Experimental and Numerical Studies of Dilution Systems for Low Emission Combustors

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Abstract

Numerical and experimental investigations of eight isothermal Jets In Cross-Flow (8-JICF) issuing radially in a round pipe are presented to assess advanced numerical approaches in industry-like configurations. The simulations are performed with Large Eddy Simulations (LES). Numerical issues such as grid resolution and physical issues like injection of turbulence in the cross-flow are addressed in this work. Detailed analyses of the LES predictions and comparisons with data underline the potential of the approach for industry-like configurations. Unsteady and averaged LES solutions of 8-JICF picture most of the single Jet In Cross-Flow (JICF) features. The Counter-rotating Vortex Pair (CVP), the wake vortices and the shear-layer vortices are clearly identified in the experiment and by LES. Spectral analysis of the LES predictions reveals proper physical behavior of the different

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structures and good energy content distribution. Characteristic Strouhal numbers are within experimental scatter. Reynolds averaged quantities (means and fluctuations) obtained by LES at several cross-stream locations are in good agreement with the experiment. Finally, jet trajectories and decay rates of cross-flow velocities and jet air scalar concentration further assert the quality of the numerical results. This last analysis reveals that 8-JICF exhibits single free JICF behaviors but in a confined environment.

Introduction

Nowadays, the optimization of combustion performance and the reduction of pollutant emissions require considerable research efforts from the gas turbine industry. The basic objectives in combustor design are to achieve easy ignition, high combustion efficiency and minimum pollutant emissions. Under such conditions two phenomena are to be controlled or avoided. First, the increase of NOx emissions due to the high levels of pressure and temperature needs to be restrained. Second, the feedback mechanism caused by the coupling between the flow and the flame may result in instabilities which originate in the periodic formation of inhomogeneous fuel pockets, the periodic shedding of large structures and the amplification of the acoustic waves. These combustion instabilities\textsuperscript{1–5} may yield dramatic damages or the failure of the combustor. Numerical methods are in that context very attractive alternatives to the expensive experimental set-ups required in such areas of research.

The aim of this work is to assess Large Eddy Simulations (LES)\textsuperscript{6–10} for the design of the next generation combustor. Contrary to the Reynolds Averaged Navier-Stokes (RANS) equations\textsuperscript{11–14} which are restricted to steady turbulent flows, LES offer numerous poten-
tial advantages and take into account combustion instabilities by solving for large flow structures while small scale effects are modeled. The implications on the numerical predictions are of importance but still need to be illustrated and validated in the context of industrial applications. For information and illustrations of the potential of each method, the reader is referred to results obtained with state-of-the-art RANS\textsuperscript{15–20} and LES\textsuperscript{21–26}. Although more academic, these configurations\textsuperscript{15–26} are of clear interest to this work.

This study constitutes a step further toward the full demonstration of LES for gas turbine engines with turbulent reacting multiphase flows. It addresses the problem of dilution jets in an industry-like configuration and aims at validating LES against experimental data gathered on a dedicated test rig located at ONERA (Toulouse). The geometry considered is typical of the dilution region of gas turbines and consists of eight isothermal JICF’s issuing radially in a round pipe. It is hereinafter designated by 8-JICF while the notation JICF will refer to a single Jet In Cross-Flow. Comprehensive experimental data for velocity and scalar concentration are gathered through advanced measurement techniques: Laser Doppler anemometry (LDA), Particle Image Velocimetry (PIV), Particle Laser-Induced Fluorescence (PLIF) and hot-wire anemometry. The effect of the grid resolution on the LES predictions is evaluated by performing the computations for two meshes: a coarse grid (grid $M_1$) and a fine grid (grid $M_2$). In complex geometries, the inlet conditions used for LES raise specific problems: a proper inlet condition should match the mean velocities but also the turbulent velocities and the acoustic impedance. In the present work, the effect of turbulence injection at the inlet is specifically tested by comparing simulations with and without turbulence injection through the main-stream inlet.

The paper starts with a brief presentation of the LES methodology followed by a descrip-
tion of the computational domain and the main flow characteristics. A section is then
dedicated to the presentation of the LES predictions to be divided into three distinct
sub-sections. First, a detailed presentation of the flow topology as predicted by LES and
observed in free JICF’s is given. Second, a spectral analysis of the major flow features
obtained in LES is compared to experimental measures obtained on 8-JICF. Finally, LES
and experimental Reynolds averaged fields of the means and fluctuations are investigated
while jet trajectories and decay rates are compared to existing correlation functions. As-
sessments of the current work and perspectives on the application of LES to industry-like
configurations are offered as concluding remarks.

**Experimental facility**

The experimental rig of 8-JICF is shown on Fig. 1 a. Dimensions and details of the experi-
mental injection system are featured on Fig. 1 b. Full optical access allows detailed analy-
yses of the various flow phenomena. Experimental data consists of LDA (Laser Doppler
Anemometry), PIV (Particle Image Velocimetry) and PLIF (Planar Laser-Induced Flu-
orescence) fields taken at various cross-stream locations in the main pipe. Point-wise
hot-wire anemometry measurements supplement the data set and are used for temporal
assessments of the main flow structures. Main-pipe inlet velocity profiles measured with
LDA complete the database.

Accuracy of the measured data is ensured through in depth cross validations of the various
diagnostics utilized. Experimental uncertainties are this way limited and clearly identified.
Access to the experimental data is made available through inquiry to the MOLECULES
Numerical simulations

Turbulent flows are known to contain large ranges of scales.\textsuperscript{14,27,28} These scales, although not clearly defined mathematically, have been repeatedly evidenced in experimental measurements or Direct Numerical Simulations (DNS) and LES.\textsuperscript{29–31} The large scale phenomena are usually associated with vortical structures whose dimensions are of the order of the domain size. The evolution of these scales is governed by the geometry of the combustion chamber and they carry most of the turbulent kinetic energy. The smaller scales have on the other hand a relatively short range of influence and are believed to behave in a more universal way. Contrary to RANS where all scales need to be modeled, LES filters out the small universal scales and aims at simulating only the dynamics of the large scales. The modeling is eased thanks to the universality of the physics governing the small scales. It yields an approach which is flexible and well suited to simulate cases encountered in the industry where large scale phenomena are crucial.

Governing equations

LES involves the spatial filtering operation:\textsuperscript{32}

\[
\overline{f(x, t)} = \int_{-\infty}^{+\infty} f(x', t) \mathcal{G}(x', x) \, dx',
\]

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where $G$ denotes the filter function and $\overline{f(x,t)}$ is the filtered value of the variable $f(x,t)$. We consider spatially and temporally invariant and localized filter functions,\textsuperscript{32} thus $G(x',x) \equiv G(x' - x)$ with properties,\textsuperscript{32,33} $G(x) = G(-x)$ and $\int_{-\infty}^{+\infty} G(x) \, dx = 1$. In the mathematical description of compressible turbulent flows with species transport, the primary variables are the density $\rho(x,t)$, the velocity vector $u_i(x,t)$, the total energy $E(x,t) \equiv e_s + 1/2 u_i u_i$, and the mass fraction of species $\alpha$, $Y_\alpha(x,t)$. The application of the filtering operation to the instantaneous transport equations yields:\textsuperscript{5}

\[
\frac{\partial \overline{\rho}}{\partial t} + \frac{\partial}{\partial x_i}(\overline{\rho \tilde{u}_i}) = 0,
\]
\[
\frac{\partial}{\partial t}(\overline{\rho \overline{u}_j}) + \frac{\partial}{\partial x_i}(\overline{\rho \tilde{u}_i \tilde{u}_j}) = -\frac{\partial \overline{\rho}}{\partial x_j} + \frac{\partial \overline{\tau}_{jk}}{\partial x_k} - \frac{\partial}{\partial x_i}(\overline{\rho \tilde{T}_{ij}}),
\]
\[
\frac{\partial}{\partial t}(\overline{\rho \tilde{E}}) + \frac{\partial}{\partial x_i}(\overline{\rho \tilde{u}_i \tilde{E}}) = -\frac{\partial \overline{\rho}}{\partial x_j} + \frac{\partial}{\partial x_j}[\overline{\tau}_{ij} - p \delta_{ij}] u_i - \frac{\partial}{\partial x_j}(\overline{\rho \tilde{Q}_j}) - \frac{\partial}{\partial x_j}(\overline{\rho \tilde{T}_{ij} u_i}),
\]
\[
\frac{\partial}{\partial t}(\overline{\rho \tilde{Y}_\alpha}) + \frac{\partial}{\partial x_i}(\overline{\rho \tilde{u}_i \tilde{Y}_\alpha}) = -\frac{\partial \overline{T}_{i}^\alpha}{\partial x_i} - \frac{\partial}{\partial x_i}(\overline{\rho \tilde{F}_{i}^\alpha}). \quad (2)
\]

In (2), one uses the Favre filtered variable,\textsuperscript{34} $\tilde{f} = \rho \overline{f}/\overline{\rho}$. The fluid follows the ideal gas law, $p = \rho RT$ and $e_s = \int_0^T C_p dT - p/\rho$, where $e_s$ is the sensible energy, $T$ stands for the temperature and $C_p$ is the fluid heat capacity at constant pressure. The viscous stress tensor, the heat diffusion vector and the molecular transport of the passive scalar read respectively:

\[
\tau_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \mu \frac{\partial u_k}{\partial x_k} \delta_{ij}, \quad q_i = -\lambda \frac{\partial T}{\partial x_i}, \quad J_i^\alpha = -\rho D^\alpha \frac{\partial Y_\alpha}{\partial x_i}. \quad (3)
\]

In (3), $\mu$ is the fluid viscosity following Sutherland’s law, $\lambda$ the heat diffusion coefficient following Fourier’s law, and $D^\alpha$ the species $\alpha$ diffusion coefficient (note that summation over repeated indices does not apply to Greek symbols). Variations of the molecular coefficients resulting from the unresolved fluctuations are neglected hereinafter so that the
various expressions for the molecular coefficients become only a function of the filtered
field.

The \( \tilde{T}_{ij}, \tilde{Q}_i, \text{and} \tilde{F}_i \) terms correspond to the so-called Sub-Grid Scale (SGS).\(^{10,35}\) The un-
resolved SGS stress tensor \( \tilde{T}_{ij} \), requires a sub-grid turbulence model. Introducing the
concept of SGS turbulent viscosity most models read:\(^{36}\)

\[
\tilde{T}_{ij} = (\bar{u}_i\bar{u}_j - \bar{u}_i\bar{u}_j) = -2 \nu_t \tilde{S}_{ij} + \frac{1}{3} \tilde{T}_{kk} \delta_{ij}, \tag{4}
\]

with,

\[
\tilde{S}_{ij} = \frac{1}{2} \left( \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) - \frac{1}{3} \frac{\partial \tilde{u}_k}{\partial x_k} \delta_{ij}. \tag{5}
\]

In (4) & (5), \( \tilde{S}_{ij} \) is the resolved strain tensor and \( \nu_t \) is the SGS turbulent viscosity. The
WALE model\(^ {37}\) (Wall Adapting Local Eddy-viscosity) expresses \( \nu_t \) as:

\[
\nu_t = (C_w \Delta)^2 \frac{(s^d_{ij} s^d_{ij})^{3/2}}{\tilde{S}_{ij} \tilde{S}_{ij}} \left( \frac{1}{2} (\tilde{g}_{ij} + \tilde{g}_{ji}) - \frac{1}{3} \tilde{g}_{kk} \delta_{ij} \right). \tag{6}
\]

In (6), \( \Delta \) denotes the filter characteristic length and is approximated by the cubic-root of
the cell volume, \( C_w \) is the model constant (\( C_w = 0.55 \)) and \( \tilde{g}_{ij} = \partial \tilde{u}_i / \partial x_j \) is the resolved
velocity gradient.

The SGS energy flux \( \tilde{Q}_i = C_p (\tilde{T}\bar{u}_i - \tilde{T} \bar{u}_i) \) and the SGS scalar flux \( \tilde{F}^\alpha_i = (\bar{u}_i \tilde{Y}_\alpha - \bar{u}_i \bar{Y}_\alpha) \)
are respectively modeled by use of the eddy diffusivity concept with a turbulent Prandtl
number \( Pr_t = 0.9 \) so that \( \kappa_t = \nu_t C_p / Pr_t \) and species SGS turbulent diffusivity \( D^\alpha_t = \frac{\nu_t C_p}{Pr_t} \)
\( \nu_t/Sc_t^\alpha \) where \( Sc_t^\alpha \) is the turbulent Schmidt number (\( Sc_t^\alpha = 0.7 \) for all \( \alpha \)):

\[
\tilde{Q}_i = -\kappa_t \frac{\partial \tilde{T}}{\partial x_i} \quad \text{and} \quad \tilde{F}_i^\alpha = -D_t^\alpha \frac{\partial \tilde{Y}_\alpha}{\partial x_i}.
\] (7)

Note that \( \tilde{T} \) is the modified filtered temperature and satisfies the modified filtered state equation,\(^{30,38-40} \) \( \bar{p} = \bar{\rho} R \tilde{T} \). Although the performances of the closures could be improved through the use of a dynamic formulation,\(^{30,31,41-43} \) they are sufficient to investigate the present configuration.

In the context of industry-like configurations the treatment of the wall turbulent boundary layer needs specific attention. With high Reynolds number flow (here \( Re = 168,000 \)), the turbulent boundary layer is thin compared to the length scales of the computational domain. To ensure a proper physical behavior of the near-wall solution in the context of LES two approaches exist. The ”resolved LES” requires a complete analysis of the grid resolution. With this approach, the inner-layer wall motions (essentially dominated by the quasi stream-wise vortices) must be sufficiently resolved; the filter and grid spacing must be of the order of \( \delta_\nu \) (\( \delta_\nu \) is the length-scale of the viscous wall region). It can be estimated that the number of grid nodes required increases as \( Re^{1.76} \) in the inner-wall region of the flow. For the outer-layer, which contains the large eddy scales, the grid resolution must resolve the turbulent kinetic energy and usually scales as \( Re^{0.4} \). The ”resolved LES” approach usually implies too large computational costs for its direct application to high Reynolds number problems.\(^{44} \) The second approach known as the ”approximate boundary condition” methodology proposes to model the whole wall region. Numerous simplifications for the near-wall LES behavior have been investigated.\(^{45-47} \) In this work, the log law model\(^{48-50} \) is assumed and the wall stresses are modeled so as to mimic the wall effects on the global flow behavior. Note that in the implementation of this approach
the wall boundary conditions use a non-zero velocity at the wall, which is determined locally using the log law and the information on the grid points directly above the wall nodes. Corrections on the temperature profile follow the same approach.

**General description of the code**

All computations are performed with a compressible Navier-Stokes code simulating unsteady flows on structured, unstructured and hybrid grids (cf. http://www.cerfacs.fr). For the prediction of unsteady turbulence, different LES sub-grid scale models have been developed including the WALE model\textsuperscript{37}, Eq. (6). The numerical discretization of the governing equations, Eqs. (2-7), uses a cell-vertex method\textsuperscript{51,52} and the discrete values of the conserved variables are stored at the cell vertices (or grid nodes). Despite the possible use of a numerical scheme offering third-order spatial and temporal accuracies (TTGC scheme\textsuperscript{53}), computations presented in this work are obtained with a second-order Lax-Wendroff scheme.

**Simulations**

Experiments for 8-JICF were performed for two values of the jet-to-mainstream momentum flux ratio, $J$ defined by:

$$J = \frac{\rho_{jet} v_{jet}^2}{\rho_{cf} v_{cf}^2}, \quad (8)$$

a low impulse case ($J = 4$) and a large one ($J = 16$). Both cases have been investigated with LES. For clarity, only the predictions obtained for $J = 16$ are presented in this work (similar topological behavior was obtained for the case $J = 4$). The flow Reynolds
number, based on the main-stream bulk velocity \( (v_{cf}) \) and the main duct diameter \( (D) \), equals 168,000 for all cases. It ensures high level of mixing and limited effects of the turbulent boundary layer. The jet Reynolds number based upon the jet diameter \( (d) \) and jet bulk velocity \( (v_{jet}) \) equals 20,500 and 41,000 for \( J = 4 \) and \( J = 16 \) respectively. Note that due to the use of equal density fluids, the jet-to-mainstream momentum flux ratio \( J \) is equivalent to the square of the velocity ratio, \( R = v_{jet}/v_{cf} \). This flow parameter is hereinafter preferred to identify each case \( (i.e. \ R = 4 \) for \( J = 16 \) and \( R = 2 \) for \( J = 4 \)).

**Computational domain**

Figure 2 illustrates the computational domain retained for the 8-JICF experimental set-up shown on Fig. 1. The length of the main pipe is 138.3 mm for a diameter of 100 mm. The jet inlet sections are located 18.3 mm downstream the main pipe inlet and 120 mm upstream the pipe outlet section. All jets are positioned 45 degrees \( (\pi/4 \text{ radians}) \) apart around the duct circumference. To reduce mesh size and computational cost, the injection system is modeled in LES by a straight injection system prolonged by a circular duct with dimensions identical to the one found in the experiment. The modeled section is circular \( (\text{see on Fig. 2 b}) \) with a contraction ratio of \( \beta = 0.811 \). The jet inlet section has a diameter of 6.92 mm and 6.1 mm at the junction with the main pipe. The aim of this modification is to allow a proper acoustic treatment of the jet injection section in the main tube.\textsuperscript{24–26}

The grid is fully unstructured and composed of tetrahedra. To control the total number of cells and reduce computational cost, mesh refinement is enforced where necessary. More specifically, injection areas, wall regions and jet trajectories must be sufficiently resolved to capture the appropriate range of length-scales. To facilitate refinement along the jet
trajectory, a correlation function is used as a guideline within the computational domain. The guiding trajectory follows the expression for a single JICF\textsuperscript{54} with low velocity ratio, $R$, 

$$
\frac{x}{d} = \left(2.351 + \frac{4}{R}\right)^{0.385} \left(\frac{1}{R}\right)^{2.6} \left(\frac{y}{d}\right)^{2.6}. 
$$

(9)

In Eq. (9), $x$ is the stream-wise direction, $y$ the wall normal direction and $d$ the jet diameter. For the grid generation, the velocity ratio, $R$ in Eq. (9), is taken to 4 irrespectively of the case simulated. Although several laws for JICF trajectories are proposed in the literature, the dependency of the LES predictions on the correlation function to be used is not addressed in this work.

The two meshes as seen on Fig. 3 are used for all the LES (i.e. $R = 2$ and $R = 4$). A coarse mesh $M_1$ and a finer one $M_2$ allow a clear assessment of the grid resolution (Table 1) and its impact on the time averaged LES predictions. The minimum cell size is located in the region where the jet meets the main-stream flow and is the same for all jets. Computational effort corresponding to an increase of resolution from meshes $M_1$ to $M_2$ yields a factor of 1.625 in the total CPU time.\textsuperscript{§}

**Boundary and initial conditions**

Dealing with a fully compressible flow solver, special attention must be paid to the treatment of acoustics through the Boundary Conditions (BC). Acoustic waves can be generated through artificial transient phenomena due to the approximated initial solution and/or the wrong treatment of the BC’s. For clarity, the set of BC’s used in the LES is listed in Table 2. They are based on the method of characteristics.\textsuperscript{55–57} All inlet condi-

\textsuperscript{§}as obtained on 32 O3800 processors of a SGI Power Challenge hosted by CINES (one flow-through time taking approximately 8 CPU hours on mesh $M_1$).
tions operate with the same type of BC (Table 2). For an inlet, the three components of the velocity vector, the temperature and species mass fractions are imposed to the desired values through a relaxing parameter.\textsuperscript{56} When this coefficient equals zero the BC is acoustically non-reflective: exiting acoustic waves leave the domain freely and no component is re-injected in the computational domain. When the coefficient is non-zero the exiting acoustic wave is partially reflected and remains within the region of computation. Furthermore zero relaxing coefficient may result in drifting mean quantities and sufficiently large relaxation coefficients are needed so that mean values remain close to their target values.\textsuperscript{58}

Turbulent pipe flow profiles are used to set the jet mean inlet velocity conditions (Patch B in Fig. 2). The resulting bulk velocity respects the mass flux as measured by ONERA. It should be noted that in the case of a no-slip wall, a zero velocity is imposed at the wall and the shape of the velocity profile issued at the intersection of the injection system and the main duct strongly depends on the grid resolution of the jet injection system. For this reason outlet jet profiles found in LES may not reproduce experimental profiles at these locations. To diminish the differences and the potential implications on the flow predictions, no-slip isothermal ($\bar{T}=300\text{ K}$) walls are used in the jet injection systems with an increased grid resolution (Patch E in Fig. 2). Other walls (Patch D in Fig. 2) follow the isothermal wall law condition described previously.

At the main duct inlet (Patch A in Fig. 2) the values of $\bar{u}, \bar{v}$ and $\bar{w}$ vary in time and space to reproduce the effect of an incoming turbulent field as observed in the experiment. The method in constructing the incoming turbulent signal is based on the Random Flow Generation (RFG) algorithm\textsuperscript{59–61} itself based on early work.\textsuperscript{62} The continuously homogeneous isotropic incoming field consists of a superposition of harmonic functions (50 modes
projected in the three directions) with characteristic length-scales prescribed by the user. Forcing the flow in such a way considerably accelerates the establishment of fully developed turbulent flows. It also ensures the presence of coherent perturbations not warranted when a pure white noise is used. Figure 4 depicts main-stream inlet profiles. All results are non-dimensionalized by the cross-flow velocity $v_{cf}$. Experimental measures are added as symbols. Measurement errors are for the mean field below 1% and around 4% for the fluctuating components. The actual response of the code with and without "injection of turbulence" are respectively represented by the solid lines and the broken lines. All LES results agree with the measurements. Deviations of the LES fluctuating profiles from the mean target fluctuating profile imposed at the BC are due to the unsteady nature of the incoming flow and the partially reflective condition.

The main duct outlet BC (Patch C in Fig. 2) simulates a far field state with given atmospheric conditions. This boundary condition is "soft"; the in-going wave is computed as a difference between the LES solution at the boundary nodes and the reference state. A "relax" coefficient allows to absorb the acoustics making the condition partially reflective. All relax coefficients used for the LES of 8-JICF are listed in Table 2 and assure low acoustic impedance of the boundary as verified by a posteriori validation of the incoming and out-going acoustic signals.58 Finally and to identify the mixing of the jets with the cross-flow fluid, oxygen issued at the jet air inlet condition is differentiated from the oxygen of the cross-flow air.

In order to proceed with the integration of the LES governing equations an initial solution is needed. To diminish computational cost and artificial effects due to the initial guess a specific methodology is employed. Two steps are followed:
1) To achieve an approached solution in a reasonable time delay, the coarse mesh ($M_1$) is first used to conduct the integration. To avoid spurious behaviors, the initial velocity is null and tends smoothly to the desired values through the inlet BC’s.

2) The converged solution obtained in 1) is interpolated on the fine mesh ($M_2$) to yield a new initial condition for proper integration of the LES equations.

Note that each step requires adaptation of the initial conditions toward physical solutions. Memory loss of the initial guesses is assured by allowing the simulation to run for approximately one flow-through time before starting statistical sampling and analysis of the LES results.

**Results and discussion**

Prior to the discussion on the LES and experimental flow topology, a brief introduction to the single JICF features is given. Jet visualizations of the 8-JICF LES predictions for the fine grid ($M_2$) and both velocity ratios follow. Vortex shedding frequencies in the near field region are investigated and gauged against experimental measurements. Then, mean velocity fields are presented along with a qualitative assessment of the grid resolution and turbulence injection effects. Finally, one assesses jet trajectories and decay rates.

**Flow features**

The single JICF configuration is widely used in many technical applications involving gas turbines, fuel injection, chimneys, etc. Compared to other shear flows (e.g. mixing layer, free jets) the JICF is the result of the complex three-dimensional interactions between
the jet and the cross-flow stream. It displays considerably more complexity with jet-like behavior in the near field and the far field. The literature proposes numerous works on the main vortical structures observed in several regions of the single JICF.\textsuperscript{63–66} Four distinct vortical structures have been identified (see Fig. 5):

1. The jet shear-layer vortices which evolve on the jet column and whose vorticity is generated at the interface between the jet and the cross-stream. Such structures are the result of the Kelvin-Helmholtz instabilities of the annular shear-layer.

2. The horseshoe vortex system, which lies on the wall around the jet exit and which is quite similar to the structures observed for flows around a cylinder wall junction.\textsuperscript{65} It also has been shown that the horseshoe relates to the shear-layer roll up and the shedding of vortices in the wake region.

3. The wake structures form downstream the jet column and persist far downstream of the exit nozzle. The fluid comes from the wall boundary layer and sheds regularly from the lee-ward side of the jet. They can be detected in the wake region as ascending vortices. These very complex three-dimensional flow patterns are strongly influenced by the jet trajectory.

4. The Counter-rotating Vortex Pair (CVP) is the dominant coherent structure of the JICF. It develops downstream and strongly depends on the boundary layer and plane wall region. The CVP plays a significant role in the far-field mixing.

In this paragraph, the single jet topology as obtained from the 8-JICF LES is presented for $R = 2$ and $R = 4$ with the help of four identification tools. Only the near-field region of one JICF out of the 8-JICF configuration is illustrated. It ensures that each JICF feature is not influenced by the neighboring JICF’s. Figures 6 a and 7 a show iso-
surfaces of jet oxygen mass fraction; inner and outer jet boundary layers are defined by a jet oxygen mass fraction of 0.1. Jet air rapidly mixes with cross-flow fluid to generate oxygen pockets. Such iso-surfaces indicate that as \( R \) increases, the jet integrity is clearly shortened in space, thus corresponding to enhanced mixing. The differences are asserted by Figs. 6 b and 7 b where iso-surfaces of low level of pressure are shown. Figures 6 c and 7 c illustrate the \( Q \)-criterion. Based upon the second invariant of the velocity gradient tensor,\(^{67}\) this criterion aims at detecting coherent structures in wall bounded turbulent flows. \( Q \) is directly related to the pressure Laplacian, \( \nabla^2 p = 2 \rho Q \), for inviscid flows and can be interpreted as the source term of pressure in the Navier-Stokes equations. On Figs. 6 d and 7 d, the strong velocity gradients present near the wall prevent a clear identification of large scale features through the stream-wise component of the vorticity vector. This drawback is circumvented with the help of the \( Q \)-criterion (Figs. 6 c and 7 c) which allows to identify structures already illustrated with the iso-surfaces of jet oxygen mass fraction.

All criteria show the jet column to deviate while it penetrates the cross-flow. This pronounced feature of the JICF (see Fig. 5 ) is present in the 8-JICF configuration. Shear-layer vortices can also be noticed for both velocity ratios. Traces of the CVP are also evidenced in the 8-JICF set-up. The major difference observed between case \( R = 4 \) and case \( R = 2 \) is the penetration angle of each individual JICF and its transition toward a fully turbulent state. Strong similarities can however be found between the two cases as reported in previous works on JICF. Based on these observations only \( R = 4 \) results are presented afterward.
The Counter-rotating Vortex Pair (CVP)

The CVP is a well known feature of the JICF configuration. This coherent structure persists and keeps evolving even in the far field region of the flow. It appears in steady and unsteady flow solutions and does not dependent on the velocity ratio $R$ and the flow Reynolds number $Re$. The CVP takes on an oval shape composed of two kidney shaped counter-rotating vortices.

Figure 8 shows the CVP as observed in the 8-JICF LES predictions for one of the eight JICF. The normalized stream-wise component of the vorticity vector, $\Omega_x$, is shown at a) $x = 1d$ and b) $x = 2d$. Dark grey iso-lines correspond to clockwise rotations while light grey iso-lines identify counter-clockwise rotations. The normalization of $\Omega_x$ by the planar maximum of $|\Omega_{z0}|$ within the CVP (outside of the wall region) is performed so as to illustrated the downstream evolution of the CVP. Through this quantity the kidney-like structures are clearly observed in the LES predictions. The decay of the structure with the downstream direction is confirmed by LES (Figs. 8 a-b). Built-in diffusive as well as dissipative forces of the system imply a decrease in the maximum of vorticity as the jet expands. It is to be noted that the rotation of the two kidney-like structures induces creation of vorticity in the near-wall region and also decreases with the downstream direction.

Comparisons of the jet oxygen concentration given by an instantaneous LES solution with PIV particle traces in the CVP region (Fig. 9 a) suggest that LES predictions and measurements do not necessarily depict well defined and coherent structures at all instants and for all jets of 8-JICF. Gross agreement can be found but no clear small or large scale pattern is identified in any jet. Finally, Fig. 9 a underlines the unsteadiness
of the flow evidenced through the asymmetries in the particle traces and the jet oxygen concentration. These asymmetries are not clearly evidenced by the averaged results (Fig. 9 b). The qualitative good agreement between LES and measurements as illustrated in Fig. 9 highlights the clear potential of LES for industry-like computations.

**Statistical analysis**

Instantaneous visualization of the LES predictions reveals the possibilities of the approach in predicting highly complex flows. Further analysis is however necessary to clearly assess LES for industrial applications. To do so, comparisons of the statistically averaged LES fields and corresponding experimental data are presented in this section. A frequency analysis of the various JICF structures identified previously is exposed, followed by statistical moments. Finally, mixing of the jet air with the main-flow is evaluated and gauged against existing correlation functions. It is of interest to underline at this point that LES animations point out that the instantaneous realizations of the JICF may deviate from their theoretical trajectory. The appearance of a weak coupling between neighboring jets is to be suspected in 8-JICF. Under these circumstances an independent treatment of all the JICF should be interpreted with care even if the time scales of the coupling appeared to be very long.

**Vortex shedding frequencies**

Vortex shedding frequencies are of clear interest to the JICF users; large vortex structures can have a substantial impact on the mixing of the jets, which is one of the primary aim of the gas turbine industrial use of JICF’s. In the experimental investigations, Fourier-spectral analysis of velocity signals (hot-wire anemometry) has been performed in two
distinct regions: the wake and the jet shear-layer. Probe’s locations have been determined based on visual investigations of the flow.

This paragraph presents the frequency acquisition method conducted on the numerical simulations. Wake and jet shear-layer vortices are studied at several probe locations where pronounced harmonics are found experimentally. In order to obtain a proper convergence of the spectral analysis, two hypotheses are needed for LES: first, the axi-symmetry of the flow is assumed although it has been pointed out that all jet penetrations are not strictly identical and independent. Second, traveling in the downstream direction along the jet column, structures are supposed to be persistent and vortex shedding frequency is assumed to be equivalent for different probe locations in the regions of interest and prior to potential vortex pairings. Under these two assumptions, averaging of the individual Fourier-spectrum is performed and compared to the measurements. The duration of the time series equals 37 ms (i.e. 6.7 convective times) and the sampling is performed at 6,000 Hz. These time-span and sampling rate enable to capture most of the flow information.

For the wake region, the analysis is based on the time series of the stream-wise component of the velocity vector, \( \tilde{u}(x, t) \) obtained on the fine grid, \( M_2 \). The same approach is used in the vortex shear-layer with \( \tilde{u}(x, t) \) being sampled. The respective spectra are presented in terms of Strouhal numbers, \( St = f L_{ref}/U_{ref} \), where \( f \) is the frequency domain (in \( s^{-1} \)), \( U_{ref} \) and \( L_{ref} \) are the reference velocity and reference length. Reference values need to be adjusted depending on the phenomena investigated. In this study, two regions are investigated: the wake region for which we take \( U_{ref} = v_{cf} \) and \( L_{ref} = d \) yielding \( St_{v_{cf}} = f d/v_{cf} \), and the jet shear-layer for which we have \( U_{ref} = v_{jet} \) and \( L_{ref} = d \) yielding \( St_{v_{jet}} = f d/v_{jet} \).

The wake region is discussed first. Few references are found on this subject in JICF’s
literature. Characteristic Strouhal numbers found for the Kármán street behind a circular
cylinder vary around $St_{v_{c,j}} = 0.21$. JICF wake vortices were studied with the use of the
smoke-wire technique\textsuperscript{63} for several velocity ratios, $R$. In the mentioned work, a degree
of repeatability is found for the case $R = 4$ and varying cross-flow Reynolds numbers
ranging from 3,800 to 11,400. The experimental Strouhal number equals 0.13 for $R = 4$.
Transitions to larger values occur around $R = 3$ and $R = 6$. In another experiment,\textsuperscript{70}
the problem is investigated for $R$ ranging from 2 to 8 and a fixed cross-flow Reynolds
equal to 8,000. The probe is located 2.7 jet diameters downstream the injection point
and $St_{v_{c,j}} = 0.15$. In 8-JICF experiments, the velocity probe is placed 6 mm from the
wall and 12 mm downstream of the jet axis. Treatment of the unsteady LES results is
obtained with five probes placed in the wake region behind the injection zone and 2 mm
apart (Fig. 10).

Vortex shedding frequencies are computed for all of the eight JICF present in the flow (*i.e.* in
all eight wake regions). Fast Fourier transforms obtained from the forty points, are
averaged and the corresponding LES Fourier-spectrum is shown on Fig. 11 a. Experimental
results are illustrated on Fig. 11 b. Broken line represents fast Fourier transforms of
the stream-wise component of the velocity vector $\tilde{u}(x, t)$, and continuous line represents a
smoothed signal (after averaging over all the realizations). LES predictions depict regular
vortex shedding at one pronounced frequency corresponding to a Strouhal number of 0.18.
Experiment yields a lower Strouhal number equal to 0.11 (Fig. 11 b).

Vortex shedding frequencies of jet shear-layer structures have received more attention from
researchers especially for acoustically excited transverse jets. Concerning unforced JICF,
recent experiments\textsuperscript{65} in both water and air suggest that periodic vortex ring roll-ups from
the nozzle appear with a Strouhal number equal to 0.295. Another experiment points out
the importance of the probe location.\textsuperscript{71} The corresponding work reports different Strouhal number for a hot-wire probe positioned from approximately 1 to 1.5 jet diameters above the upstream edge of the jet exit. According to their observations, higher harmonics appear with increasing value of the cross-stream speed while keeping the velocity ratio constant ($R = 4$): $St_{v_{jet}} = 0.721$ with $v_{jet} = 4.8$ m/s and $St_{v_{jet}} = 0.828$ for $v_{jet} = 8$ m/s. The impact of the downstream location of the measuring point was also addressed.\textsuperscript{72} To do so, the authors register velocity signals at three positions in the jet shear-layer of a JICF with cross-flow Reynolds number of 27,500 and a velocity ratio of 6. The reported Strouhal numbers are found to decrease with increasing downstream position. Finally, Direct Numerical Simulations (DNS) assess the influence of the wall boundary layer.\textsuperscript{73} The numerical experiments are performed for different velocity ratios and one non-dimensionalized boundary layer thickness $\delta_{BL}/d = 0.5$. For $R = 5.4$ their finding is: $St_{v_{jet}} = 0.545$.

Figure 12 shows the three probe locations used to analyze LES predictions in the jet shear-layer region. The probes are positioned close to the upstream edge of the jet shear-layer to help discern the large structure frequencies. In the experiment, the probe is placed 6 mm from the wall and 3 mm upstream of the jet axis. LES predictions are compared against experimental data on Fig. 13. Spectrum’s shape is qualitatively well reproduced. The high response observed at low frequencies probably corresponds to the CVP’s formation. For both cases, the spectra seem to be masked by broad-band behavior with no discernible dynamics in the range $0 < St_{v_{jet}} < 0.07$. This can be explained by the fact that probes are located slightly inside the jet and reveal internal details of the CVP’s structure. For $0.07 < St_{v_{jet}} < 0.27$, the LES spectrum remains stable and reveals a small peak around a Strouhal number value of 0.24. The same general
behavior is revealed in the experiment but over a much larger range, $0.2 < St_{v_{jet}} < 0.4$. A sustained plateau may also be visible in LES for $St_{v_{jet}} = 0.35$ and which would correspond to the plateau at $St_{v_{jet}} = 0.7$ in the experiment. Finally, spectra go down sharply in the high frequency range.

Many reasons can explain the differences found in the peak detections derived from LES predictions or experimental results. First, the jet injection system is different in LES (Fig. 2) and in the experiments (Fig. 1). Short ducts with contraction ratios equal to 0.811 are employed for LES in order to reduce the computational cost and to facilitate the treatment of the acoustic field at the jet inlet boundary. Geometric differences imply that jet exit velocity profiles do not exactly represent the experimental conditions. Probe location is another difficulty as reported in previous works. Most importantly, the grid resolution where the jet meets the cross-flow along with the LES model used do not guarantee proper prediction of natural unstable modes of the shear-layer. This implies pronounced Reynolds number and LES model effects in the numerical approach. Note that such issues are commonly encountered in LES of transitioning flows and can hardly be solved without very dense grids and advanced sub-grid models. Such issues are however outside the scope of this work and further investigations are being conducted on the subject. Finally, statistical convergence of the LES spectra can not be as reported experimentally where thousands of spectra are available.

Despite the potential limitations of the proposed analysis, LES yields promising results even in the context of the industry-like configuration of this study. Indeed, the shape of the spectra are very well reproduced at several locations in the flow, indicating that the physics governing LES is similar to the one reported by experiments. Peak Strouhal numbers are however poorly predicted by LES. Nevertheless, and despite the uncertainties
associated with the flow configuration and the numerical approach, LES predictions are very encouraging.

Mean fields

Having qualitatively assessed the potential of LES in reproducing the dynamics of industry-like configurations, a more quantitative analysis of the LES predictions needs to be performed. To fulfill this task a statistical approach is adopted to construct averaged fields to be compared with ONERA’s measurements. The averaging procedure corresponds to a Reynolds average of the instantaneous LES predictions. The time of integration is in this case approximately four flow-through times (based on the main duct bulk velocity and the computational length, \(i.e.\) 25 m/s and 138.3 mm). The importance of the mesh resolution on the first and second moments is assessed at this occasion. LES predictions obtained with and without turbulence injection are presented as well. The comparisons with experimental measurements are performed at three cross-planes localized 2\(d\), 5\(d\) and 10\(d\) downstream the jet injection plane.

The analysis focuses on the study of the mean and fluctuating stream-wise component of the velocity predicted by LES. Results are non-dimensionalized by the cross-flow mean bulk velocity \(v_{cf}\). The ordinate follows the duct radius (0 at the center-line and maximum at the duct wall) and is scaled by \(D\) the main-pipe diameter. Figure 14 compares profiles of the non-dimensional mean axial velocity against experimental data. Both grid resolutions (grids \(M_1\) and \(M_2\)) are considered for the comparison of the LES with and without ”turbulence injection” (denoted by ”TI” on the graph). Very good agreement is found when comparing fine grid LES predictions with the measured data. For all planes the jet penetration is well reproduced as well as the jet wake region. Effects due to the
increased mesh resolution in the jet near field are readily observed and result from three highly coupled phenomena. First, local increase of the mesh resolution allows one to enrich the LES flow solution with smaller and better resolved small scale structures. The downstream JICF region is in turn "more turbulent" and the large scale energy content is better represented. Second, associated with this grid refinement, is also a less dissipative LES. Improved levels of dissipation originate from a more efficient application of the LES model. The flow structures influenced by the model are closer to the required assumption for the use of the SGS model, *i.e.* local isotropy. Third and last, smaller cell volumes usually go with decreased dissipative effects induced by the numerical scheme itself. Consequently and as observed in Fig. 14, the mean stream-wise profiles are expected to be sharper (less diffused) in regions of improved energetic content, *i.e.* $2d$ and $5d$. These results illustrate the necessity to properly resolve regions of importance for LES to yield good quality predictions. Iterative approach with adaptation of the local mesh resolution (as performed here) may therefore be required when no clear information can be inferred for the flow to be simulated. Finally, when turbulence injection (TI) is used, no significant change is observed when compared to other LES predictions. The intensity of the perturbations at the main inlet probably does not influence such turbulent shear flows. The importance of the cross-flow turbulence is in this context not clear even though it is well known that such an approach may significantly improve the prediction in more academic configurations.

Figure 15 displays the mean axial component of the velocity vector for one eighth of the cross-sectional planes located at $2d$, $5d$ and $10d$. Each LES field shown in the figure is time-averaged over two flow-through times and spatially averaged over the eight jets to yield one section as obtained in the experiment. The color-map is identical for both set
of results. At $x = 2d$, LES results depict a reasonably sized jet and a croissant-like shape linked to the passage of the vortex shear-layer structure. Going further downstream the overall agreement remains and confirms the observations of Fig. 14. Finally LES shows a slight asymmetry in the far field of the jet ($10d$ downstream the injection plane). Note that experimental measures are obtained for only half a jet and then symmetrized. Comparisons need therefore to be performed with care. Mean planar velocity vectors are also shown in Fig. 14 to illustrate the fluid motion in the $2d$, $5d$ and $10d$ planes. This latter quantity evidences the induced motion due to the CVP.

The non-dimensionalized fluctuation of the stream-wise velocity component $u'/v_{cf}$ is depicted on Fig. 16. LES predictions are in good agreement with measurements and the profiles are very well represented: in the CVP’s region, strong turbulent activity is found as observed in the experiment at the three downstream locations. In the wall region, the fluctuation is well predicted by LES indicating the proper behavior of the wall model. The injection of turbulence at the inlet improves the predictions of $u'/v_{cf}$ along the center-line of the main pipe ($r/D = 0$). Implications due to the mesh refinement are similar to the results observed for the mean stream-wise component of the velocity vector (Fig. 15).

It is important to underline at this point that although the identification of the various peak Strouhal numbers could not be predicted accurately, experimental mean values are still very well reproduced by LES. This further asserts the potential of LES for industrial applications and can be explained mainly by the reproduction in LES of the complex spatial and temporal physical phenomena observed experimentally.
Jet trajectories and decay rates

JICF jet trajectories have received a lot of attention from researchers as well as from the industry. Often based on the locus of the maximum stream-wise component of the velocity vector, the jet path can also be defined as the locus of the maximum jet scalar concentration in this same plane. Many experimental and theoretical works on free JICF’s indicate that the jet path can be expressed in self-similar variables for a large range of jet-to-mainstream momentum flux ratio (at least for free JICF). Some of the experimental correlations resulting from these previous works are given in Table 3.

LES results are gauged for both definitions against free JICF’s experimental correlations as obtained for a velocity ratio of $R = 4$. Figures 17 a-b respectively show jet paths in self-similar coordinates based on the stream-wise component of the velocity vector and the scalar concentration maximum as functions of the non-dimensionalized downstream directions. Agreement with experimental measurements on 8-JICF is very good and validates the LES results. When compared with analytical expressions, the present results are found to under-predict the proposed correlation functions over the entire length of the domain. Such results highlight the impact of the geometric differences between 8-JICF and a single free JICF. Confinement is expected to play a determining role in the latter while deliberately nonexistent in the former. Focusing on the criterion constructed from the velocity field (Fig. 17 a), the initial jet deflection observed experimentally in the range $0 < x/d < 5$ and the downstream region $x/d > 10$ are very well replicated for both mesh resolutions (grids $M_1$ and $M_2$), with and without turbulence injection (TI). LES and measures for 8-JICF are closer to the third and fifth correlation function.77 For the jet oxygen maximum concentration (Fig. 17 b) jet penetration is found to be closer to the first expression78 for the scalar and LES reproduce the correlation in the near field region. The
mesh resolution and the turbulence injection do not seem to influence the jet trajectory.

Figures 18 a-b respectively display the decays of the maximum stream-wise component of velocity and the maximum jet oxygen scalar concentration along the respective trajectories shown on Fig. 17. The abscissa corresponds to the distance traveled along the jet trajectory from the injection point (denoted here by $s$). The decays are presented in a non-dimensional form as indicated in the literature.\textsuperscript{68,79} Figure 18 suggests power law decays as observed in free JICF’s. In the near field region, $\ln(s/Rd) < 1$, the maximum of velocity decreases to a $-1$ power as encountered for free turbulent jets. For $\ln(s/Rd) > 1$ (i.e. $s \approx 15 d$), a transition occurs and the decay rate tends toward the value of $-2/3$ as reported for free JICF.\textsuperscript{80,81} These changes in slope are often related to the CVP which increases the rate of mixing and the decay rate of the maximum value of the stream-wise velocity component. Fig. 18 b corroborates the previous findings, especially for the case performed with turbulence injection. The near field jet-like decay is less pronounced and mixing seems accentuated when LES is closely analyzed. Modeling effects would need to be assessed to further validate the approach. The influence of the grid resolution is not significant in predicting the velocity and scalar concentration decay rates.

**Conclusions**

This work assesses the potential of the Large Eddy Simulations (LES) approach in the context of an industry-like configuration. The study focuses on a typical dilution zone in gas-turbine geometries. More specifically LES are performed for eight isothermal Jets In Cross-Flow (JICF) issuing radially in a round pipe. Experimental measurements and detailed diagnostics allow a clear assessment of the LES predictions. Special attention is
devoted to the mesh resolution and the effect of turbulence injection in the cross-flow. The influence of the grid resolution and the addition of turbulence at the main duct inlet do not imply significant changes in the predictions. Spectral analysis of the various time dependent structures usually present in a free JICF proves LES to be a very powerful tool to predict unsteady behavior. Indeed large vortical structures such as the Counter-rotating Vortex Pair’s (CVP’s), the shear-layer vortices and the wake vortices are readily observed in LES. Peak Strouhal numbers are within the experimentally reported values. Experimental means (first moments and fluctuations) are very well reproduced at various locations of the main duct pipe. Finally, jet trajectories and decay rates are found to be in agreement with experimental measures. Comparisons with experimental and various existing correlation functions for free JICF indicate that confinement effects are present in the studied geometry and further assert the overall potential of the LES approach for industrial applications.

Acknowledgements

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Figure Captions

Fig. 1 Experimental set-up of ONERA: (a) view of the experimental test section and (b) its schematic representation.

Fig. 2 Computational domain considered for LES of the isothermal Jets in Cross-Flow. a) Patches representation and b) geometry of the jet injection system. All given distances are in millimeters.

Fig. 3 Detailed view of the refinement used to generate a) the coarse mesh $M_1$, and b) the fine mesh $M_2$.

Fig. 4 Non-dimensional profiles of a), the mean stream-wise component of the velocity vector and b), its mean fluctuating component as functions of the non-dimensional radius of the main duct, $r/D$. Broken lines: case without ”turbulence injection”, solid lines: case with ”turbulence injection”, circles: measurements.

Fig. 5 Vortex system in a JICF. 63

Fig. 6 Jet visualizations (grid $M_2$, case $R = 2$): a) iso-surface of jet oxygen mass fraction $\tilde{Y}_{O_2} = 0.1$, b) iso-surface of pressure $\bar{p} = 1.013$ bars, c) iso-surface of Q-criterion $Q = 9 \times 10^6$ s$^{-2}$, d) iso-surface of the stream-wise component of the vorticity vector $\Omega_x = 3 \times 10^3$ s$^{-1}$. The iso-surface is shaded with respect to the local value of the stream-wise component of the velocity vector (light shades for low values and dark shades for large values).

Fig. 7 Jet visualizations (grid $M_2$, case $R = 4$): a) iso-surface of jet oxygen mass fraction $\tilde{Y}_{O_2} = 0.1$, b) iso-surface of pressure $\bar{p} = 1.013$ bars, c) iso-surface
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Fig. 8 Levels of the normalized stream-wise vorticity $\Omega_x$, a) plane $x = 1d$, b) plane $x = 2d$.

Fig. 9 Visualization of the CVP’s as obtained from (a) instantaneous and (b) mean fields. Each picture displays experimental views of PIV particle traces (top) and LES views of jet oxygen concentration (bottom). Dark gray relates to large values of the concentrations and light gray corresponds to low values. The cross-stream plane used for visualization is located (a) $1d$ and (b) $5d$ downstream of the jet injection plane.

Fig. 10 Probe locations in the LES study of the wake region (dimensions are in millimeters): a) lateral view and b) top view.

Fig. 11 Vortex shedding frequencies analysis in the wake region. a) LES predictions, b) measurements.

Fig. 12 Probe locations in the LES study of the jet shear layer region (dimensions are in millimeters and shown for the plane $z = 0$).

Fig. 13 Vortex shedding frequencies analysis in the jet shear layer region. a) LES predictions, b) measurements.

Fig. 14 Comparisons of the non-dimensionalized mean stream-wise component of the velocity vector, $u/v_{cf}$, as obtained from the LES predictions and
experimental measurements (circles). Long dashed lines: grid $M_1$ without $TI$, dotted lines: grid $M_2$ without $TI$, and solid lines: grid $M_2$ with $TI$.

Fig. 15 Non-dimensionalized fields of the mean axial component of the velocity vector, $u/v_{cf}$, obtained with LES, left column and measured by ONERA, right column. The comparison is obtained for three downstream locations, a) $x = 2d$, b) $x = 5d$, c) $x = 10d$.

Fig. 16 Comparisons of the non-dimensionalized fluctuation of the stream-wise velocity component, $u'/v_{cf}$, as predicted by LES and obtained in the experiment (circles). Long dashed lines: grid $M_1$ without $TI$, dotted lines: grid $M_2$ without $TI$, solid lines grid $M_2$ with $TI$.

Fig. 17 Jet trajectories obtained by LES and the self-similar expressions given in Table 3: a) maximum of the stream-wise component of velocity and, b) maximum of scalar concentration.

Fig. 18 Jet decay rates as predicted by LES: a), maximum stream-wise component of the stream-wise component of velocity and b), maximum scalar concentration. Solid lines represent the decay laws.
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Table 1 Mesh characteristics.

Table 2 Boundary conditions.

Table 3 Self-similar expressions for a free JICF’s trajectories based on jet velocities and jet oxygen concentration. Note: for Patrick’s expression $n$ is equal to 0.38 for the velocity and 0.34 for the concentration; for Hasselbrink’s expression $c_{e_j} = 0.32$ and $(3/c_{ew})^{(1/3)} = 1.6$. 
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Tables

Table 1: Mesh characteristics.

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Table 2: Boundary conditions.

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### Velocity based trajectory

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<td>Patrick$^{78}$ (1967)</td>
<td>$6 &lt; R &lt; 50$</td>
<td>$\frac{y}{d} = R^{0.85} \left( \frac{x}{d} \right)^n$</td>
</tr>
<tr>
<td>Margason$^{77}$ (1968)</td>
<td>$2 &lt; R &lt; 10$</td>
<td>$\frac{x}{d} = \frac{1}{4} \left( \frac{1}{R} \right)^2 \left( \frac{y}{d} \right)^3$</td>
</tr>
<tr>
<td>Kamotani$^{82}$ (1972)</td>
<td>$3.87 &lt; R &lt; 7.75$</td>
<td>$\frac{y}{d} = 0.89 R^{0.94} \left( \frac{x}{d} \right)^{0.36}$</td>
</tr>
<tr>
<td>Chassaing$^{54}$ (1974)</td>
<td>$2.37 &lt; R &lt; 6.35$</td>
<td>$\frac{x}{d} = \left( 2.351 + \frac{4}{R} \right)^{0.385} \left( \frac{1}{R} \right)^{2.6} \left( \frac{y}{d} \right)^{2.6}$</td>
</tr>
<tr>
<td>Fearn$^{83}$ (1978)</td>
<td>$3 &lt; R &lt; 10$</td>
<td>$\frac{y}{d} = 0.975 R^{0.9085} \left( \frac{x}{d} \right)^{0.3385} \left( \frac{1}{R} \right)^{1/2}$</td>
</tr>
<tr>
<td>Hasselbrink$^{80}$ (2004)</td>
<td>$1 &lt;&lt; R$ (near field)</td>
<td>$\frac{y}{Rd} = \left( \frac{2}{c_{ej} \frac{d}{R}} \right)^{1/2}$</td>
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<tr>
<td>Hasselbrink$^{80}$ (2004)</td>
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<td>$\frac{y}{Rd} = \left( \frac{3}{c_{ew} \frac{d}{R}} \right)^{1/3}$</td>
</tr>
</tbody>
</table>

### Scalar concentration based trajectory

<table>
<thead>
<tr>
<th>Author</th>
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</tr>
<tr>
<td>Karagozian$^{84}$ (1986)</td>
<td>not specified</td>
<td>$\frac{y}{d} = 0.527 R^{1.178} \left( \frac{x}{d} \right)^{0.314}$</td>
</tr>
</tbody>
</table>