0 L)}[2n_1 cos(k_0 l).sin(\omega_0\tau_1) + n_2.sin(\omega_0\tau_2)] \tag{3.53}$$

Features of the configuration

To compare analytics with soundtube1_5d results, the following features are chosen:

- $T = 2000\,K$
- $P = 1.013\,bars$
- $S = 7 \times 10^{-4}\,m^2$
- $W = 0.02897\,kg/mol$
- $\gamma = 1.4$
- $L = 0.75\,m$
- $n_1 = 0.01$
- $n_2 = 0.015$
- $\tau_1 = 1 \times 10^{-5}\,s$
- $\tau_2 = 2.8 \times 10^{-5}\,s$

3.7.2 soundtube1_5D input file

```
[Geometry]

TubeNumber=          3
ReflectionNumber=    2
IntersecNumber=      2


[Reflection1]
TubeReflect=                1 , -
Refl_numorder_denomorder= 0 , 0
R_num0=                     1.0 , 0.0
R_denom0=                   1.0 , 0.0

[Reflection2]
TubeReflect=                3 , +
Refl_numorder_denomorder= 0 , 0
R_num0=                     -1.0 , 0.0
R_denom0=                   1.0 , 0.0

[Tube1]
temperature=       2000.0
pressure=          1.013E5
weight=            0.02897
gamma=             1.4
length=            0.25
surface=           3E-4
coordxyz_minus=    0.0 , 0.0 , 0.0
coordxyz_plus=     0.25 , 0.0 , 0.0
```

```
[Tube2]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.25
surface=          3E-4
coordxyz_minus=   0.25 , 0.0 , 0.0
coordxyz_plus=    0.50 , 0.0 , 0.0

[Tube3]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.25
surface=          3E-4
coordxyz_minus=   0.50 , 0.0 , 0.0
coordxyz_plus=    0.75 , 0.0 , 0.0


[Intersection1]
N_numorder_denomorder=    0 , 0
n_num0=                   0.01 , 0.0
n_denom0=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 1.0E-5 , 0.0
tau_denom0=               1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                  0.0 , 0.0
np_denom0=                1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=                1.0E-5 , 0.0
taup_denom0=              1.0 , 0.0

connectTubeNum=        2

connectTube1=          1 , +
connectTube2=          2 , -
flame_ref_tube=        1 , +

[Intersection2]
N_numorder_denomorder=    0 , 0
n_num0=                   0.015 , 0.0
n_denom0=                 1.0 , 0.0
```

```
Tau_numorder_denomorder=  0 , 0
tau_num0=                2.8E-5 , 0.0
tau_denom0=              1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                 0.0 , 0.0
np_denom0=               1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=               1.0E-5 , 0.0
taup_denom0=             1.0 , 0.0

connectTubeNum=        2

connectTube1=          2 , +
connectTube2=          3 , -
flame_ref_tube=        1 , +


[Simulation]

option=      modeshape3d

LowerLeft_domain=      1.0 , -100.0
UpperRight_domain=     2000.0 , 100.0

LS_tube=               2
LS_relative_position=  0.15
LS_amplitude=          1.0
LS_phase=              0.0
LS_beginfreq=          1.0
LS_endfreq=            2000.0
LS_resol_forcing=      2.0

modeshape_freq=        899.2 , 0.45
modeshape_resol=       0.001
modeshape_time=        0.019
modeshape_TubeNum=     2
modeshape_Tube1=       1 , -
modeshape_Tube2=       2 , -


[Output]

Resultfile=              in-out/flam_epaiss.res
geometryfile=            in-out/flam_epaiss_geom.dat
fluxResultfile=          in-out/flam_epaissFForcing.dat
totenergyResultfile=     in-out/flam_epaissEForcing.dat
modeshapefile=           in-out/modes/flam_epaiss.mode
```

```
xmgrcmdfile=            xmgrmodecmd.dat
meshplot3d_file=        in-out/modes/flam_epaiss_mesh3D.g
solplot3d_file=         in-out/modes/flam_epaiss_sol3D.q
```

3.7.3 soundtube1_5D results and comparison with analytical results

Modal method

mode1	Re(f):2.97289384475941e+02	Im(f):-5.17788415780247e-02
mode2	Re(f):8.99236555906445e+02	Im(f):4.45502312056407e-01
mode3	Re(f):1.49419363395483e+03	Im(f):-1.08061192337921e-01

Hybrid method

mode1_5D[1]:	Re(f)=2.972896e+02	Im(f)=-5.181963e-02
mode1_5D[2]:	Re(f)=8.992432e+02	Im(f)=4.495891e-01
mode1_5D[3]:	Re(f)=1.494194e+03	Im(f)=-1.071503e-01

Energyforcing_dom method

mode 1:	Re(f)=2.972894e+02	Im(f)=-5.177884e-02
mode 2:	Re(f)=8.992366e+02	Im(f)=4.455023e-01
mode 3:	Re(f)=1.494194e+03	Im(f)=-1.080612e-01

Fluxforcing method

The result of the forcing method is illustrated in the figure 3.14.

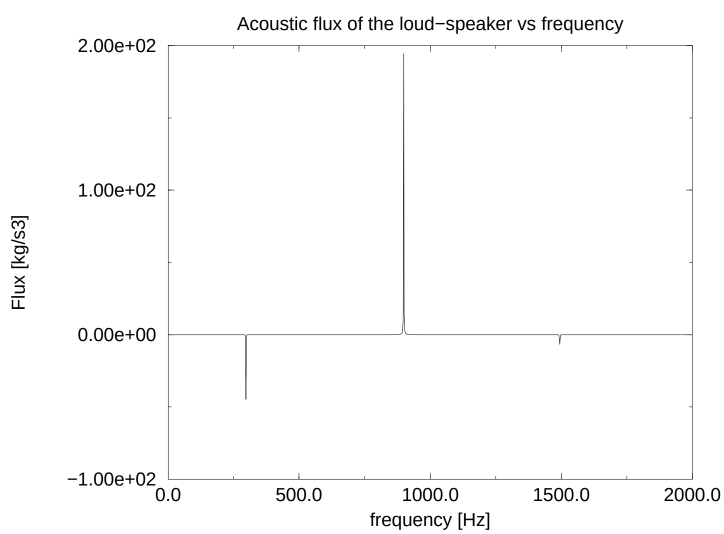


Figure 3.14: Acoustic flux of the loudspeaker evaluated with “fluxforcing”. Case of the tube with thick flame

Analytical results

The modes without combustion are calculated following the relation 3.3 but with $L = 0.75$:
$f_{01} = 298.813\,Hz$ and Im(f_{01})=0
$f_{02} = 896.4374\,Hz$ and Im(f_{02})=0
$f_{03} = 1494.062\,Hz$ and Im(f_{03})=0
In this situation, the relations 3.52, 3.53 give the following results:

- mode 1:
  $\Re(\delta f)$ =-1.5359 so Re(f)=297.277 Hz
  $\Im(\delta f)$ =-0.0529 so Im(f)=-0.0529 Hz

- mode 2:
  $\Re(\delta f)$=2.8180 so Re(f)=899.2554 Hz
  $\Im(\delta f)$=0.4482 so Im(f)=0.4482 Hz

- mode 3:
  $\Re(\delta f)$=0.1312 so Re(f)=1494.193 Hz
  $\Im(\delta f)$=-0.1081 so Im(f)=-0.1081 Hz

3.8 Three tubes in a "T" configuration without flame

3.8.1 General features and analytical results

The configuration of the figure 3.17 is considered.

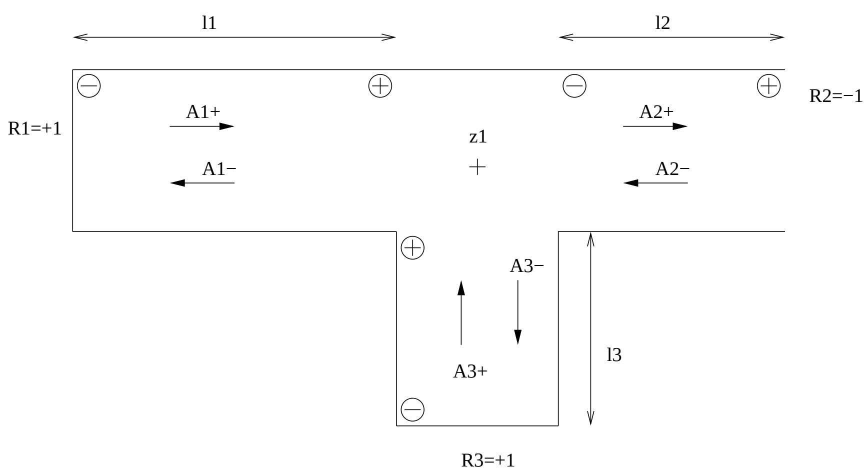


Figure 3.15: "T" configuration without flame

To find more easily an analytical solution, the following simplifications are applied:

- Same mean temperature in all the tubes:

$$T_1 = T_2 = T_3 = T$$

- Same section in all the tubes:

$$S_1 = S_2 = S_3 = S$$

- Same mean pressure in all the tubes:

$$P_1 = P_2 = P_3 = P$$

Reflection relations

According to these hypothesis and the definition of a reflection coefficient, the reflection relations are in the current case:

- 1^{st} reflection condition:

$$R_1 = 1 \Longrightarrow A_1^+ = A_1^- \tag{3.54}$$

- 2^{nd} reflection condition:

$$R_2 = -1 \Longrightarrow A_2^+ e^{i.2.k.l_2} = -A_2^- \tag{3.55}$$

- 3^{rd} reflection condition:

$$R_3 = 1 \Longrightarrow A_3^+ = A_3^- \tag{3.56}$$

Jump-relations for pressure

These relations are in the current case:

- First jump-relation for pressure:

$$p_1'(z_1) = p_2'(z_1) \Longrightarrow A_1^+.e^{i.k.l_1} + A_1^-.e^{-i.k.l_1} = A_2^+ + A_2^-$$

  By using the relations 3.54 and 3.55 and by supposing $sin(kl_2) \neq 0$ it becomes:

$$A_2^+ = A_1^+.\frac{i.e^{-i.k.l_2} cos(kl_1)}{sin(kl_2)} \tag{3.57}$$

- Second jump-relation for pressure:

$$p_1'(z_1) = p_3'(z_1) \Longrightarrow A_1^+.e^{i.k.l_1} + A_1^-.e^{-i.k.l_1} = A_3^+.e^{i.k.l_3} + A_3^-.e^{-i.k.l_3}$$

  By using the relations 3.54 and 3.56 and by supposing $cos(kl_3) \neq 0$ it becomes:

$$A_3^+ = A_1^+.\frac{cos(kl_1)}{cos(kl_3)} \tag{3.58}$$

Jump-relation for speed

The general jump-relation for speed is applied in the current case. It is reminded that the orientation of each tube is essential in this relation.

$$S.u'_1(z_1) - S.u'_2(z_1) + S.u'_3(z_1) = 0$$

By introducing the “Riemann invariants”, it yields:

$$A_1^+.e^{i.k.l_1} - A_1^-.e^{-i.k.l_1} - A_2^+ + A_2^- + A_3^+.e^{i.k.l_3} - A_3^-.e^{-i.k.l_3} = 0$$

By using firstly the relations 3.54, 3.55, 3.56 and secondly the relations 3.57 and 3.58, the precedent equation can be simplified as follow:

$$A_1^+.[sin(kl_1) - cos(kl_1).cotan(kl_2) + cos(kl_1).tan(kl_3)] = 0 \tag{3.59}$$

Characteristic function

If $A_1^+ \neq 0$ and if $cos(kl_1) \neq 0$ then the relation 3.59 leads to the following characteristic function:

$$tan(kl_1) - cotan(kl_2) + tan(kl_3) = 0 \tag{3.60}$$

Solve this equation is not trivial and not necessary to compare the soundtube1_5D solutions with analytics. Instead of, results given by soundtube1_5D are introduced in equation 3.60 to verify if they satisfy it.

Features of the configuration

To compare analytics with soundtube1_5d results, the following features are chosen:

- $T = 2000\,K$
- $P = 1.013\,bars$
- $S = 0.03\,m^2$
- $W = 0.02897\,kg/mol$
- $\gamma = 1.4$
- $l_1 = 0.25\,m$
- $l_2 = 0.35\,m$
- $l_3 = 0.2\,m$

3.8.2 soundtube1_5D input file

```
[Geometry]

TubeNumber=          3
ReflectionNumber=    3
```

```
IntersecNumber=      1


[Reflection1]
TubeReflect=     1 , -
Refl_numorder_denomorder= 0 , 0
R_num0=                  1.0 , 0.0
R_denom0=                1.0 , 0.0

[Reflection2]
TubeReflect=     2 , +
Refl_numorder_denomorder= 0 , 0
R_num0=                  -1.0 , 0.0
R_denom0=                1.0 , 0.0

[Reflection3]
TubeReflect=     3 , -
Refl_numorder_denomorder= 0 , 0
R_num0=                  1.0 , 0.0
R_denom0=                1.0 , 0.0


[Tube1]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.25
surface=          3E-2
coordxyz_minus=   0.0 , 0.0 , 0.0
coordxyz_plus=    0.25 , 0.0 , 0.0

[Tube2]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.35
surface=          3E-2
coordxyz_minus=   0.25 , 0.0 , 0.0
coordxyz_plus=    0.60 , 0.0 , 0.0

[Tube3]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.2
surface=          3E-2
```

```
coordxyz_minus=   0.25 , 0.2866 , 0.0
coordxyz_plus=    0.25 , 0.0866 , 0.0

[Intersection1]
N_numorder_denomorder=    0 , 0
n_num0=                   0.0 , 0.0
n_denom0=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 0.0 , 0.0
tau_denom0=               1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                  0.0 , 0.0
np_denom0=                1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=                0.0 , 0.0
taup_denom0=              1.0 , 0.0

connectTubeNum=        3

connectTube1=          1 , +
connectTube2=          2 , -
connectTube3=          3 , +
flame_ref_tube=        1 , +

[Simulation]

option=       modeshape3d

LowerLeft_domain=      1.0 , -100.0
UpperRight_domain=     2000.0 , 100.0

LS_tube=               2
LS_relative_position=  0.15
LS_amplitude=          1.0
LS_phase=              0.0
LS_beginfreq=          1.0
LS_endfreq=            2000.0
LS_resol_forcing=      2.0

modeshape_freq=        308.185 , 0.0
modeshape_resol=       0.01
modeshape_time=        0.019
modeshape_TubeNum=     2
modeshape_Tube1=       1 , -
modeshape_Tube2=       3 , +
```

```
[Output]

Resultfile=               in-out/tconfig.res
geometryfile=             in-out/tconfig_geom.dat
fluxResultfile=           in-out/tconfig_FluxForcing.dat
totenergyResultfile=      in-out/tconfig_totEForcing.dat
modeshapefile=            in-out/modes/tconfig.mode
xmgrcmdfile=              xmgrmodecmd.dat
meshplot3d_file=          in-out/modes/tconfigmesh3D.g
solplot3d_file=           in-out/modes/tconfigsol3D.q
```

3.8.3 soundtube1_5D results and comparison with analytical results

Modal method

mode1	Re(f):3.08185562303430e+02	Im(f):-3.20991295045994e-15
mode2	Re(f):9.85546974375448e+02	Im(f):2.98529281906877e-14
mode3	Re(f):1.22985400446902e+03	Im(f):1.85165833108401e-14
mode4	Re(f):1.96300768743214e+03	Im(f):-1.77689469965228e-14

Hybrid method

mode1_5D[1]:	Re(f)=3.081854e+02	Im(f)=-3.959956e-14
mode1_5D[2]:	Re(f)=9.855470e+02	Im(f)=0.000000e+00
mode1_5D[3]:	Re(f)=1.229865e+03	Im(f)=1.187987e-13
mode1_5D[4]:	Re(f)=1.963008e+03	Im(f)=0.000000e+00

Energyforcing_dom method

mode 1:	Re(f)=3.081856e+02	Im(f)=1.057769e-14
mode 2:	Re(f)=9.855470e+02	Im(f)=-6.235264e-15
mode 3:	Re(f)=1.229854e+03	Im(f)=-1.940787e-14
mode 4:	Re(f)=1.963008e+03	Im(f)=4.250708e-14

Energyforcing method

The figure 3.16 shows the result of the “energyforcing” for this case.

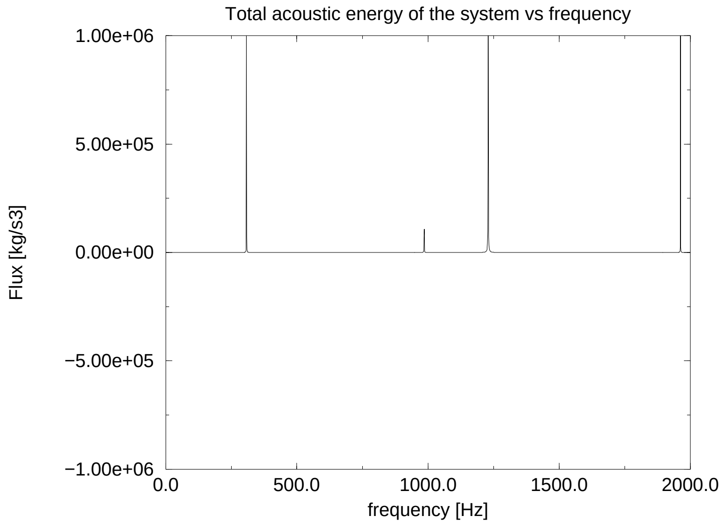


Figure 3.16: Acoustic energy in the case of the “T” configuration without flame

Analytical results

In this case, explicit relations for eigen frequencies can not be obtained but the characteristic function is available. As a consequence, soundtube results are introduced into the characteristic function (relation 3.60) to verify if they satisfy it. Since the results of all the methods are very similar, only the results of the modal method are intorduced into the equation 3.60.

- mode 1: $f_0 = 308.1856\,Hz$ and $k = \frac{2\pi f_0}{c}$

$$tan(kl_1) - cotan(kl_2) + tan(kl_3) = 3.5 \times 10^{-7}$$

- mode 2: $f_0 = 985.547\,Hz$ and $k = \frac{2\pi f_0}{c}$

$$tan(kl_1) - cotan(kl_2) + tan(kl_3) = 3 \times 10^{-6}$$

- mode 3: $f_0 = 1229.854\,Hz$ and $k = \frac{2\pi f_0}{c}$

$$tan(kl_1) - cotan(kl_2) + tan(kl_3) = -1 \times 10^{-6}$$

- mode 4: $f_0 = 1963.008\,Hz$ and $k = \frac{2\pi f_0}{c}$

$$tan(kl_1) - cotan(kl_2) + tan(kl_3) = 1.8 \times 10^{-6}$$

3.9 Three tubes in a “T” configuration with flame

3.9.1 General features and analytical results

The configuration of the figure 3.17 is considered.

To find more easily an analytical solution, the following simplifications are applied:

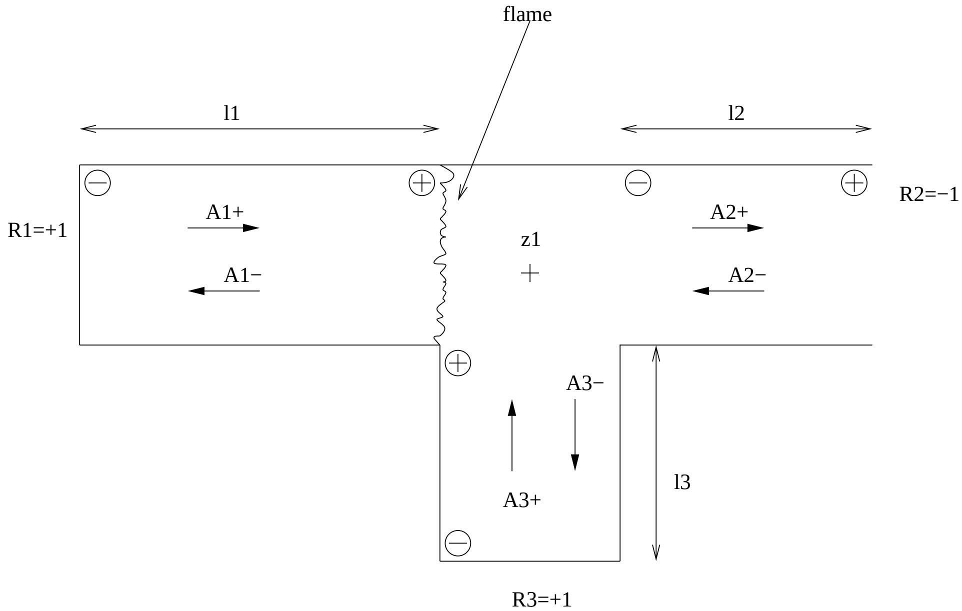


Figure 3.17: “T” configuration with flame, without heat release and with a fuel line

- Same mean temperature in all the tubes (no heat release by the flame):

$$T_1 = T_2 = T_3 = T$$

- Same section in all the tubes:

$$S_1 = S_2 = S_3 = S$$

- Same mean pressure in all the tubes:

$$P_1 = P_2 = P_3 = P$$

Reflection relations

According to these hypothesis and the definition of a reflection coefficient, the reflection relations are in the current case:

- 1^{st} reflection condition:

$$R_1 = 1 \Longrightarrow A_1^+ = A_1^- \tag{3.61}$$

- 2^{nd} reflection condition:

$$R_2 = -1 \Longrightarrow A_2^+ e^{i.2.k.l_2} = -A_2^- \tag{3.62}$$

- 3^{rd} reflection condition:

$$R_3 = 1 \Longrightarrow A_3^+ = A_3^- \tag{3.63}$$

Jump-relations for pressure

These relations are in the current case:

- First jump-relation for pressure:

$$p_1'(z_1) = p_2'(z_1) \Longrightarrow A_1^+.e^{i.k.l_1} + A_1^-.e^{-i.k.l_1} = A_2^+ + A_2^-$$

  By using the relations 3.61 and 3.62 and by supposing $sin(kl_2) \neq 0$ it becomes:

$$A_2^+ = A_1^+.\frac{i.e^{-i.k.l_2}cos(kl_1)}{sin(kl_2)} \tag{3.64}$$

- Second jump-relation for pressure:

$$p_1'(z_1) = p_3'(z_1) \Longrightarrow A_1^+.e^{i.k.l_1} + A_1^-.e^{-i.k.l_1} = A_3^+.e^{i.k.l_3} + A_3^-.e^{-i.k.l_3}$$

  By using the relations 3.61 and 3.63 and by supposing $cos(kl_3) \neq 0$ it becomes:

$$A_3^+ = A_1^+.\frac{cos(kl_1)}{cos(kl_3)} \tag{3.65}$$

Jump-relation for speed

The general jump-relation for speed is applied in the current case. It is reminded that the orientation of each tube is essential in this relation.

$$S.u_1'(z_1) + S.n.e^{i.\omega.\tau}.u_1'(z_1) + S.n_p.e^{i.\omega.\tau_p}.p_1'(z_1) - S.u_2'(z_1) + S.u_3'(z_1) = 0$$

By introducing the “Riemann invariants”, it yields:

$$\begin{gathered} A_1^+.e^{i.k.l_1}(\rho.c.n_p.e^{i.\omega.\tau_p} + 1 + n.e^{i.\omega.\tau}) + A_1^-.e^{-i.k.l_1}(\rho.c.n_p.e^{i.\omega.\tau_p} - (1 + n.e^{i.\omega.\tau})) \\ -A_2^+ + A_2^- + A_3^+.e^{i.k.l_3} - A_3^+.e^{-i.k.l_3} = 0 \end{gathered}$$

By using firstly the relations 3.61,3.62,3.63 and secondly the relations 3.64 and 3.65, the precedent equation can be simplified as follow:

$$A_1^+.[cos(kl_1).\rho.c.n_p.e^{i.\omega.\tau_p} + i.sin(kl_1).(1+n.e^{i.\omega.\tau}) - i.cos(kl_1).cotan(kl_2) + i.cos(kl_1).tan(kl_3)] = 0 \tag{3.66}$$

Characteristic function

If $A_1^+ \neq 0$ and if $cos(kl_1) \neq 0$ then the relation 3.66 leads to the following characteristic function:

$$f(k,n) = \rho.c.n_p.e^{i.\omega.\tau_p} + i.(tan(kl_1).n.e^{i.\omega.\tau} + i.(tan(kl_1) - cotan(kl_2) + tan(kl_3)) = 0 \tag{3.67}$$

To find analytical solutions to this equation, more simplifications are needed:

- First, n and n_p are considered very small ($<< 1$), thus values for eigen frequencies are closed to ones in the case without flame. Consequently Taylor expansions can be used.

- Second,

$$\begin{gathered} n = \rho.c.n_p \\ \tau = \tau_p \end{gathered}$$

With these new hypothesis, the expansion of the characteristic function around the values $k=k_0$ and $n=0$ (k_0 corresponds to an eigen mode of the configuration without flame) yields:

$$f(k_0+\delta k, 0+n) \approx f(k_0,0) + \delta k.\frac{\partial f}{\partial k}(k_0,0) + n.\frac{\partial f}{\partial n}(k_0,0) = 0$$

- Calculus of $f(k_0,0)$:

$$f(k_0,0) = tan(k_0 l_1) - cotan(k_0 l_2) + tan(k_0 l_3) = 0$$

- Calculus of $\frac{\partial f}{\partial k}(k_0,0)$:

$$\frac{\partial f}{\partial k} = n.\frac{\partial}{\partial k}(e^{ikc\tau}(1+i.tan(kl_1))) + i.\frac{\partial}{\partial k}(tan(kl_1) - cotan(kl_2) + tan(kl_3))$$

  So,

$$\frac{\partial f}{\partial k}(k_0,0) = i.\frac{\partial}{\partial k}(tan(k_0 l_1) - cotan(k_0 l_2) + tan(k_0 l_3))$$

- Calculus of $\frac{\partial f}{\partial n}(k_0,0)$:

$$\frac{\partial f}{\partial n} = \frac{\partial}{\partial n}(n.e^{ikc\tau}.(1+i.tan(kl_1))) + \frac{\partial}{\partial n}(i.(tan(kl_1) - cotan(kl_2) + tan(kl_3)))$$

$$\frac{\partial f}{\partial n} = e^{ikc\tau}.(1+i.tan(kl_1))$$

  So,

$$\frac{\partial f}{\partial n}(k_0,0) = e^{ik_0 c\tau}.(1+i.tan(k_0 l_1))$$

Finally, the calculus of $f(k_0+\delta k, 0+n)$ leads to:

$$\delta k = \frac{(n.e^{ik_0 c\tau}.(i - tan(k_0 l_1))}{l_1.(1+tan^2(k_0 l_1)) + l_2.(1+cotan^2(k_0 l_2)) + l_3.(1+tan^2(k_0 l_3))}$$

The frequency shift of the eigen mode, in comparison with the case without combustion, on real and imaginary parts can be deduced:

$$\delta f_r = \delta k_r.\frac{c}{2\pi} = \frac{c}{2\pi}.\frac{-n.cos(\omega_0\tau).tan(k_0 l_1) - nsin(\omega_0\tau)}{l_1.(1+tan^2(k_0 l_1)) + l_2.(1+cotan^2(k_0 l_2)) + l_3.(1+tan^2(k_0 l_3))} \quad (3.68)$$

$$\delta f_i = \delta k_i.\frac{c}{2\pi} = \frac{c}{2\pi}.\frac{n.cos(\omega_0\tau) - nsin(\omega_0\tau).tan(k_0 l_1)}{l_1.(1+tan^2(k_0 l_1)) + l_2.(1+cotan^2(k_0 l_2)) + l_3.(1+tan^2(k_0 l_3))} \quad (3.69)$$

Features of the configuration

To compare analytics with soundtube1_5d results, the following features are chosen:

- $n = \rho c n_p = 0.01$
- $\tau = \tau_p = 10^{-5}\, s$

- $T = 2000\,K$
- $P = 1.013\,bars$
- $S = 7 \times 10^{-4}\,m^2$
- $W = 0.02897\,kg/mol$
- $\gamma = 1.4$
- $l_1 = 0.25\,m$
- $l_2 = 0.35\,m$
- $l_3 = 0.2\,m$

3.9.2 soundtube1_5D input file

```
[Geometry]

TubeNumber=          3
ReflectionNumber=    3
IntersecNumber=      1


[Reflection1]
TubeReflect=     1 , -
Refl_numorder_denomorder= 0 , 0
R_num0=                   1.0 , 0.0
R_denom0=                 1.0 , 0.0

[Reflection2]
TubeReflect=     2 , +
Refl_numorder_denomorder= 0 , 0
R_num0=                   -1.0 , 0.0
R_denom0=                 1.0 , 0.0

[Reflection3]
TubeReflect=     3 , -
Refl_numorder_denomorder= 0 , 0
R_num0=                   1.0 , 0.0
R_denom0=                 1.0 , 0.0

[Reflection4]
TubeReflect=     4 , +
Refl_numorder_denomorder= 0 , 0
R_num0=                   1.0 , 0.0
R_denom0=                 1.0 , 0.0

[Tube1]
```

```
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.25
surface=          3E-2
coordxyz_minus=   0.0 , 0.0 , 0.0
coordxyz_plus=    0.25 , 0.0 , 0.0

[Tube2]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.35
surface=          3E-2
coordxyz_minus=   0.25 , 0.0 , 0.0
coordxyz_plus=    0.60 , 0.0 , 0.0

[Tube3]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.2
surface=          3E-2
coordxyz_minus=   0.25 , 0.2866 , 0.0
coordxyz_plus=    0.25 , 0.0866 , 0.0


[Intersection1]
N_numorder_denomorder=    0 , 0
n_num0=                   0.01 , 0.0
n_denom0=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 1.0E-5 , 0.0
tau_denom0=               1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                  0.01 , 0.0
np_denom0=                1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=                1.0E-5 , 0.0
taup_denom0=              1.0 , 0.0

connectTubeNum=       3
```

```
connectTube1=           1 , +
connectTube2=           2 , -
connectTube3=           3 , +
flame_ref_tube=         1 , +

[Simulation]

option=       modeshape3d

LowerLeft_domain=       1.0 , -100.0
UpperRight_domain=      2000.0 , 100.0

LS_tube=                2
LS_relative_position=   0.24
LS_amplitude=           1.0
LS_phase=               0.0
LS_beginfreq=           1.0
LS_endfreq=             2000.0
LS_resol_forcing=       2.0

modeshape_freq=         307.52 , 1.06
modeshape_resol=        0.001
modeshape_time=         0.019
modeshape_TubeNum=      2
modeshape_Tube1=        1 , -
modeshape_Tube2=        2 , -

[Output]

Resultfile=               in-out/tflam.res
geometryfile=             in-out/tflam_geom.dat
fluxResultfile=           in-out/tflam_FluxForcing.dat
totenergyResultfile=      in-out/tflam_totEForcing.dat
modeshapefile=            in-out/modes/tflam.mode
xmgrcmdfile=              xmgrmodecmd.dat
meshplot3d_file=          in-out/modes/tflammesh3D.g
solplot3d_file=           in-out/modes/tflamsol3D.q
```

3.9.3 soundtube1_5D results and comparison with analytical results

Modal method

mode1	Re(f):3.07520962306743e+02	Im(f):1.05903645501529e+00
mode2	Re(f):9.86078465870147e+02	Im(f):1.17858427166319e-01
mode3	Re(f):1.22991758281190e+03	Im(f):4.94382880240250e-02
mode4	Re(f):1.96230183280806e+03	Im(f):1.57705721551234e+00

Hybrid method

mode1_5D[1]:	Re(f)=3.075266e+02	Im(f)=1.285341e+00
mode1_5D[2]:	Re(f)=9.860783e+02	Im(f)=6.001217e-02
mode1_5D[3]:	Re(f)=1.229924e+03	Im(f)=5.977145e-01
mode1_5D[4]:	Re(f)=1.962307e+03	Im(f)=1.541549e+00

Energyforcing_dom method

mode 1:	Re(f)=3.075210e+02	Im(f)=1.059036e+00
mode 2:	Re(f)=9.860785e+02	Im(f)=1.178584e-01
mode 3:	Re(f)=1.229918e+03	Im(f)=4.943829e-02
mode 4:	Re(f)=1.962291e+03	Im(f)=1.570058e+00

Fluxforcing method

The figure 3.18 shows the results of the “fluxforcing” option.

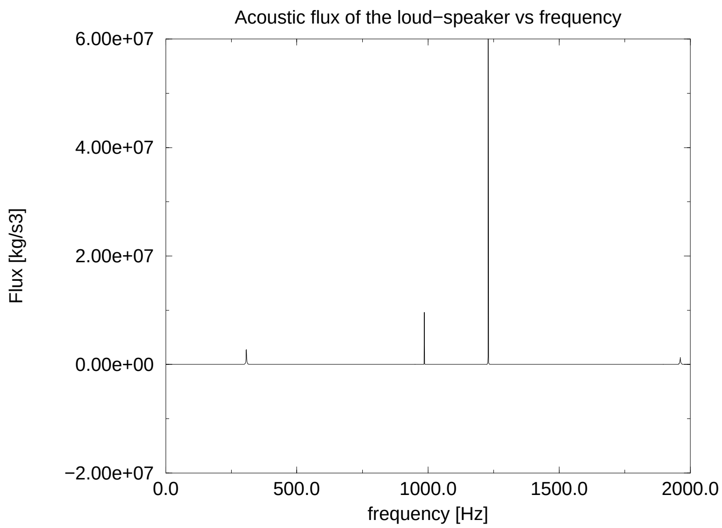


Figure 3.18: Acoustic flux of the loudspeaker in the case of the “T” configuration with flame

Analytical results

The modes without combustion are the modes of the example 8, that is to say:
$f_{01} = 308.1856\,Hz$ and Im(f_{01})=0
$f_{02} = 985.547\,Hz$ and Im(f_{02})=0
$f_{03} = 1229.854\,Hz$ and Im(f_{03})=0
$f_{04} = 1963.008\,Hz$ and Im(f_{04})=0
In this situation, the relations 3.68, 3.69 give the following results:

- mode 1:
  $\Re(\delta f)$ =-0.6657 so Re(f)=307.520 Hz
  $\Im(\delta f)$ =1.0632 so Im(f)=1.0632 Hz

- mode 2:
  $\Re(\delta f)$=0.5336 so Re(f)=986.081 Hz
  $\Im(\delta f)$=0.1182 so Im(f)=0.1182 Hz

- mode 3:
  $\Re(\delta f)$=0.0636 so Re(f)=1229.904 Hz
  $\Im(\delta f)$=0.0495 so Im(f)=0.0495 Hz

- mode 4:
  $\Re(\delta f)$=-0.7091 so Re(f)=1962.299 Hz
  $\Im(\delta f)$=1.5816 so Im(f)=1.5816 Hz

3.10 Tube in a "ring" configuration without flame

3.10.1 General features and analytical results

The configuration of the figure 3.19 is considered. Although it can be viewed as an one-dimensional device, it couldn't be treated with the former release of the code named "sound-tube". That's why this configuration is considered 1.5 dimensional.

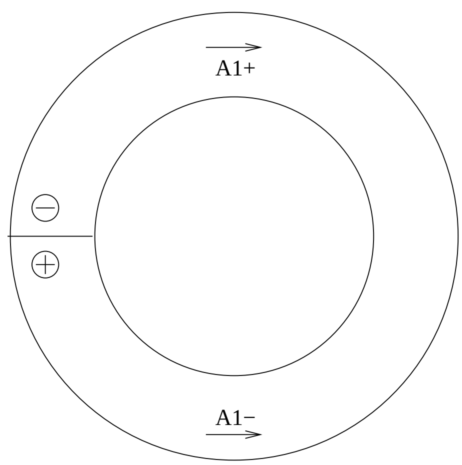


Figure 3.19: "Ring" configuration without flame

The features of this configuration are:

- Mean temperature T
- Mean pressure P
- Section S
- Length l

Jump-relation for pressure

The general relation customized to the current case yields:

$$p'_1(z_1^+) = p'_1(z_1^-) \Longrightarrow A_1^+.e^{i.k.l} + A_1^-.e^{-i.k.l} = A_1^+ + A_1^-$$

This relation can also be written as follow:

$$sin(\frac{kl}{2}).(A_1^+.e^{\frac{i.k.l}{2}} - A_1^-.e^{\frac{-i.k.l}{2}}) = 0$$

Here a new hypothesis is retained:

$$sin(\frac{kl}{2}) \neq 0 \tag{3.70}$$

As a consequence, the following equation can be deduced from the pressure continuity:

$$A_1^- = A_1^+.e^{i.k.l} \tag{3.71}$$

Jump-relation for speed

In this case, it becomes:

$$S.u'_1(z_1^+) - S.u'_1(z_1^-) = 0 \Longrightarrow A_1^+.e^{i.k.l} - A_1^-.e^{-i.k.l} - A_1^+ + A_1^- = 0$$

Thanks to the precedent hypothesis (relation 3.70) it yields:

$$A_1^+.e^{\frac{i.k.l}{2}} + A_1^-.e^{\frac{-i.k.l}{2}} = 0$$

By combining this last eqaution with relation 3.71, it yields the following contrary conclusion:

$$A_1^+.e^{\frac{i.k.l}{2}} = 0 \Longrightarrow IMPOSSIBLE$$

This last assumption is not acceptable.
As a consequence, the hypothesis 3.70 can not be satisfied, so the eigen frequencies are given by:

$$sin(\frac{k_0 l}{2}) = 0 \Longrightarrow k_0.l = 2.m.\pi,\ m \in \mathbb{N} \tag{3.72}$$

Features of the configuration

To compare analytics with soundtube1_5d results, the following features are chosen:

- $T = 2000\,K$
- $P = 1.013\,bars$
- $S = 0.03\,m^2$
- $W = 0.02897\,kg/mol$
- $\gamma = 1.4$
- $l = 1.0\,m$

3.10.2 soundtube1_5D input file

```
[Geometry]

TubeNumber=         1
ReflectionNumber=   0
IntersecNumber=     1


[Reflection1]
TubeReflect=     1 , -
Refl_numorder_denomorder= 0 , 0
R_num0=                   1.0 , 0.0
R_denom0=                 1.0 , 0.0

[Reflection2]
TubeReflect=     2 , +
Refl_numorder_denomorder= 0 , 0
R_num0=                   -1.0 , 0.0
R_denom0=                 1.0 , 0.0

[Tube1]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           1.0
surface=          3E-2
coordxyz_minus=   0.0 , 0.0 , 0.0
coordxyz_plus=    0.0 , 0.0 , 0.0

[Intersection1]
N_numorder_denomorder=    0 , 0
n_num0=                   0.0 , 0.0
n_denom0=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 0.0 , 0.0
tau_denom0=               1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                  0.0 , 0.0
np_denom0=                1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=                0.0 , 0.0
taup_denom0=              1.0 , 0.0

connectTubeNum=        2
```

```
connectTube1=          1 , +
connectTube2=          1 , -
flame_ref_tube=        1 , +

[Simulation]

option=        energyforcing_dom

LowerLeft_domain=      1.0 , -100.0
UpperRight_domain=     2000.0 , 100.0

LS_tube=               1
LS_relative_position=  0.15
LS_amplitude=          1.0
LS_phase=              0.0
LS_beginfreq=          1.0
LS_endfreq=            2000.0
LS_resol_forcing=      2.0

modeshape_freq=        896.43 , 0.0
modeshape_resol=       0.001
modeshape_time=        0.019
modeshape_TubeNum=     1
modeshape_Tube1=       1 , -

[Output]

Resultfile=              in-out/ring.res
geometryfile=            in-out/ring_geom.dat
fluxResultfile=          in-out/ring_FluxForcing.dat
totenergyResultfile=     in-out/ring_totEForcing.dat
modeshapefile=           in-out/modes/ring.mode
xmgrcmdfile=             xmgrmodecmd.dat
meshplot3d_file=         in-out/modes/ringmesh3D.g
solplot3d_file=          in-out/modes/ringsol3D.q
```

3.10.3 soundtube1_5D results and comparison with analytical results

Modal method

mode1	Re(f):8.96437393240598e+02	Im(f):-1.54451567444292e-13
mode2	Re(f):1.79287478648120e+03	Im(f):4.20079092495351e-13

Hybrid method

This method can not be applied in this case since there is no reflection coefficient.

Energyforcing_dom method

mode 1: Re(f)=8.964374e+02 Im(f)=3.073009e-13
mode 2: Re(f)=1.792875e+03 Im(f)=-3.156686e-13

Energyforcing method

The figure 3.20 shows the results of the “energyforcing” option.

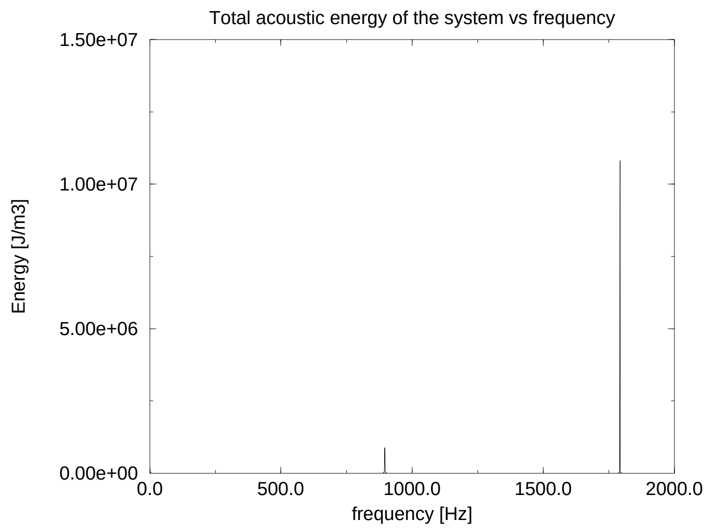


Figure 3.20: Acoustic energy in the case of the “ring” configuration

Analytical results

In this situation, the relation 3.72 give the following results:

mode 1: Re(f)=f_r=896.4374 Im(f)=f_i=0
mode 2: Re(f)=f_r=1792.875 Im(f)=f_i=0

3.11 Tube in a “cross” configuration without flame

3.11.1 General features and analytical results

The configuration of the figure 3.21 is considered. To find more easily an analytical relation, the following simplifications are applied:

- Same mean temperature in all the tubes:

$$T_1 = T_2 = T_3 = T_4 = T$$

- Same mean pressure in all the tubes:

$$P_1 = P_2 = P_3 = P_4 = P$$

- Same section in all the tubes:

$$S_1 = S_2 = S_3 = S_4 = S$$

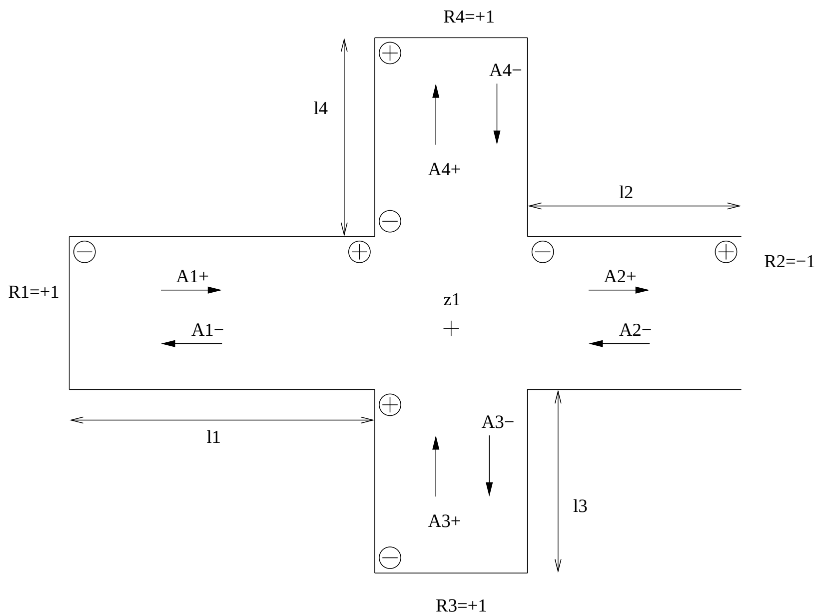


Figure 3.21: “Cross” configuration without flame

Reflection relations

According to these hypothesis and the definition of a reflection coefficient, the reflection relations are in the current case:

- 1^{st} reflection condition:
$$R_1 = 1 \Longrightarrow A_1^+ = A_1^- \tag{3.73}$$
- 2^{nd} reflection condition:
$$R_2 = -1 \Longrightarrow A_2^+ e^{i.2.k.l_2} = -A_2^- \tag{3.74}$$
- 3^{rd} reflection condition:
$$R_3 = 1 \Longrightarrow A_3^+ = A_3^- \tag{3.75}$$
- 4^{th} reflection condition:
$$R_4 = 1 \Longrightarrow A_4^+ e^{i.2.k.l_4} = A_4^- \tag{3.76}$$

Jump-relations for pressure

These relations are in the current case:

- First jump-relation for pressure:
$$p'_1(z_1) = p'_2(z_1) \Longrightarrow A_1^+.e^{i.k.l_1} + A_1^-.e^{-i.k.l_1} = A_2^+ + A_2^-$$
By using the relations 3.73 and 3.74 and by supposing $sin(kl_2) \neq 0$ it becomes:
$$A_2^+ = A_1^+.\frac{i.e^{-i.k.l_2} cos(kl_1)}{sin(kl_2)} \tag{3.77}$$

- Second jump-relation for pressure:

$$p'_1(z_1) = p'_3(z_1) \Longrightarrow A_1^+.e^{i.k.l_1} + A_1^-.e^{-i.k.l_1} = A_3^+.e^{i.k.l_3} + A_3^-.e^{-i.k.l_3}$$

By using the relations 3.73 and 3.75 and by supposing $cos(kl_3) \neq 0$ it becomes:

$$A_3^+ = A_1^+.\frac{cos(kl_1)}{cos(kl_3)} \tag{3.78}$$

- Third jump-relation for pressure:

$$p'_1(z_1) = p'_4(z_1) \Longrightarrow A_1^+.e^{i.k.l_1} + A_1^-.e^{-i.k.l_1} = A_4^+ + A_4^-$$

By using the relations 3.73 and 3.76 and by supposing $cos(kl_4) \neq 0$ it becomes:

$$A_4^+ = A_1^+.\frac{e^{-i.k.l_4}cos(kl_1)}{cos(kl_4)} \tag{3.79}$$

Jump-relation for speed

The general jump-relation for speed is applied in the current case. It is reminded that the orientation of each tube is essential in this relation.

$$S.u'_1(z_1) - S.u'_2(z_1) + S.u'_3(z_1) - S.u'_4(z_1) = 0$$

By introducing the “Riemann invariants”, it yields:

$$A_1^+.e^{i.k.l_1} - A_1^-.e^{-i.k.l_1} - A_2^+ + A_2^- + A_3^+.e^{i.k.l_3} - A_3^+.e^{-i.k.l_3} - A_4^+ + A_4^- = 0$$

By using firstly the relations 3.73, 3.74, 3.75, 3.76 and secondly the relations 3.77, refex8press3 and 3.79, the precedent equation can be simplified as follow:

$$A_1^+.[sin(kl_1) - cos(kl_1).cotan(kl_2) + cos(kl_1).tan(kl_3) + cos(kl_1).tan(kl_4)] = 0 \tag{3.80}$$

Characteristic function

If $A_1^+ \neq 0$ and if $cos(kl_1) \neq 0$ then the relation 3.80 leads to the following characteristic function:

$$tan(kl_1) - cotan(kl_2) + tan(kl_3) + tan(kl_4) = 0 \tag{3.81}$$

Solve this equation is not trivial and not necessary to compare the soundtube1_5D solutions with analytics. Instead of, results given by soundtube1_5D are introduced in equation 3.81 to verify if they satisfy it.

Features of the configuration

To compare analytics with soundtube1_5d results, the following features are chosen:

- $T = 2000\,K$
- $P = 1.013\,bars$
- $S = 0.03\,m^2$

- $W = 0.02897\,kg/mol$
- $\gamma = 1.4$
- $l_1 = 0.25\,m$
- $l_2 = 0.35\,m$
- $l_3 = 0.2\,m$
- $l_4 = 0.4\,m$

3.11.2 soundtube1_5D input file

```
[Geometry]

TubeNumber=           4
ReflectionNumber=     4
IntersecNumber=       1


[Reflection1]
TubeReflect=                 1 , -
Refl_numorder_denomorder= 0 , 0
R_num0=                      1.0 , 0.0
R_denom0=                    1.0 , 0.0

[Reflection2]
TubeReflect=                 2 , +
Refl_numorder_denomorder= 0 , 0
R_num0=                      -1.0 , 0.0
R_denom0=                    1.0 , 0.0

[Reflection3]
TubeReflect=                 3 , -
Refl_numorder_denomorder= 0 , 0
R_num0=                      1.0 , 0.0
R_denom0=                    1.0 , 0.0

[Reflection4]
TubeReflect=                 4 , +
Refl_numorder_denomorder= 0 , 0
R_num0=                      1.0 , 0.0
R_denom0=                    1.0 , 0.0

[Tube1]
temperature=        2000.0
pressure=           1.013E5
weight=             0.02897
```

```
gamma=            1.4
length=           0.25
surface=          3E-2
coordxyz_minus=   0.0 , 0.0 , 0.0
coordxyz_plus=    0.25 , 0.0 , 0.0

[Tube2]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.35
surface=          3E-2
coordxyz_minus=   0.25 , 0.0 , 0.0
coordxyz_plus=    0.60 , 0.0 , 0.0

[Tube3]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.2
surface=          3E-2
coordxyz_minus=   0.25 , -0.2866 , 0.0
coordxyz_plus=    0.25 , -0.0866 , 0.0

[Tube4]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.4
surface=          3E-2
coordxyz_minus=   0.25 , 0.0866 , 0.0
coordxyz_plus=    0.25 , 0.4866 , 0.0


[Intersection1]
N_numorder_denomorder=    0 , 0
n_num0=                   0.0 , 0.0
n_denom0=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 0.0 , 0.0
tau_denom0=               1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                  0.0 , 0.0
np_denom0=                1.0 , 0.0
```

```
Taup_numorder_denomorder= 0 , 0
taup_num0=                0.0 , 0.0
taup_denom0=              1.0 , 0.0

connectTubeNum=        4

connectTube1=          1 , +
connectTube2=          2 , -
connectTube3=          3 , +
connectTube4=          4 , -
flame_ref_tube =       1 , +

[Simulation]

option=       energyforcing

LowerLeft_domain=      1.0 , -20.0
UpperRight_domain=     2000.0 , 100.0

LS_tube=               2
LS_relative_position=  0.17
LS_amplitude=          1.0
LS_phase=              0.0
LS_beginfreq=          1.0
LS_endfreq=            2000.0
LS_resol_forcing=      2.0

modeshape_freq=        234.1  , 0.0
modeshape_resol=       0.01
modeshape_time=        0.019
modeshape_TubeNum=     2
modeshape_Tube1=       3 , -
modeshape_Tube2=       4 , -

[Output]

Resultfile=               in-out/cross.res
geometryfile=             in-out/cross_geom.dat
fluxResultfile=           in-out/cross_FluxForcing.dat
totenergyResultfile=      in-out/cross_totEForcing.dat
modeshapefile=            in-out/modes/cross.mode
xmgrcmdfile=              xmgrmodecmd.dat
meshplot3d_file=          in-out/modes/crossmesh3D.g
solplot3d_file=           in-out/modes/crosssol3D.q
```

3.11.3 soundtube1_5D results and comparison with analytical results

Modal method

mode1	Re(f):2.34095380140721e+02	Im(f):4.08833767298616e-15
mode2	Re(f):6.56960831416439e+02	Im(f):6.11267779389330e-15
mode3	Re(f):9.88878853041694e+02	Im(f):-1.24399251380061e-14
mode4	Re(f):1.22844457676004e+03	Im(f):1.84844489826468e-14
mode5	Re(f):1.56590631379724e+03	Im(f):1.80332032372472e-14

Hybrid method

mode1_5D[1]:	Re(f)=2.340951e+02	Im(f)=0.000000e+00
mode1_5D[2]:	Re(f)=6.569606e+02	Im(f)=2.639971e-14
mode1_5D[3]:	Re(f)=9.888789e+02	Im(f)=0.000000e+00
mode1_5D[4]:	Re(f)=1.228466e+03	Im(f)=-5.279941e-14
mode1_5D[5]:	Re(f)=1.565907e+03	Im(f)=-2.639971e-14

energyforcing_dom method

mode 1:	Re(f)=2.340954e+02	Im(f)=2.287732e-14
mode 2:	Re(f)=6.569608e+02	Im(f)=1.526419e-15
mode 3:	Re(f)=9.888789e+02	Im(f)=-2.282694e-14
mode 4:	Re(f)=1.228445e+03	Im(f)=-4.397936e-15
mode 5:	Re(f)=1.565906e+03	Im(f)=-3.984808e-15

Energyforcing method

The figure 3.22 shows the results of the “energyforcing” option.

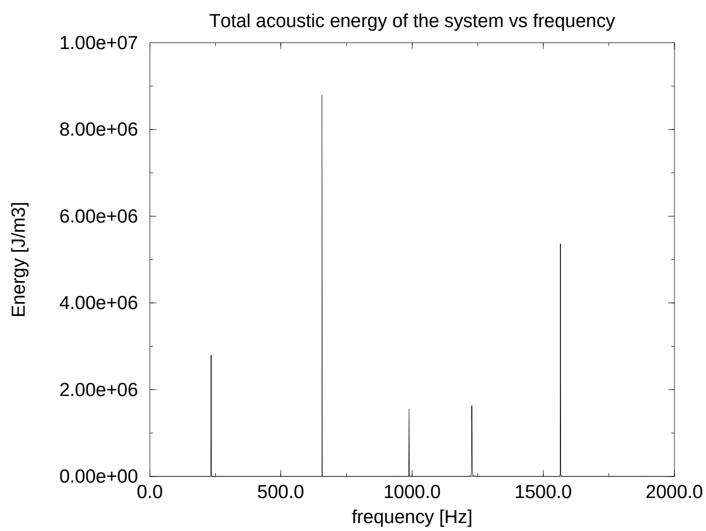


Figure 3.22: Acoustic energy in the case of a “cross” configuration

Analytical results

In this case, explicit relations for eigen frequencies can not be obtained but the characteristic function is available. As a consequence, soundtube results are introduced into the characteristic function (relation 3.81) to verify if they satisfy it. Since the results of all the methods are very similar, only the results of the modal method are intorduced into the equation 3.81.

- mode 1: $f_0 = 234.095\,Hz$ and $k = \frac{2\pi f_0}{c}$

$$tan(kl_1) - cotan(kl_2) + tan(kl_3) + tan(kl_4) = -6 \times 10^{-6}$$

- mode 2: $f_0 = 656.961\,Hz$ and $k = \frac{2\pi f_0}{c}$

$$tan(kl_1) - cotan(kl_2) + tan(kl_3) + tan(kl_4) = 9 \times 10^{-6}$$

- mode 3: $f_0 = 988.879\,Hz$ and $k = \frac{2\pi f_0}{c}$

$$tan(kl_1) - cotan(kl_2) + tan(kl_3) + tan(kl_4) = 1 \times 10^{-5}$$

- mode 4: $f_0 = 1228.445\,Hz$ and $k = \frac{2\pi f_0}{c}$

$$tan(kl_1) - cotan(kl_2) + tan(kl_3) + tan(kl_4) = 9 \times 10^{-5}$$

- mode 5: $f_0 = 1565.906\,Hz$ and $k = \frac{2\pi f_0}{c}$

$$tan(kl_1) - cotan(kl_2) + tan(kl_3) + tan(kl_4) = -1 \times 10^{-5}$$

3.12 A first attempt of “turbine” configuration

3.12.1 General features

The configuration of the figure 3.23 is considered. The figure 3.24 gives the “soundtube1_5d modelization”. There is no combustion in this example. In this example, the analytical solution is not available. Only the possibility of soundtube1_5d are tested in this case.

The values of the different parameters of the configuration are:

- $T_a = 300\,K$, $T_b = 1000\,K$, $T_c = 2000\,K$
- $P = 1.013\,bars$
- $S_a = 2 \times 10^{-3}\,m^2$, $S_b = 7.8 \times 10^{-3}\,m^2$
- $W = 0.02897\,kg/mol$
- $\gamma = 1.4$
- $l_e = 0.18\,m$
- $L_e = 0.267\,m$

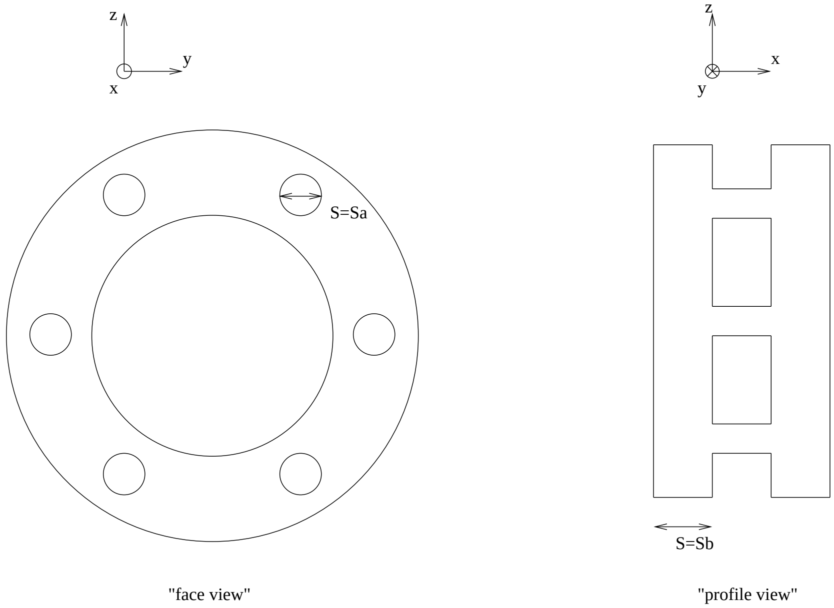


Figure 3.23: “Turbine” configuration used with soundtube1_5d

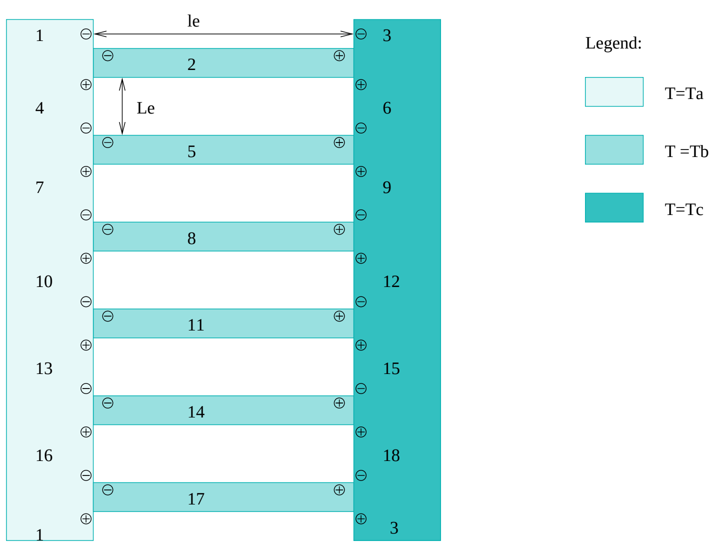


Figure 3.24: Soundtube1_5d modelization of the “Turbine” configuration

3.12.2 soundtube1_5D input file

```
[Geometry]

TubeNumber=          18
ReflectionNumber=    0
IntersecNumber=      12

[Tube1]
temperature=       300.0
```

```
pressure=        1.013E5
weight=          0.02897
gamma=           1.4
length=          0.267
surface=         7.8E-3
coordxyz_minus=  0.134 , 0.231 , 0.0
coordxyz_plus=   0.401 , 0.231 , 0.0

[Tube2]
temperature=     1000.0
pressure=        1.013E5
weight=          0.02897
gamma=           1.4
length=          0.18
surface=         2E-3
coordxyz_minus=  0.134 , 0.231 , 0.0
coordxyz_plus=   0.134 , 0.231 , 0.18

[Tube3]
temperature=     2000.0
pressure=        1.013E5
weight=          0.02897
gamma=           1.4
length=          0.267
surface=         7.8E-3
coordxyz_minus=  0.134 , 0.231 , 0.18
coordxyz_plus=   0.401 , 0.231 , 0.18

[Tube4]
temperature=     300.0
pressure=        1.013E5
weight=          0.02897
gamma=           1.4
length=          0.267
surface=         7.8E-3
coordxyz_minus=  0.0 , 0.0 , 0.0
coordxyz_plus=   0.134 , 0.231 , 0.0

[Tube5]
temperature=     1000.0
pressure=        1.013E5
weight=          0.02897
gamma=           1.4
length=          0.18
surface=         2E-3
coordxyz_minus=  0.0 , 0.0 , 0.0
coordxyz_plus=   0.0 , 0.0 , 0.18

[Tube6]
```

```
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.267
surface=          7.8E-3
coordxyz_minus=   0.0 , 0.0 , 0.18
coordxyz_plus=    0.134 , 0.231 , 0.18

[Tube7]
temperature=      300.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.267
surface=          7.8E-3
coordxyz_minus=   0.134 , -0.231 , 0.0
coordxyz_plus=    0.0 , 0.0 , 0.0

[Tube8]
temperature=      1000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.18
surface=          2E-3
coordxyz_minus=   0.134 , -0.231 , 0.0
coordxyz_plus=    0.134 , -0.231 , 0.18

[Tube9]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.267
surface=          7.8E-3
coordxyz_minus=   0.134 , -0.231 , 0.18
coordxyz_plus=    0.0 , 0.0 , 0.18

[Tube10]
temperature=      300.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.267
surface=          7.8E-3
coordxyz_minus=   0.401 , -0.231 , 0.0
coordxyz_plus=    0.134 , -0.231 , 0.0
```

```
[Tube11]
temperature=      1000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.18
surface=          2E-3
coordxyz_minus=   0.401 , -0.231 , 0.0
coordxyz_plus=    0.401 , -0.231 , 0.18

[Tube12]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.267
surface=          7.8E-3
coordxyz_minus=   0.401 , -0.231 , 0.18
coordxyz_plus=    0.134 , -0.231 , 0.18

[Tube13]
temperature=      300.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.267
surface=          7.8E-3
coordxyz_minus=   0.534 , 0.0 , 0.0
coordxyz_plus=    0.401 , -0.231 , 0.0

[Tube14]
temperature=      1000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.18
surface=          2E-3
coordxyz_minus=   0.534 , 0.0 , 0.0
coordxyz_plus=    0.534 , 0.0 , 0.18

[Tube15]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.267
surface=          7.8E-3
coordxyz_minus=   0.534 , 0.0 , 0.18
coordxyz_plus=    0.401 , -0.231 , 0.18
```

```
[Tube16]
temperature=      300.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.267
surface=          7.8E-3
coordxyz_minus=   0.401 , 0.231 , 0.0
coordxyz_plus=    0.534 , 0.0 , 0.0

[Tube17]
temperature=      1000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.18
surface=          2E-3
coordxyz_minus=   0.401 , 0.231 , 0.0
coordxyz_plus=    0.401 , 0.231 , 0.18

[Tube18]
temperature=      2000.0
pressure=         1.013E5
weight=           0.02897
gamma=            1.4
length=           0.267
surface=          7.8E-3
coordxyz_minus=   0.401 , 0.231 , 0.18
coordxyz_plus=    0.534 , 0.0 , 0.18

[Intersection1]
N_numorder_denomorder=    0 , 0
n_num0=                   0.0 , 0.0
n_denom0=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 0.0 , 0.0
tau_denom0=               1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                  0.0 , 0.0
np_denom0=                1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=                0.0 , 0.0
taup_denom0=              1.0 , 0.0

connectTubeNum=        3
```

```

connectTube1=          1 , -
connectTube2=          2 , -
connectTube3=          4 , +
flame_ref_tube=        1 , -

[Intersection2]
N_numorder_denomorder=    0 , 0
n_num0=                   0.0 , 0.0
n_denom0=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 0.0 , 0.0
tau_denom0=               1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                  0.0 , 0.0
np_denom0=                1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=                0.0 , 0.0
taup_denom0=              1.0 , 0.0

connectTubeNum=        3

connectTube1=          2 , +
connectTube2=          3 , -
connectTube3=          6 , +
flame_ref_tube=        2 , +

[Intersection3]
N_numorder_denomorder=    0 , 0
n_num0=                   0.0 , 0.0
n_denom0=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 0.0 , 0.0
tau_denom0=               1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                  0.0 , 0.0
np_denom0=                1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=                0.0 , 0.0
taup_denom0=              1.0 , 0.0

connectTubeNum=        3
```

```
connectTube1=          4 , -
connectTube2=          5 , -
connectTube3=          7 , +
flame_ref_tube=        4 , -

[Intersection4]
N_numorder_denomorder=    0 , 0
n_num0=                   0.0 , 0.0
n_denom0=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 0.0 , 0.0
tau_denom0=               1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                  0.0 , 0.0
np_denom0=                1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=                0.0 , 0.0
taup_denom0=              1.0 , 0.0

connectTubeNum=        3

connectTube1=          5 , +
connectTube2=          6 , -
connectTube3=          9 , +
flame_ref_tube=        5 , +

[Intersection5]
N_numorder_denomorder=    0 , 0
n_num0=                   0.0 , 0.0
n_denom0=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 0.0 , 0.0
tau_denom0=               1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                  0.0 , 0.0
np_denom0=                1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=                0.0 , 0.0
taup_denom0=              1.0 , 0.0

connectTubeNum=        3

connectTube1=          7 , -
```

```
connectTube2=          8 , -
connectTube3=          10 , +
flame_ref_tube=        7 , -

[Intersection6]
N_numorder_denomorder=    0 , 0
n_num0=                   0.0 , 0.0
n_denom0=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 0.0 , 0.0
tau_denom0=               1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                  0.0 , 0.0
np_denom0=                1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=                0.0 , 0.0
taup_denom0=              1.0 , 0.0

connectTubeNum=        3

connectTube1=          8 , +
connectTube2=          9 , -
connectTube3=          12 , +
flame_ref_tube=        8 , +

[Intersection7]
N_numorder_denomorder=    0 , 0
n_num0=                   0.0 , 0.0
n_denom0=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 0.0 , 0.0
tau_denom0=               1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                  0.0 , 0.0
np_denom0=                1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=                0.0 , 0.0
taup_denom0=              1.0 , 0.0

connectTubeNum=        3

connectTube1=          10 , -
connectTube2=          11 , -
```

```
connectTube3=           13 , +
flame_ref_tube=         10 , -

[Intersection8]
N_numorder_denomorder=    0 , 0
n_num0=                   0.0 , 0.0
n_denom0=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 0.0 , 0.0
tau_denom0=               1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                  0.0 , 0.0
np_denom0=                1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=                0.0 , 0.0
taup_denom0=              1.0 , 0.0

connectTubeNum=         3

connectTube1=           11 , +
connectTube2=           12 , -
connectTube3=           15 , +
flame_ref_tube=         11 , +

[Intersection9]
N_numorder_denomorder=    0 , 0
n_num0=                   0.0 , 0.0
n_denom0=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 0.0 , 0.0
tau_denom0=               1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                  0.0 , 0.0
np_denom0=                1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=                0.0 , 0.0
taup_denom0=              1.0 , 0.0

connectTubeNum=         3

connectTube1=           13 , -
connectTube2=           14 , -
connectTube3=           16 , +
```

```
flame_ref_tube=        13 , -

[Intersection10]
N_numorder_denomorder=    0 , 0
n_num0=                   0.0 , 0.0
n_denom0=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 0.0 , 0.0
tau_denom0=               1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                  0.0 , 0.0
np_denom0=                1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=                0.0 , 0.0
taup_denom0=              1.0 , 0.0

connectTubeNum=        3

connectTube1=          14 , +
connectTube2=          15 , -
connectTube3=          18 , +
flame_ref_tube=        14 , +

[Intersection11]
N_numorder_denomorder=    0 , 0
n_num0=                   0.0 , 0.0
n_denom0=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 0.0 , 0.0
tau_denom0=               1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                  0.0 , 0.0
np_denom0=                1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=                0.0 , 0.0
taup_denom0=              1.0 , 0.0

connectTubeNum=        3

connectTube1=          16 , -
connectTube2=          17 , -
connectTube3=          1 , +
flame_ref_tube=        16 , -
```

```

[Intersection12]
N_numorder_denomorder=    0 , 0
n_num0=                   0.0 , 0.0
n_denom0=                 1.0 , 0.0

Tau_numorder_denomorder=  0 , 0
tau_num0=                 0.0 , 0.0
tau_denom0=               1.0 , 0.0

Np_numorder_denomorder=   0 , 0
np_num0=                  0.0 , 0.0
np_denom0=                1.0 , 0.0

Taup_numorder_denomorder= 0 , 0
taup_num0=                0.0 , 0.0
taup_denom0=              1.0 , 0.0

connectTubeNum=        3

connectTube1=          17 , +
connectTube2=          18 , -
connectTube3=          3 , +
flame_ref_tube=        17 , +

[Simulation]

option=     modeshape3d

LowerLeft_domain=      1.0 , -20.0
UpperRight_domain=     2000.0 , 20.0

LS_tube=               4
LS_relative_position=  0.18
LS_amplitude=          1.0
LS_phase=              0.0
LS_beginfreq=          1.0
LS_endfreq=            2000.0
LS_resol_forcing=      2.0

modeshape_freq=        1452.5 , 0.0
modeshape_resol=       0.01
modeshape_time=        0.02
modeshape_TubeNum=     6
modeshape_Tube1=       1 , +
modeshape_Tube2=       4 , +
modeshape_Tube3=       7 , +
modeshape_Tube4=       10 , +
modeshape_Tube5=       13 , +
```

```
modeshape_Tube6=       16 , +

[Output]

Resultfile=              in-out/smallturb.dat
geometryfile=            in-out/smalltrub_geom.dat
totenergyResultfile=     in-out/smallturb_totEForcing.dat
fluxResultfile=          in-out/smallturb_flux1_5D.dat
modeshapefile=           in-out/modes/smallturb.mode
xmgrcmdfile=             xmgrmodecmd.dat
meshplot3d_file=         in-out/modes/smallturb_mesh3D.g
solplot3d_file=          in-out/modes/smallturb_sol3D.q
```

3.12.3 soundtube1_5D results

Modal method

In such a complicated case, modal method can not give, currently, reliable resuts. So, this method is not adviced in this sort of case.

Hybrid method

This method can not be applied for this case since there is no reflection boundary. Indeed, this method uses a forcing at a “entry” of the system, that is to say a boundary (reflection condition).

Energyforcing_dom method mode 1: Re(f)=2.856439e+02 Im(f)=-1.209283e-13

mode 2:	Re(f)=3.070250e+02	Im(f)=1.705909e-14
mode 3:	Re(f)=4.695187e+02	Im(f)=-3.275880e-13
mode 4:	Re(f)=5.905884e+02	Im(f)=1.992805e-13
mode 5:	Re(f)=6.501661e+02	Im(f)=1.191429e-15
mode 6:	Re(f)=6.792502e+02	Im(f)=3.747685e-14
mode 7:	Re(f)=8.593596e+02	Im(f)=7.655350e-14
mode 8:	Re(f)=1.070995e+03	Im(f)=-4.593839e-13
mode 9:	Re(f)=1.107459e+03	Im(f)=1.051792e-14
mode 10:	Re(f)=1.235482e+03	Im(f)=-1.119317e-15
mode 11:	Re(f)=1.300332e+03	Im(f)=-6.756055e-15
mode 12:	Re(f)=1.452518e+03	Im(f)=-6.397276e-14
mode 13:	Re(f)=1.467140e+03	Im(f)=-2.289618e-14
mode 14:	Re(f)=1.590887e+03	Im(f)=1.008439e-13
mode 15:	Re(f)=1.803388e+03	Im(f)=-2.900549e-14
mode 16:	Re(f)=1.814095e+03	Im(f)=-1.146402e-13
mode 17:	Re(f)=1.859048e+03	Im(f)=-1.364803e-13
mode 18:	Re(f)=1.950498e+03	Im(f)=8.194328e-15

Energyforcing method

The results given by the forcing method (with option “energyforcing”) can be seen on figure 3.25 and more precisely on figure 3.26. On such a configuration, the time needed by soundtube1_5d to find eigen frequencies with forcing methods is about 30 seconds on Sun station (500 MHz, 128 $Mbytes$ of RAM).

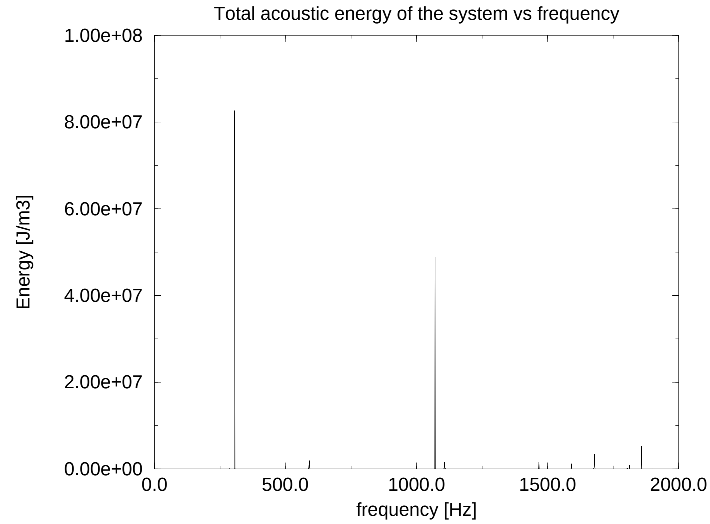


Figure 3.25: Acoustic energy in the case of a “turbine” configuration

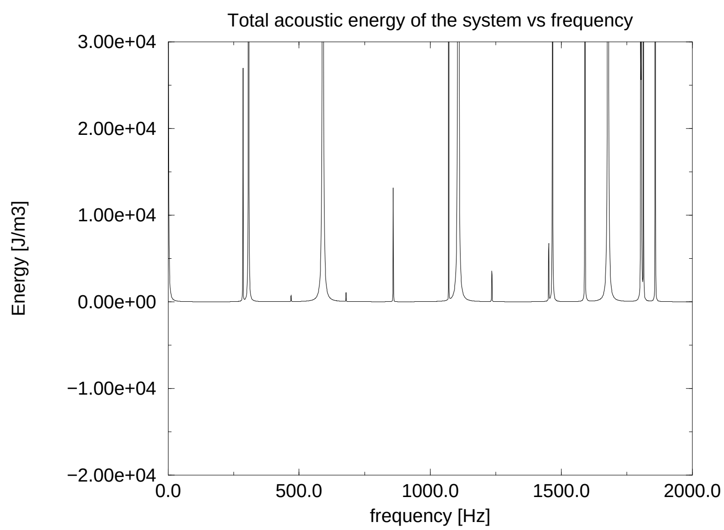


Figure 3.26: Acoustic energy in the case of a “turbine” configuration (Zoom)

Chapter 4

Tools of visualization

This chapter deals with the use of the different "macros". These are tools to facilitate the visualization of graphical results such given by the options "energyforcing", "fluxforcing","modeshape" or "modeshape3d".

4.1 Visualization of the results obtained with fluxforcing method

4.1.1 Purpose of the tool "xmgrFluxForcing"

"xmgrFluxForcing" provides a visualization of the results of the "fluxforcing" option. It uses the graphic software XMGR. In a nutshell, it displays the curve which represents the variations of the loudspeaker's acoustic flux with the (real) forcing frequency.

4.1.2 Utilization of the tool "xmgrFluxForcing"

After the execution of soundtube1_5d with the option "fluxforcing" (as reminded in chapter 2), user just has to execute "xmgrFluxForcing" with the name of the input file:

xmgrFluxForcing $input\,file\,name$

The results of the "fluxforcing" method appear at screen.

4.1.3 Auxiliaries files needed

"xmgrFluxForcing" needs also other files that, normaly, can be ignored by basic user. They are:

- xmgrFluxForcing.pl (perl script)
- xmgrFF_cmd.tmp (necessary to XMGR)
- xFluxForcing.par (parameter file of XMGR)

4.2 Visualization of the results obtained with energyforcing method

4.2.1 Purpose of the tool “xmgrEnergyForcing”

“xmgrEnergyForcing” provides a visualization of the results of the energyforcing option. It uses the graphic software XMGR. In a nutshell, it displays the curve which represents the variation of the total acoustic energy with forcing frequency.

4.2.2 Utilization of the tool “xmgrEnergyForcing”

After the execution of soundtube1_5d with the option energyforcing (as reminded in chapter 2), user just has to execute “xmgrEnergyForcing” with the name of the input file:

xmgrEnergyForcing *input file name*

The results of the “energyforcing” method appears at screen.

4.2.3 Necessary auxiliaries files

Likewise with “xmgrEnergyForcing”, “xmgrEnergyForcing” needs also other files that, normaly, can be ignored by basic user. They are:

- xmgrEnergyForcing.pl (perl script)
- xmgrEF_cmd.tmp (necessary to XMGR)
- xEForcing.par (parameter file of XMGR)

4.3 1D visualization of a mode with “xmgrmodeshape”

4.3.1 Purpose of the tool “xmgrmodeshape”

The macro “xmgrmodeshape” gives the results of “modeshape” option at screen that is to say the 1D visualization of a mode. The quantities to characterize the mode and which display at screen are:

- $Re(p'(x))$
- $Re(u'(x))$
- $arg(p'(x))$
- $arg(u'(x))$

4.3.2 Utilization of the tool "xmgrmodeshape"

Before visualizing a given mode shape with "xmgrmodeshape", user has firstly to specify different parameters in the input file as it is reminded in chapter 2, i.e:

- the frequency of the mode ("modeshape_freq")
- the spatial resolution ("modeshape_resol")
- the time at which the mode is observed ("modeshape_time"). This parameter induces a phase.
- the total number of tubes in which the modes are visualized ("modeshape_TubeNum").
- the specification of these tubes ("shapemode_Tube $number of the tube$").

Once it is done, user has to run soundtube1_5d with the option "modeshape".

After that, "xmgrmodeshape" can display the results. To be executed, it needs only the name of the input file:

xmgrmodeshape $input file name$

Thus, the variations of $Re(p'(x))$, $Re(u'(x))$, $arg(p'(x))$, $arg(u'(x))$ along the path selected appear at screen.

4.3.3 Auxiliaries files needed

"xmgrmodeshape" needs also other files that, normaly, can be ignored by basic user. They are:

- xmgrmodeshape
- xmgrmodeshape.pl (Perl script)
- xmgrmodeshape.par (parameter file for XMGR)
- the file specified by the section "xmgrcmdfile" in the input file

4.4 3D visualization of a mode

To visualize a mode in 3D, there is no particular macro. User has just to use a software of visualization such as "ENSIGHT" and select the corresponding files to use. These are written by soundtube1_5d in plot3d format: a mesh file and a solution file.

4.4.1 Parameters needed

First, user has to specify the spatial coordinate of each tube in the input file. Indeed, since the acoustic model is one dimensionnal, the only spatial parameters needed are the coordinates of the center of the sides - and + of each tube. As it is explained in the chapter 2, these arguments in the input file are named "coordxyz_plus" and "coordxyz_minus". The figure 4.1 illustrates this point.

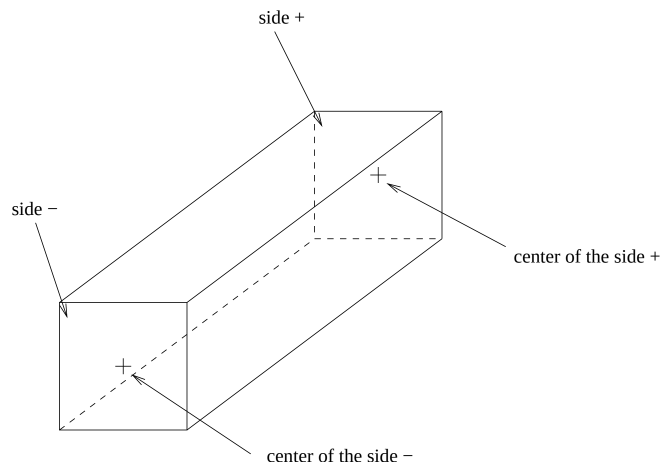


Figure 4.1: 3D visualization of a tube

As shown by the figure 4.1, a tube always appears with square section.

Second, user has to specified the different parameters which characterize the mode to visualize. These are:

- the frequency of the mode ("modeshape_freq")
- the spatial resolution ("modeshape_resol")
- the time at which the mode is observed ("modeshape_time"). This parameter induces a phase.

Finally, user has to select the option "modeshape3d" and run soundtube1_5d.

4.4.2 Visualization of the variables

The classical variables in a plot3d format are the *density*, the *energy*, the $X-momentum$, the $Y-momentum$ and the $Z-momentum$. The acoustic variables visualized replace the precedent variables in the solution file as follow:

- *density* stands for $Re(p')$
- *energy* stands for $|Re(p')|$
- $X-momentum$ stands for $arg(p')$

- $Y - momentum$ stands for $Re(u')$
- $Z - momentum$ stands for $arg(u')$

So, to visualize one of this variable, user has firstly to create the mesh by selecting the corresponding mesh file, and secondly selecting the equivalent “plot3d” variable.

4.4.3 files needed

As it has already explained, two files are needed to a 3D visualization of the modes: a mesh file and a solution file. The names of these two files is specified in the input file. Thus, “meshplot3d_file” indicates the name of the mesh file and “solplot3d_file” indicates the name of the solution file. These two files are written by soundtube1_5d when the option “modeshape3d” is selected

Bibliography

[1] T. Poinsot, D. Veynante. *Theoretical and numerical combustion*. Edwards 2001.

[2] Landau, Lifchitz *Physique theorique, Mecanique des fluides*. Ellipses 1994.