Anisothermal Wall Functions for RANS and LES of Turbulent Flows With Strong Heat Transfer

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1 Introduction

Various types of flows (e.g. in aeroengines, nuclear reactors) present very large temperature differences, implying density variations. In those flows, the correct prediction of thermal fluxes at the walls is a crucial problem, because materials can be submitted to contraction and dilatation phenomena that can damage them. As it is well known in the field of LES and DNS, the computational cost of wall bounded flows strongly depends on the modeling used for the walls, thus on the mesh refinement in the near wall regions. Consequently, DNS or wall-resolved LES can only be used for moderate friction Reynolds numbers, while using low order wall-modeling allows to resolve high Reynolds number flows with a reasonable cost.

Reliable wall-modeling in highly anisothermal configurations at low Mach numbers (i.e. under low compressible fluid approximation) has not been reached yet, the main reason being that the wall fluxes depend on all the details of the turbulent flow in the vicinity of the solid boundaries and that measuring such details for high temperature flows is very challenging from an experimental point of view. Still, relevant reference data are required to support / test the physical assumptions made during the development of new wall functions. In this context, DNS appears as a good candidate to generate such data.

DNS results from Nicoud [1, 2] in combination with the use of the Van Driest tranformation [3, 4] are used in Section 2 to derive an anisothermal wall function for RANS. Section 3 describes how the anisothermal properties of this wall function can be integrated in a more complex wall model, namely the TBLE model [5], in the framework of LES. Both models have been implemented in the Trio_U CEA (French Nuclear Energy Agency) code and first results have been produced.

2 Derivation of an anisothermal wall function for RANS simulations

2.1 About the Van Driest transformation

The Van Driest transformation [3] is based on the calculation of a modified dimensionless wall coordinate U_{VD}^{+} by:

$$U_{VD}^{+} = \int_{0}^{U^{+}} \left(\frac{\overline{\rho}}{\rho_w}\right)^{1/2} \mathrm{d}U^{+} \tag{1}$$

Note that the mass-weighted quantities are considered here, based on the Favre decomposition [6]. The dimensionless wall coordinate U^{+} stands for $\widetilde{U}/U_{\tau}$, where U_{τ} is the friction velocity.

This transformation has been verified in various turbulent channel flow, particularly in the DNS results of Nicoud [1, 2], even in the case of very high temperature gradients, to match the standard logarithmic law [7], so that one can admit that:

$$U_{VD}^{+} = \frac{1}{\kappa} \ln y^{+} + C \qquad \text{with: } y^{+} = yU_{\tau}/\nu_w \tag{2}$$

The DNS results of a periodic turbulent channel flow with variable density, at a friction Reynolds number $Re_{\tau} = 180$, from Nicoud [1, 2] show that the ratio between the local temperature $\widetilde{T}$ and the wall temperature T_w is a simple linear function of the product $Pr_t B_q U^{+}$. Pr_t denotes the turbulent Prandtl number, and $B_q = T_{\tau}/T_w$ (T_{τ} is the friction temperature):

$$\frac{T_w}{\widetilde{T}} = \frac{1}{C_1 - \mathrm{Pr}_t B_q U^{+}} \tag{3}$$

where C_1 is a constant. Even if $C_1 = 1$ has been proven to be a reasonable choice [8], this constant will be expressed by identification later on in this section. At this point, for algebraic reasons, we choose C_1 as (K being a constant):

$$C_1 = 1 - \mathrm{Pr}_t B_q \times K \tag{4}$$

Inserting Eq. (3) in the Van Driest tranformation, and using the notation $\Delta T = (\widetilde{T} - T_w)/T_w$, we can integrate Eq. (1):

$$U_{VD}^{+} = \frac{2\,(U^{+} + K)}{\Delta T} \left[\sqrt{1 + \Delta T} - \sqrt{1 + \frac{K \Delta T}{U^{+} + K}}\right] = \frac{1}{\kappa} \ln y^{+} + C \tag{5}$$

2.2 Determination of the constant K

When $\Delta T \to 0$, it is trivial to demonstrate that the anisothermal wall function recovers the standard law for the velocity, for all values of K. In terms of temperature, the standard law valid for the quasi-isothermal cases is Kader's law [9] (obtained from a flat plate configuration). In the logarithmic layer, the Kader formula leads to:

$$T^+ = 2.12 \ln y^+ + \beta(\mathrm{Pr}) \qquad (6)$$

$$\text{with:} \quad \beta(\mathrm{Pr}) = \left(3.85\mathrm{Pr}^{1/3} - 1.3\right)^2 + 2.12\ln(\mathrm{Pr})$$

The identification of the two functions given by the relation $T^+ = \mathrm{Pr}_t(U^+ + K)$ from Eq. (3) and Eq. (6) has been made at $y^+ = 100$, in an arbitrary way. We find:

$$K = K(Pr) = -C_{u,vd} + \frac{\beta(\mathrm{Pr})}{\mathrm{Pr}_t} + \left(\frac{2.12}{\mathrm{Pr}_t} - \frac{1}{\kappa}\right)\ln(100) \qquad (7)$$

The constant K has then be expressed, and the anisothermal wall function that has been derived here can be described in its final form by the set of three equations: Eqs. (3)–(5)–(7).

2.3 Expression of the turbulent kinetic energy k and dissipation ϵ

In a RANS simulation, using the $k-\epsilon$ model, the use of a wall function implies that the values of $\widetilde{k}$ and $\overline{\epsilon}$ are adapted from the wall characteristics, namely U_τ and the wall-normal velocity gradient.

In the anisothermal regime, the expression of the turbulent kinetic energy is:

$$\widetilde{k} = \frac{\rho_w}{\overline{\rho}} \frac{U_\tau^2}{\sqrt{C_\mu}} \qquad (8)$$

This expression of $\widetilde{k}$ is the standard expression for the turbulent kinetic energy in the first off-wall point, multiplied by the density ratio, which is simply $1 + \Delta T$. Consequently, in quasi-isothermal flows, the density ratio tending to unity, the expression of $\widetilde{k}$ recovers the standard one.

As the logarithmic law is not valid in anisothermal flows, this velocity gradient is recalculated from Eq. (5). Noting $B = 1 + (K(Pr)U_\tau \Delta T)/(\widetilde{U} + K(Pr)U_\tau)$, the dissipation can be expressed as:

$$\overline{\epsilon} = \frac{\rho_w}{\overline{\rho}} U_\tau^2 \left[\kappa y \left(\frac{2\sqrt{1+\Delta T}}{\Delta T U_\tau} - \frac{2B^{1/2}}{\Delta T U_\tau} + \frac{K(Pr)B^{-1/2}}{\widetilde{U} + K(Pr)U_\tau}\right)\right]^{-1} \qquad (9)$$

In the vicinity of $\Delta T \to 0$, the density ratio has yet been identified to tend to unity. The parenthesis term of Eq. (9) in a quasi-isothermal regime becomes U_τ^{-1}. It is consequently found that the dissipation recovers the standard expression.

2.4 Academic test case and results

The main academic test case investigated to validate the anisothermal wall function is a turbulent bi-periodic channel flow. Firstly, the considered flow is quasi-isothermal, i.e. submitted to a small temperature gradient near the wall. Then, three anisothermal cases, with increasing temperature gradients, are presented. The change of the temperature gradient is obtained by changing the wall temperature ratio T_2/T_1, T_1 being the bottom wall temperature (cold) and T_2 the top wall temperature (hot).

The friction Reynolds number $Re_\tau = yU_\tau/\nu_w$ is equal to $20,000$, which is a high enough value to generate a turbulent flow representative of engineering applications. Unfortunately, few reliable experimental or numerical results are available for such a value. The first off-wall point, where the wall function applies, is located around $y^+ = 100$. A total of 33 nodes composes the wall normal refinement.

The simulations were performed with the CEA homemade CFD Trio_U code [10], validated elsewhere, following a Low Mach number method, that is in a Quasi-Compressible approach (QC), accounting for dilatable fluids without solving the acoustics.

- **Quasi-isothermal case:**

Here, we verify that the anisothermal wall function really recovers a standard behaviour. The temperature ratio is close to 1 ($T_2/T_1 = 1.01$). The dimensionless U^+ and T^+ profiles across the channel flow are displayed in Fig. 1. No difference between the different calculations is detected. Moreover, the velocity profiles fit well the standard logarithmic law. The same result is obtained for the temperature, comparing the results to Kader's law.

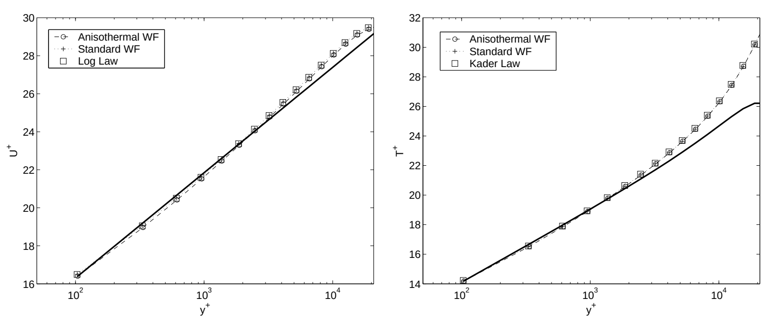

Fig. 1. Dimensionless velocity and temperature profiles in the quasi-isothermal case

- **Anisothermal cases:**

Three anisothermal flows were investigated, with $T_2/T_1 = 1.33$, 1.66 and 4. The values of the friction velocity and friction temperature are listed in Table 1. The standard wall function over-estimates the wall heat flux, which is an established fact. The coupling between the dynamics and thermal properties is observed by the significant changes of U_τ in the different cases. Fig. 2 shows the dimensionless velocity and temperature profiles for the case $T_2/T_1 = 4$. Note that the logarithmic and Kader's laws are not valid in this case, but are shown as a reference.

For large temperature gradients, the change of wall-modeling leads to significant differences in the T^+ profiles, thus in the assessment of q_w.

		Anisothermal WF	Standard WF	Relative error (%)
$T_2/T_1 = 1.01$	U_τ	0.4039	0.4024	0.40
	T_τ	-0.0483719	-0.0484043	0.07
$T_2/T_1 = 1.33$	U_τ	0.3925	0.3982	1.46
	T_τ	-1.40449	-1.43095	1.88
$T_2/T_1 = 1.66$	U_τ	0.3807	0.3934	3.34
	T_τ	-2.50365	-2.58128	3.10
$T_2/T_1 = 4$	U_τ	0.3105	0.3657	17.8
	T_τ	-6.94711	-7.29286	4.98

Table 1. U_τ and T_τ for the four different temperature ratio T_2/T_1

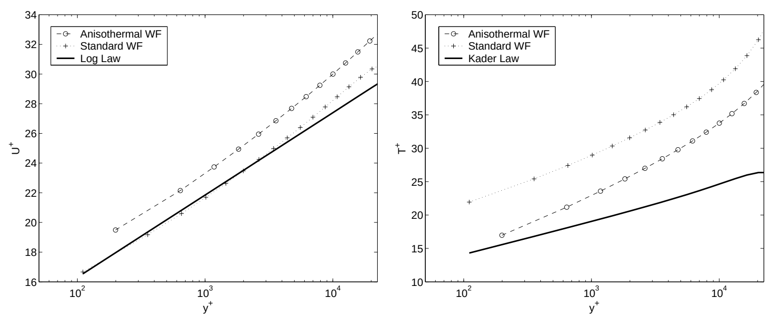

Fig. 2. Dimensionless velocity and temperature profiles, case $T_2/T_1 = 4$

3 Adaptation of the TBLE model to anisothermal flows

3.1 The TBLE model and its anisothermal version

The TBLE consists in embedding a one-dimensional fine grid between the wall and the first off-wall coarse mesh point. It was introduced in 1996 by Balaras [5] and has been increasingly investigated by other researchers [11, 12, 13]. Thanks to the 1-D mesh refinement, the wall shear stress is evaluated directly by the velocity gradient at the wall, after reaching the converged solution of the boundary layer equations in the fine mesh.

The TBLE method is based on the resolution of four equations: two for the streamwise and spanwise momentum components, one for the temperature and the state equation. If y is the wall-normal direction, the set of equations reads:

$$P_0 = \rho R T \tag{10}$$

$$\frac{\partial \rho U_k}{\partial t} = F_k + \frac{\partial}{\partial y}\left[(\mu + \mu_t)\frac{\partial U_k}{\partial y}\right] \tag{11}$$

$$\rho c_p \frac{\partial T}{\partial t} = F_{th} + \frac{\partial}{\partial y}\left[(\lambda + \lambda_t)\frac{\partial T}{\partial y}\right] \tag{12}$$

P_0 is the thermodynamic pressure of the Low Mach number approach, which is constant over the whole domain and appears in the state equation (10). The subscript k denotes the velocity component ($k = 1$ or 3). F_k and F_{th} denote the source terms for the momentum and the temperature.

In order to adapt the TBLE model to high temperature gradient configuration, the mixing length turbulence model basically used [5] needs to be adjusted so that the momentum equation recovers the Van Driest transformation. Without temporal and source terms, Eq. (11) reads (in the logarithmic region, where $\mu \ll \mu_t$):

$$\frac{\mathrm{d}}{\mathrm{d}y}\left(\mu_t \frac{\mathrm{d}U}{\mathrm{d}y}\right) = 0 \tag{13}$$

Eq. (13) is known to correspond to the logarithmic law. As this standard law is not valid in the anisothermal regime, we need to use an adapted turbulence model (noted μ_{t_a}), so that Eq. (2) is true, that is equivalent to:

$$\frac{\mathrm{d}}{\mathrm{d}y}\left(\mu_{t_a} \frac{\mathrm{d}U_{VD}}{\mathrm{d}y}\right) = 0 \tag{14}$$

The direct differentiation of Eq. (1) gives a simple relation between U and U_{VD} and, hence, Eq. (14) can be rewritten as:

$$\frac{\mathrm{d}}{\mathrm{d}y}\left(\mu_t \frac{1}{\sqrt{1 + \varDelta T}} \frac{\mathrm{d}U}{\mathrm{d}y}\right) = 0 \tag{15}$$

Eq. (15) finally shows that a suitable expression for μ_{t_a} is a mixing length model formulation, multiplied by an anisothermal factor, function of the temperature difference between the local TBLE mesh point considered and the wall.

3.2 Preliminary results and expectations

A first step in the use of this dilatable version of the TBLE model has been to test it off-code. Note that its implementation in the Trio_U code has been done, even if no results can be discussed at the moment.

From a linear profile that is imposed between the first off-wall LES point and the wall onto the fine mesh, the TBLE model converges towards a turbulent profile as given by the solution of the simplified Navier-Stokes and energy equations.

In a quasi-isothermal regime, the dimensionless velocity and temperature profiles were analysed to check the ability to recover respectively the standard logarithmic law [7] and the Kader formula [9]. Fig. 3 shows that the TBLE model fits very well these two reference laws.

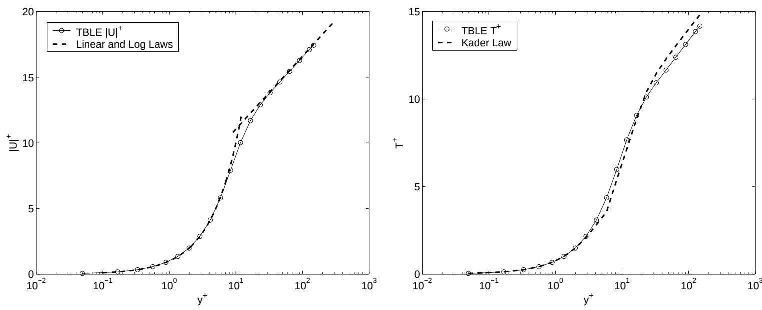

Fig. 3. Dimensionless velocity (left) and temperature (right) profiles given by the TBLE model after convergence

Our expectations concerning the dilatable version of the TBLE model is that it gives at least the same results as the anisothermal wall function in high temperature gradient cases. Also, we expect that the additional cost will remain low.

4 Conclusions and future work

In this paper, we present an original method for the derivation of anisothermal wall models. A first one dedicated to RANS simulations is an anisothermal wall function in the classical law-of-the wall concept, while the second is a zonal approach particularly suited for LES. As the Van Driest transformation has been proven to be valid even in very high temperature gradient cases, particularly by the use of DNS results, we except that these two anisothermal models will give valuable results in strong heat transfer configurations, compared to the classical wall models.

The TBLE approach, in the framework of LES, is meant to provide information of the unsteady near-wall behaviour of the velocity and temperature fields. But obviously, the unstationarity that can be captured by the wall function is limited to a certain frequency that is unknown for the moment. We propose to investigate in a further work this frequency limit, via direct simulation of a channel flow, by scanning a range of frequencies of passive scalar temporal variations and analyzing the impact on the near-wall behaviour. Such a work is under progress.

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