Measurements of Transfer Functions using Large Eddy Simulations: theoretical framework and first tests.

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Introduction

Thermoacoustic instabilities in combustion systems involve the acoustic characteristics of the combustor and air supply. They result from a coupling between resonant acoustic waves and fluctuating heat release rate. Acoustic waves propagate through the combustor and the reflected waves influence the combustion process. A global numerical framework is presented in this article to predict combustion instabilities in premixed flames. In this approach, the system is modelled as a network of acoustic elements. Each element may be described by its transfer matrix between inlet and outlet variable fluctuations. For flame transfer estimation two methods can be used. The first one, called PAA for Pure Acoustic Analysis, is based on the knowledge of the acoustic velocity and pressure at different points for the matrix transfer construction. The second method is based on the response of heat release to an imposed fluctuation of acoustic velocity. From this point of view, an estimation of the heat release rate and the knowledge of a reference velocity at a given point are needed. This approach illustrated by the 'n-τ' model introduced by Crocco & Cheng[2] has been useful in the past and is still of current interest. These two approaches are formally identical in a 1D case and the PAA formulation can be formulated as a function of the n-tau indices and geometrical parameters. Large Eddy Simulation (LES) could be one of the most appropriate techniques to determine transfer function in burners. Indeed in LES, fluctuations of heat release rate and velocity associated to large scale motion are spatially and temporally resolved. Finally, an estimation by LES of the predicted transfer function is compared to experimental results in a non premixed combustion chamber.

Principle of method

Fundamental equations

Equations of acoustics are derived from the main conservation equations: mass, momentum and energy (for reacting flow) or isentropic relation (for non-reacting flow), using the following assumptions (see [8]):

- zero volume forces and zero volume heat source,
- negligible viscous forces,
- linear acoustics (acoustic fluctuations are small compared to reference quantities: $p' \ll p_0$, $\rho' \ll \rho_0$, $u' \ll c_0$, where c_0 is the sound speed),
- the mean flow is small.

All investigations are done assuming longitudinal harmonic waves propagating in ducts. Then, linearizing Navier-Stokes equations leads to Eq. (1) and (2). Their derivation may be found in [8].

$$\frac{\partial u'}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial x} \quad (1)$$

$$\frac{1}{\rho_0 c_0^2}\frac{\partial p'}{\partial t} + \frac{1}{S}\frac{\partial S u'}{\partial x} = \frac{\gamma - 1}{\rho_0 c_0^2}\dot{\omega}'_T \quad (2)$$

where $\dot{\omega}'_T$ is the unsteady heat release rate and S(x) is the variable cross section.

Acoustic jump conditions for thin flames

Integrating Equations (1) and (2) from $x = x^-_{j+1}$ to $x = x^+_{j+1}$ and taking the limit when x go to x_{j+1} leads to the following jump relations (Ref. [8]):

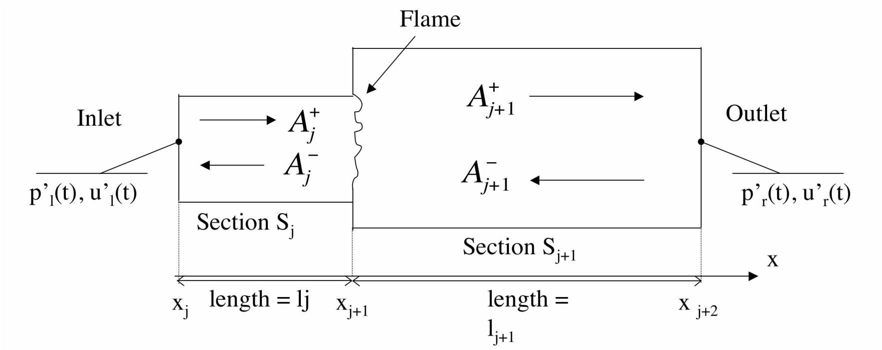

Figure 1: Jump conditions through a thin premixed flame.

$$p'_j(x_{j+1}) = p'_{j+1}(x_{j+1}) \quad (3)$$

$$S_{j+1}u'_{j+1}(x_{j+1}) - S_j u'_j(x_{j+1}) = \frac{\gamma - 1}{\rho_{j+1}c_{j+1}^2}\dot{\Omega}'_T \quad (4)$$

where ρ_{j+1} and c_{j+1} are respectively the mean density and the sound speed in section j+1 and $\dot{\Omega}'_T$ is the total unsteady heat release rate produced by the flame. Assuming constant mean temperature in each duct, the acoustic pressure and velocity signals in duct j+1 are:

$$p'_{j+1}(x,t) = (A^+_{j+1}e^{ik(x-x_{j+1})} + A^-_{j+1}e^{-ik(x-x_{j+1})})e^{i\omega t} \quad (5)$$

$$u'_{j+1}(x,t) = \frac{1}{\rho_{j+1} c_{j+1}} (A^{+}_{j+1} e^{ik(x-x_{j+1})} - A^{-}_{j+1} e^{-ik(x-x_{j+1})}) e^{i\omega t} \quad (6)$$

$$\dot{\Omega}'_T = \Omega e^{-i\omega t} \quad (7)$$

where k=ω/c is the wave number and ω is the pulsation.

Transfer matrix

A transfer matrix between the wave amplitudes in sections j+1 and j can then be derived for the interface of Fig. 1. The reader is referred to [8] for complete information.

$$\begin{pmatrix} A^{+}_{j+1} \\ A^{-}_{j+1} \end{pmatrix} = T_j \begin{pmatrix} A^{+}_{j} \\ A^{-}_{j} \end{pmatrix} + O_j \quad (8)$$

with

$$T_j = \frac{1}{2} \begin{bmatrix} e^{ikl_j}(1+\Gamma_j) & e^{-ikl_j}(1-\Gamma_j) \\ e^{ikl_j}(1-\Gamma_j) & e^{-ikl_j}(1+\Gamma_j) \end{bmatrix} \quad \text{and} \quad O_j = \frac{1}{2} \frac{\rho_{j+1} c_{j+1}}{S_{j+1}} \begin{pmatrix} \frac{\gamma - 1}{\rho_j c_j^2} \Omega \\ -\frac{\gamma - 1}{\rho_j c_j^2} \Omega \end{pmatrix} \quad (9)$$

where $l_j = x_{j+1} - x_j$ and Γ_j is defined as:

$$\Gamma_j = \frac{\rho_{j+1} c_{j+1} S_j}{\rho_j c_j S_{j+1}} \quad (10)$$

T_j is the transfer matrix and O_j is the source term due to combustion.

Two techniques are found in the combustion community to evaluate the transfer matrix:

1. The n-τ formulation, which provides a model for $\dot{\Omega}'_T$
2. The Purely Acoustic Analysis (PAA) which considers only the matrix transfer M, such as: $\begin{pmatrix} p'_N \\ u'_N \end{pmatrix} = M \begin{pmatrix} p'_1 \\ u'_1 \end{pmatrix}$.

For both methods, a perturbation is generated by forcing the inlet and /or the outlet flow rate at a fixed frequency to study the flame response.

The n-τ formulation

This model is based on the assumption that Ω depends only on the longitudinal wave amplitudes, $\Omega = \Omega(A^{+}_{N}, A^{-}_{N}, A^{+}_{1}, A^{-}_{1})$. Such a model gives only a basic description of the reality, but it allows building a useful simple function. In fact, the heat release fluctuations depend on many factors, like turbulence or chemical effects. Besides, this model is not valid if transverse modes exist. Lastly, in the n-τ model effects of pressure fluctuations on combustion are neglected compared to velocity fluctuations. For instance, a perturbation of the inlet flow rate will generate a vortex in the combustion chamber that will burn and modify the heat release rate after a time delay τ and an amplification n. The simplest law between $\dot{\Omega}'_T$ and u' is usually written following the suggestion of Crocco[2]:

$$\dot{\Omega}'_T(t) = S_1 \frac{\rho_1 c_1^2}{\gamma - 1}.n.u'(x_1, t-\tau) \tag{11}$$

where $\dot{\Omega}'_T$ is the total heat release fluctuation of the flame and u' is the velocity fluctuation measured at the flame holder.

This model represents the flame function transfer of the thin flame. It can be extended to a thick flame assuming that it is composed of successive thin reaction zones. In this case, the combustion chamber is divided in a series of ducts and the model is applied in each duct. Such a model is useful to predict flame instabilities for longitudinal modes but in real combustors, it is difficult to determine the transfer function. Large Eddy Simulation (LES) and careful experiments are the most appropriate numerical techniques to obtain n and τ.

The purely Acoustic Analysis (PAA)

A thermoacoustic system can be represented by a network of acoustic elements. In the present study, the whole system is composed of an injector, a burner with or without flame and a part of exhaust pipe. The burner itself is considered as a duct and may be divided in successive elements. Each of them gives a simple linear relation between the acoustic quantities on both sides of the element. For the present method, the system or a part of it is considered as "black box" (Ref. [7] and [9]). Two measurements ports are located upstream and downstream the combustion system as seen on Fig 2.

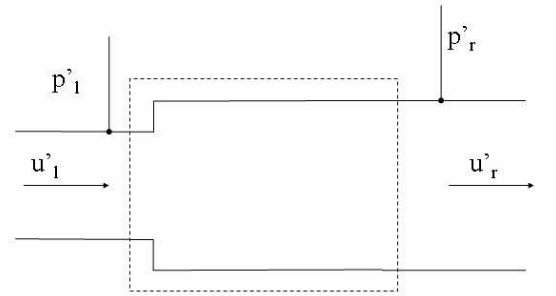

Figure 2: A burner modeled as an acoustic two-port.

The subscripts *l* and *r* indicate left and right pressure and velocity.

In the spectral domain, the two-ports can be described by the equation:

$$\begin{pmatrix} p'_r \\ u'_r \end{pmatrix} = \underbrace{\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}}_{M} \begin{pmatrix} p'_l \\ u'_l \end{pmatrix} \tag{12}$$

In this formulation, p', u' are complex data as well as the unknown components of the matrix M. The system is underestimated: it is necessary to provide two different acoustic test states to resolve the four linear independent equations with a unique solution for the coefficients of M:

$$
\begin{pmatrix} p'^{(1)}_r \\ u'^{(1)}_r \\ p'^{(2)}_r \\ u'^{(2)}_r \end{pmatrix} = \begin{bmatrix} M_{11} & M_{12} & 0 & 0 \\ M_{21} & M_{22} & 0 & 0 \\ 0 & 0 & M_{11} & M_{12} \\ 0 & 0 & M_{21} & M_{22} \end{bmatrix} \begin{pmatrix} p'^{(1)}_l \\ u'^{(1)}_l \\ p'^{(2)}_l \\ u'^{(2)}_l \end{pmatrix} \qquad (13)
$$

where superscripts (1) and (2) refer to two different measurement states. The choice of the states (1) and (2) offers various possibilities. For example, (1) can be a situation where upstream forcing is applied while (2) corresponds to downstream forcing [9].

Contrary to the n-τ formulation, the flame response is measured locally via acoustic fluctuations of velocity and pressure: no measurement of heat release is required. For 1D acoustic waves, it is possible to show that these two formulations are identical. However in reacting cases, the PAA measurement method requires an array of microphones which can be used in burnt gases.

Equivalence of n-τ and PAA methods in low-speed reacting flows

This section shows that both methods (PAA and n-τ) are equivalent. Starting from the n-τ model and using Fig.1, the matrix M of the PAA model is reconstructed as follows. For the n-τ formulation, the velocities fluctuations are measured at $x = x_{j+1}$. For sinusoidal fluctuations and in terms of harmonic amplitude Eq. (11) is written:

$$
\frac{\gamma - 1}{\rho_j c_j}\Omega = \frac{n}{\rho_j c_j} S_j e^{i\omega\tau}(A_j^+ e^{ikl_j} - A_j^- e^{-ikl_j}) \qquad (14)
$$

Using Eq. (9), and replacing Ω by expression (14) gives

$$
O_j = K_{n,\tau}\begin{pmatrix} A_j^+ e^{ikl_j} - A_j^- e^{-ikl_j} \\ -A_j^+ e^{ikl_j} + A_j^- e^{-ikl_j} \end{pmatrix} \qquad (15)
$$

with

$$
K_{n,\tau} = \frac{1}{2}\frac{\rho_{j+1} c_{j+1}}{\rho_j c_j}\frac{S_j}{S_{j+1}} n.e^{i\omega\tau} \qquad (16)
$$

Finally, combining (15) in system (8) gives:

$$
\begin{pmatrix} A_{j+1}^+ \\ A_{j+1}^- \end{pmatrix} = H \begin{pmatrix} A_j^+ \\ A_j^- \end{pmatrix} \qquad (17)
$$

with

$$
H = T + K_{n,\tau}\begin{bmatrix} e^{ikl_j} & -e^{-ikl_j} \\ -e^{ikl_j} & e^{-ikl_j} \end{bmatrix} \qquad (18)
$$

The PAA provides the transfer matrix M of velocity and pressure fluctuations between sections S_j (upstream the flame) and S_{j+1} (downstream the flame). Using Eq. (5) and (6), the measured variables are p'_l, u'_l, p'_r and u'_r can be expressed as functions of the wave amplitudes as:

$$
p'_l = A_j^+ + A_j^- \quad \text{and} \quad p'_r = A_{j+1}^+ e^{ikl_{j+1}} + A_{j+1}^- e^{ikl_{j+1}} \qquad (19)
$$

$$u'_l = \frac{1}{\rho_j c_j}(A_j^+ - A_j^-) \quad and \quad u'_r = \frac{1}{\rho_{j+1} c_j + 1}(A_{j+1}^+ e^{ikl_{j+1}} - A_{j+1}^- e^{ikl_{j+1}}) \tag{20}$$

Then, the transfer matrix M used in the PAA model can be expressed in terms of the geometry of the burner and the n-τ parameters:

$$\begin{pmatrix} p'_r \\ u'_r \end{pmatrix} = \underbrace{\begin{bmatrix} e^{ikl_{j+1}} & e^{-ikl_{j+1}} \\ \dfrac{e^{ikl_{j+1}}}{\rho_{j+1} c_{j+1}} & -\dfrac{e^{-ikl_{j+1}}}{\rho_{j+1} c_{j+1}} \end{bmatrix} H \begin{bmatrix} 1 & 1 \\ \dfrac{1}{\rho_j c_j} & -\dfrac{1}{\rho_j c_j} \end{bmatrix}^{-1}}_{\text{M}} \begin{pmatrix} p'_l \\ u'_l \end{pmatrix} \tag{21}$$

showing that the two methods are formally equivalent even though their practical implementation differs.

Comparison between LES and experimental results

Experimental configuration

The experimental configuration has been developed to validate numerical simulations. A non premixed combustion chamber has been arranged in the EM2C laboratory at Ecole Centrale Paris. This geometry is based (like in aeronautical gas turbines) on a coaxial injector (fuel stream in air flow). The combustion chamber is specially designed both for easy optical diagnostics (two dimensional configuration) and easy numerical simulations (simple grid meshes). Propane is injected through two slots in a coflowing air stream (Fig. 3**)** and two backward facing steps ensure the flame stabilization. The burner is 300 mm in length, 80 mm in width and 100 mm high. Depending on the fuel and air flow rates, the turbulent flame may be anchored in the vicinity of the propane injectors lifted and stabilized by the recirculation zones induced by the facing steps. In order to investigate the flame response to longitudinal acoustic perturbations, the air flow is modulated by plane waves induced by two loudspeakers plugged in the inlet duct (Fig. 3). The lifted flame regime corresponds to larger powers and is closer to industrial cases than the anchored flame: in this article, only the flame response of the lifted flame regime is studied.

n-τ measurements and calibration procedure

Hurle[2] had shown that, for a given equivalence ratio, CH emission is proportional to heat release rate at fixed equivalence ratio. For variable equivalence ratios, the link between CH emission and heat release is not as simple. However, normalizing the emission signal using its stoichiometric value, the error remains small when a bijection is assumed between CH emission and heat release rate. In fact, for low variations of the equivalence ratio around the stoichiometric value ($0.8 < \phi < 1.2$), this error does not exceed 15 percent and the CH radical emission is relevant to measure the heat release rate. CH radical emissions are measured using a 10 mm width vertical slot moving along the axis of the combustion chamber (Fig. 3) allowing a spatial analysis of the flame transfer function [10].

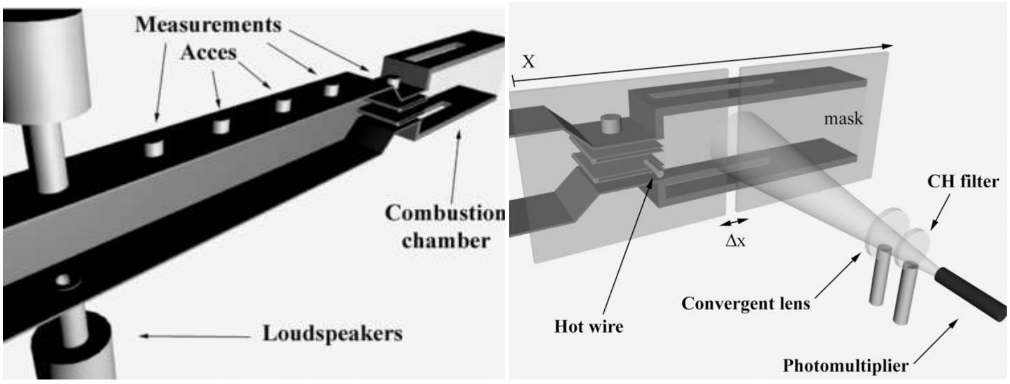

Figure 3: Combustion chamber and experimental setup used for LES validation and measurements of n-τ indices.

Relation (11) still holds but the time delay τ and the interaction index n become functions of the downstream location x. n and τ are determined using Fourier transforms of the velocity and CH radical emission signals using Eq. (11). The local time delay τ is the time needed for a velocity perturbation at the burner inlet to interact with the flame whereas the local interaction index, n, reveals the spatial location of the flame response in the burner. Heat release rates as a function of the downstream location x are estimated as in the experiment by integration over vertical slots (Fig. 3). The scaling between the photomultiplier signal S and the heat release rate Ω has been done assuming that:

- The propane burns homogeneously inside the combustion chamber (i.e. the same amount of fuel burns in each slot). This assumption is rough but the emission rms over the thirty slots (N=30) does not exceed 20 % of the mean signal.
- Non linear variations between CH radical emission and heat release are neglected.

So the heat release rate Ω is linked to the CH emission signal S by the relation:

$$\Omega(x,t) = S(x,t)/\alpha \text{ With } \alpha = \frac{\dot{m}_{c3H8}}{N} Q \qquad (22)$$

where N is the number of measurements slots, $\dot{m}_{c3H8}$ is the propane mass flow rate and Q the heat of the propane/air reaction. Finally the heat release rate Ω can be used to calculate the local n-τ indices.

LES computation

The objective is now to investigate the ability of large eddy simulation to reproduce the time delay τ and the interaction index n. Large eddy simulations of the burner are performed modeling unresolved Reynolds stresses using the filtered Smagorinsky model and describing the combustion by the dynamically thickened flame model DTFLES [5]. The flame front cannot be resolved in LES and the combustion is mainly a subgrid scale phenomenon. In the DTFLES model, the flame thickness may be increased by a factor F keeping constant the laminar flame speed by increasing molecular diffusivities by a factor F and decreasing the reaction rate by the same factor F. To account for the flame turbulence interaction at the subgrid scale level Colin et al.[1] have developed an efficiency function, E, depending on the turbulence and flame characteristics. Légier et al [5] have also shown that the thickening factor F may depend on spatial location. Then, the flame is thickened by increasing F in the reaction zone whereas outside the flame F=1 leaving the molecular mixing unaffected. An acoustic perturbation is added by applying forcing at inlet according to the Inlet Wave Modulation method [4]. The principle of this technique is to modulate the acoustic wave entering the chamber, instead of the velocity, while letting the outgoing wave leave the domain without reflection. The advantage of this technique is to avoid resonance. To compute the n-τ indices, the axial velocity and the integrated heat release versus x are recorded during 20 acoustic cycles in the simulation [Fig. 4**].**

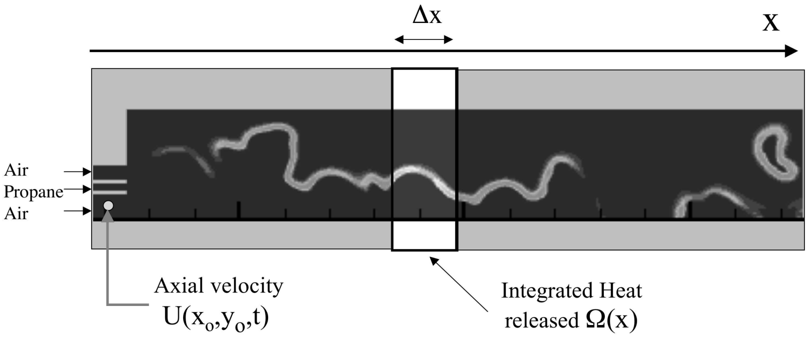

Figure 4: Measurement of n-τ indices in LES. Lifted flame regime

Comparison between LES and experiment

Various acoustic frequencies, belonging both to the rig longitudinal modes range and to the natural low frequency broad band noise in the combustion chamber, have been experimentally tested. Acoustic forcing cases at 200 Hz and 350 Hz are presented here. Local indices n and τ are plotted versus the downstream location x for both excitation frequencies on Fig. 5**.** First, for both frequencies the n-τ parameters estimated with Large Eddy Simulation are in good agreement with experimental results. The local time delay increases with the axial position x showing a convective effect of the reactive coherent structures induced by the acoustic forcing. Moreover, the delay prediction is relatively accurate because the phase shift between experimental and numerical results is much smaller than the acoustic period T_e. The maximum n value is obtained around 50 mm after the backward facing step because the flame is lifted. Some discrepancies between the experimental and numerical results can be explained by the simplified calibration procedure used for the calibration of the interaction index n [Eq. (22)]. Recent 3D numerical results are very promising and will allow determining the contribution of streamwise fluctuations in the flame response.

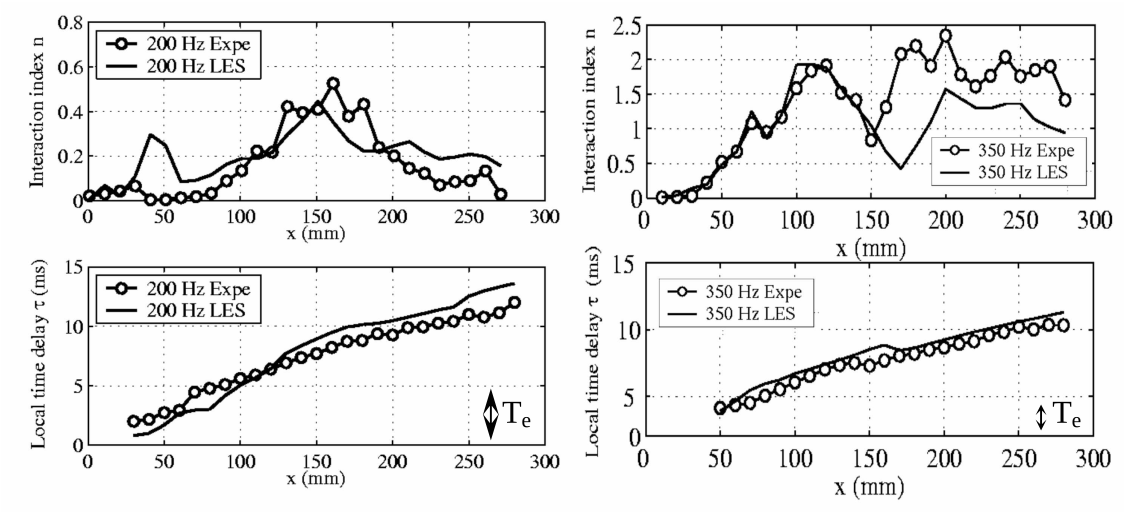

Figure 5: Measurement of n-τ indices in LES

6 References

1. Colin, O., Ducros, F. Veynate, D & Poinsot T. (2000) "A thickened flame model for large eddy simulations of turbulent combustion". *Phys. Fluids* 12, 1854-1863.

2. Crocco, L., CHENG, S. (1956) "Theory of combustion instabilities in liquid propellant rocket motors", volume Agerdograph N° 8 Butterworths Science.

3. Hurle, I., Price , R., Sugden T., and Thomas, A. (1968) *Proc.Roy.Soc.London* A, 303-409

4. Kaufmann, A. (2001) "Flow forcing techniques for numerical simulation of combustion instabilities." *Combustion and Flame*, 131(4):371-385

5. Legier, J.P., Poinsot. P and Veynante, D. (2001) "Dynamically thickened flame Large Eddy Simulation model for premixed and non-premixed turbulent combustion." In *Proceeding of the Summer Program*, pages 157-168, Center for Turbulence research, NASA Ames/Stanford University, USA, 2000.

6. Mullin, T. (1993) "The Nature of Chaos", *Oxford Science Publications.*

7. Paschereit, C.O., Flohr, P., Schuermans B. (2001) "Prediction of combustion oscillations in gas turbine combustors." 39th *AIAA* Aerospace Sciences Meeting and Exhibit.

8. Poinsot, T. and Veynante, D. (2001) "*Theoretical and numerical combustion* ", R.T. Edwards.

9. Schuermans, B.B.H, Polifke, W., Paschereit, C.H. (1999) "Measurement of transfer matrices and source terms of premixed flames", ASME Turbo Expo '99, Indianapolis, Indiana, USA.

10. Varoquie, B., Legier, J.P, Lacas, F., Veynante, D. & Poinsot T. (2002). "Experimental analysis and large eddy simulation to determine the response of non premixed flame submitted to acoustic forcing". *29th Int Symposium on Combustion*. Sapporo, Japan.