Numerical Simulation of Rotating Stall Inception in an Axial Compressor

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The prediction of the surge line position is a key parameter for motorists. Moreover, the development of aerodynamic instabilities in compressors is not fully understood and thus cannot be efficiently controlled. These subjects are the main reasons of this paper. It deals with the 3D simulation and the investigation of a rotating stall inception phenomenon in a complete stage of an axial compressor. The computed configuration shows three cells of low axial momentum that extend on the full span of the compressor. Each cell moves around the compressor annulus at 55\% of the rotor speed. A comparison with experimental data indicates that the stability limit is well computed whereas the number of cells is not correctly predicted by the simulation. Numerical results underline the competition between the tip leakage flow and the main flow during the stall inception process. A first local instability develops near the casing and is interpreted as the precursor of the modal rotating stall.

\textbf{Nomenclature}

\textbf{Latin letters}

\begin{tabular}{ll}
C & number of rotating stall cells \\
\( f \) & frequency \\
\( h \) & radial position \\
\( H \) & compressor span \\
\( \text{He} \) & helicity \\
\( m \) & spatial interaction mode \\
\( N_{R,S} \) & number of blades (R: rotor, S: stator) \\
\( \text{Pi} \) & total pressure \\
\end{tabular}

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$P_s = \text{static pressure}$

$r = \text{radius}$

$R = \text{gas constant } (c_p/c_v = 287.0 \text{ J.K}^{-1})$

$R_{k, S_k} = \text{modal structure induced by a rotor or a stator blade row}$

$Q = \text{mass flow}$

$Q_n = \text{nominal mass flow } (10.5 \text{ kg.s}^{-1})$

$t = \text{time}$

$T = \text{rotor passing period } (2\pi/\Omega_{\text{rotor}})$

$T_i = \text{total temperature}$

$U_t = \text{friction velocity}$

$V_{x, \theta, z} = \text{absolute velocity components}$

$w = \text{Hamming’s function}$

$W_{x, \theta, z} = \text{relative velocity components}$

$y^+ = \text{non dimensional distance to the wall } (y U_t/\nu)$

**Greek letters**

$\theta = \text{azimuth}$

$\gamma = \text{gas constant } (c_p/c_v = 1.4)$

$\lambda = \text{throttle parameter}$

$\rho = \text{density}$

$\Omega = \text{rotation speed}$

$\nu = \text{kinematic velocity}$

**Subscript / superscript**

$0 = \text{upstream state}$

$out = \text{outlet value (upstream the throttle position)}$

$\text{inf} = \text{infinite value (downstream the throttle position)}$

$\sim = \text{dimensional value}$
I. Introduction

ERODYNAMIC instabilities are one of the most important domain for the design of compressors. These flow phenomena have been studied for 50 years but the basic mechanisms are not yet fully understood. Many experimental studies (Emmons, Day) have characterized these instabilities which occur around the maximum of the total-to-total pressure ratio. Usually the first instability encountered in most compressors is the rotating stall. This phenomenon is characterized by the development of one or more cells of stalled flow which rotate at a fraction of the rotor speed. This instability is responsible of a performance reduction and of large vibrations that can damage blades. Moreover, it can lead to surge, a more energetic instability characterized by large axial mass flow variations.

To keep the system safe, motorists use a security margin called the surge margin. Unfortunately this approach leads to choose an operating point with lower performances compared to the maximum of efficiency. Mainly, two approaches are possible to increase performances. The first one is to reduce the surge margin. But to operate closer to surge line means to increase the probability of stall inception. Thus, this method is reliable only if precursors can be detected before instabilities occur (Garnier). The second approach is to shift the stability limit toward lower mass flow. Shifting stability limit needs a good understanding of the physical mechanisms that are responsible for stall inception. Then it is possible to control the flow and increase the stability, for example with casing treatments (Brignole).

In that context, an efficient prediction of the stability limit is very useful for motorists to design compressors at higher pressure ratio and to improve the system in terms of weight. Analytical models (Moore) can give valuable information about the pressure ratio or mass flow rate evolution during rotating stall or surge but do not accurately estimate the stability limit. According to these models, the stability limit of a compressor is the maximum of the pressure ratio of its characteristic. Unfortunately, the stability limit is not always observed at that point. Some phenomena as separation in the intake (Colin) are responsible of inlet distortion that can lead to rotating stall or surge, even if the operating point is on the negative slope of the characteristic. One way to improve the prediction of stall inception point is the use of Computational Fluid Dynamics. Previous studies have shown that this approach is able to simulate aerodynamic instabilities (Niazi). For example, basic mechanisms of rotating stall can be captured using a quasi 3D time marching Navier-Stokes flow solver (Gourdain). During the past few years, other numerical studies were also carried out in order to explore the role of different elements on the loss of stability. For example,
Bergner\textsuperscript{9} and Inoue\textsuperscript{10} have highlighted the role of the tip leakage flow on the stall inception and He\textsuperscript{11} has shown that rotor/stator interactions can modify the stability limit and the configuration of rotating stall.

The present work focuses on two aspects: the simulation of rotating stall with a 3D flow solver and the improvement of knowledge about this instability. The originality of this study is to perform a simulation on the whole annulus of a compressor stage. Moreover a sufficient physical time is computed to observe the complete development of the rotating stall phenomenon. Thanks to this simulation, the role of the 3D effects and of the tip leakage flow can be investigated. A description of the compressor is provided in a first part. The numerical method, the compressor model and the boundary conditions are described in the second part. Results from the calculation on a parallel computer are presented and compared with experimental data in a third part. Then the different mechanisms that play a role in the development of rotating stall are explained in a last part. A time-spectral representation is performed to study the transient flow from the stable operating point toward the first unstable operating point. A simple model based on an extension of the Tyler and Sofrin\textsuperscript{12} relation is also used to understand the flow structure during rotating stall. The aim of the model is to predict Fourier modes and frequencies which are the result of the interaction between rotating stall cells and the blade rows of the compressor.

II. Description of the compressor

The test case considered for this study is the Cme2 compressor (Faure\textsuperscript{13}, Fig. 1). This axial subsonic compressor designed by SNECMA is located at the LEMFI laboratory (Orsay, France). It is composed of a single stage with one rotor of 30 blades and one stator of 40 blades. Table 1 gives some information about the aerodynamic performances and the main geometric characteristics. The measured nominal rotation speed is 6330±14 rpm, which corresponds to a relative tip Mach number of 0.534, so the flow is perfectly subsonic in the entire compressor.

![Fig. 1 View of the compressor Cme2 (left) and of the experimental facility (right)]
The tip clearance is 0.8% of the rotor span. At the nominal operating point, the characteristics of the flow are $10.5 \pm 0.1 \, \text{kg} \cdot \text{s}^{-1}$ for the mass flow and 1.152 for the pressure ratio. This kind of low power compressor is well suited to study instabilities because the risk of mechanical failure is low. More precisely, due to the performances of the system, the instability which develops is the rotating stall and no surge phenomenon is exhibited. The compressor test facility is equipped with a Laser Doppler Velocimeter (LDV) and hot cross-film probes. Experimental investigations at design, off-design and during rotating stall were carried out at the LEMFI (Michon\textsuperscript{14}).

Table 1. Compressor description

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal pressure ratio</td>
<td>1.152</td>
</tr>
<tr>
<td>Nominal mass flow</td>
<td>$10.5 , \text{kg} \cdot \text{s}^{-1}$</td>
</tr>
<tr>
<td>Measured efficiency</td>
<td>0.92</td>
</tr>
<tr>
<td>Nominal rot. speed</td>
<td>6330 rpm</td>
</tr>
<tr>
<td>Blade Passing Frequency</td>
<td>3165 Hz</td>
</tr>
<tr>
<td>Rotor speed at tip</td>
<td>$182.0 , \text{m} \cdot \text{s}^{-1}$</td>
</tr>
<tr>
<td>Shaft power</td>
<td>180 kW</td>
</tr>
<tr>
<td>Tip radius</td>
<td>0.275 m</td>
</tr>
<tr>
<td>Number of rotor blades</td>
<td>30</td>
</tr>
<tr>
<td>Number of stator blades</td>
<td>40</td>
</tr>
<tr>
<td>Rotor blade chord</td>
<td>84 mm</td>
</tr>
<tr>
<td>Stator blade chord</td>
<td>77 mm</td>
</tr>
<tr>
<td>Tip clearance</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Hub to tip ratio</td>
<td>0.78</td>
</tr>
</tbody>
</table>

III. Numerical simulation

A. 3D flow solver and numerical parameters

The governing equations are the Reynolds Averaged Navier-Stokes equations (RANS) that describe the conservation of mass, momentum and energy of the fluid. This system of equations is solved with the \textit{elsA} software developed by ONERA and CERFACS (Cambier\textsuperscript{15}). Equations are integrated with a compressible finite volume formulation method on structured meshes in the reference frame of each row. The simulation of rotating stall can only be performed with unsteady calculations, so the cost in terms of CPU time and memory is a major parameter for this kind of application. A preliminary unsteady simulation with a 3D simplified configuration has been performed to validate numerical choices (Gourdain\textsuperscript{16}).
In order to reduce CPU cost, a wall law approach is used. The minimum cell size is equal to 40 μm that corresponds to a mean computed distance $y^+$ of 20. An explicit method is well known to be less memory consumer compared to a full implicit algorithm. For this reason, an explicit 4 steps Runge Kutta scheme is applied for temporal integration. However this scheme is coupled with an Implicit Residual Smoothing algorithm to speed up the convergence. The method is of second order time accuracy.

Convective fluxes are computed thanks to a 2nd order centered scheme with artificial scalar dissipation (Jameson\textsuperscript{17}). The system of equations is then closed with the one-equation Spalart-Allmaras turbulence model (Spalart\textsuperscript{18}). Due to the mean value of the Reynolds number ($\text{Re}=5 \times 10^5$), the flow is assumed to be fully turbulent. Thus no transition model is applied. With these parameters, only 10,000 iterations by rotor revolution are required to ensure a stable computation.

B. Meshing strategy and boundary conditions

The meshing strategy is based on a multi-block topology (Fig. 2). One passage is divided into H-grids and one O-grid around the blade. The tip clearance is taken into account thanks to a single H mesh with 11 points in the radial direction. To allow more flexibility on the design of the mesh, a boundary condition with non-matching points is used at the interface between the O grid of the blade and the H mesh of the tip gap. The number of mesh points for one passage is about 500,000 for one rotor channel and 400,000 for one stator channel. The node distribution in one passage is described in table 2. In order to avoid numerical difficulties like mass flow distortion on the inlet boundary condition, the mesh extends up to 0.3 diameter upstream the rotor blades and 0.2 diameter downstream the stator blades. The rotation speed of the rotor is fixed at the nominal rotation speed (6330 rpm). At the inlet, the boundary condition is an axial injection condition. Uniform stagnation pressure $P_i$ and temperature $T_i$ are imposed (without any perturbation). Classical downstream boundary conditions as a uniform static pressure are not sufficient to simulate an operating point near the pressure ratio peak. When the slope of the characteristic becomes zero, the calculation is not able to reach the fixed downstream pressure. A solution is to use a boundary condition with a higher degree of freedom. To set the operating point, the experimental facility is equipped with a throttle. The chosen boundary condition is thus based on the behaviour of an idealized throttle. A simple quadratic law is applied at the outlet to determine a static pressure $P_{out}$ thanks to Eq.(1). The mass flow used to fix the static pressure value corresponds to the mean mass flow computed inside the calculation domain on the outlet window. Then a radial equilibrium condition is applied to compute a static pressure along the compressor span with Eq.(2). The main
advantage of this method is its capacity to define a temporal evolution of the static pressure used on the outlet boundary condition. In the case of unsteady flows (like rotating stall), this method avoids numerical instabilities and captures better physical behaviours of the flow.

\[ P_{S_{out}} = P_{inf} + \lambda Q^2 \]  

\[ \frac{\partial P_s}{\partial r} = \rho \frac{V_\theta^2}{r} \]  

A 1D non reflective condition is associated with the inlet and outlet boundary conditions. Thanks to this model, all points of the compressor characteristic can be simulated (including the maximum of pressure ratio). The position of the operating point on the characteristic is fixed only by the value of the throttle parameter \( \lambda \).

| Table 2  Node distribution in one passage (i\times j\times k) |
|-----------------|-----------------|
| **Rotor**       | **Stator**       |
| O-blade         | 183×19×67       | 171×19×57       |
| H-upstream      | 33×57×67        | 9×47×57         |
| H-channel       | 59×21×67        | 67×25×57        |
| H-downstream    | 15×51×67        | 39×41×57        |
| H-tip clearance | 93×7×11         | -               |
| **Total**       | 500.415         | 395.922         |

Fig.2 Partial view of the 3D mesh (1 point over 2)
C. Simulation of the whole stage

The main reason to simulate the complete stage of the compressor is that no flow periodicity is assumed during the rotating stall. Unfortunately, it was not possible to simulate the complete experimental facility for practical reasons of computational time. Thus, the simulated lengths of the inlet and outlet ducts represent only a part of the experimental geometry. The mesh described in the table 2 is duplicated in order to simulate the flow in all passages (30 rotor channels and 40 stator channels). It leads to a mesh of 31 millions of nodes. For one rotor revolution, the CPU time with 4 processors NEC-SX6 is about 260 hours and the RAM memory consumption reaches 30 GB. The table 3 shows the performances of the parallel approach with respect to a single processor. For this application, the mean speed-up with four processors is equal to three.

### Table 3 Performances of the parallel computation

<table>
<thead>
<tr>
<th>Number of processors</th>
<th>CPU time (μs/it/point)</th>
<th>Mflops</th>
<th>Mflops by processor</th>
<th>Speed-up</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.14</td>
<td>2194</td>
<td>2194</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>0.72</td>
<td>6521</td>
<td>1630</td>
<td>2.97</td>
</tr>
</tbody>
</table>

The simulation starts from a converged steady field at near stall conditions. Then the throttle parameter is increased to obtain an operating point beyond the surge line. Only a single unsteady simulation is performed for the whole stage (i.e. one operating point). With the CPU time available (4000 hours), 14 revolutions of the rotor have been simulated. Then the capacity of the model to simulate the stall inception is estimated in the next part thanks to a comparison with experimental data.

IV. Comparison with the experimental data

In some compressors, the experimental time for the development of aerodynamic instabilities can be very long (more than hundreds of revolutions). Obviously, it is not possible to reproduce such a time with a numerical simulation. Thus to estimate the position of the surge line, unsteady calculations are performed on a tenth of the whole geometry. The operating point is assumed to be stable if a periodic state is reached at the end of the simulation (10 rotor revolutions). A first validation of the numerical model is obtained by a comparison between the experimental characteristic and the numerical one. The total-to-total pressure ratio is plotted on Fig. 3 with respect to the mass flow. Despite of an important discrepancy at the nominal operating point, the pressure ratio is well
estimated for low mass flow rates, especially around the pressure peak. The last stable operating point is found at $Q/Q_n=0.828$. Experimentally, the stability limit is found at $Q/Q_n=0.816\pm0.009$. Thus the stall inception point is well estimated by the simulation since the difference between experiments and simulation is only 1% of the nominal mass flow rate.

Fig. 3 Comparison between 3D calculations and measurements

Fig. 4 Static pressure signal registered during the simulation (rotor/stator interface, $h/H=80\%$)
During the unsteady simulation of the whole stage, a static pressure signal is registered at the rotor/stator interface at 80% of the compressor span (Fig. 4). For convenience, results shown in this paper are presented in a non-dimensional form

\[ P_S = \frac{\bar{P}_S}{P_{i_0}}, \quad \tilde{V}_{s,\theta,z} = \frac{\bar{V}_{s,\theta,z}}{\sqrt{\gamma R T_i}} \]

First instabilities are detected during the second rotor revolution. The signal Fig. 4 corresponds to a period of time ranging from 8 to 14 rotations. It shows that different phenomena occur during the simulation. Until the 9\textsuperscript{th} revolution, the signal exhibits a first high frequency phenomenon that is not linked to the rotor passing period. Then a transient occurs during the 9\textsuperscript{th} rotation and the flow evolves toward a configuration with a lower frequency. The LDV facility is able to perform 3D measurements but the quality of the results depends on the flow seeding. During the rotating stall, the flow is deviated around the rotating stall cells. Inside cells, there are very poorly seeded regions. Hot cross-film probes are used to improve the measurements accuracy. For example, Fig. 5 shows an axial velocity signal obtained by a probe located near the casing (at 90\% of the rotor span) slightly upstream the rotor. Each peak of the signal is linked to the passage of one cell. This periodic signal demonstrates the stability of the rotating stall phenomenon in this compressor. Figure 6 presents a comparison of two axial velocity flow fields upstream the rotor blades, near the leading edge. The experimental field is obtained from the phase-averaged of the axial velocity component registered by probes. The computed field is an instantaneous axial velocity flow field obtained at the end of the simulation (14\textsuperscript{th} revolution).

**Fig. 5 Axial velocity signal registered by a hot wire probe located near the casing upstream the rotor**
The position of a rotating stall cell is pointed out by a region of low axial momentum (<0.19 on Fig. 6). The capacity to predict properly the rotating stall configuration with a numerical simulation is linked to a good estimation of the number of cells. Unfortunately, Fig. 6 indicates clearly that this number is not well predicted by the simulation. Experimentally one cell moving at 40% of the rotor speed is observed, whereas three cells are shown by the numerical simulation. The table 4 compares the experimental and the numerical data during rotating stall. Although the number of cells is not the same, some similarities exist in both cases. Firstly, about a third of the circumference is blocked and each cell extends on the whole compressor span. Experiments show a region of high axial velocity (>0.31 on Fig. 6) inside the cell and near the casing. The 3D simulation shows also a small region of high axial velocity near the casing but located on the left part of each cell. Secondly, the total pressure ratio during rotating stall is reasonably well predicted by the simulation (difference is about 8%).

It is not very surprising that differences exist between the simulation and the experimental measurements. The authors’ opinion concerning these differences is that the computed flow domain is too short, especially the length of the downstream duct and the presence of four struts upstream the rotor in the experiments. Previous works have already shown a strong influence of the system in the selection of the most destabilizing mode (Spakovszky). For example, a study that considers a simplified geometry of the compressor has been performed *a posteriori* to assess the influence of ducts (Gourdain). It indicates that longer downstream duct leads to rotating stall with fewer cells.
The axial velocity flow field from a simulation with a long downstream duct is shown on Fig. 7. Only one cell is observed.

![Flow field image](image)

**Fig. 7 Instantaneous flow field of static pressure (simulation with long ducts, results from [19])**

<table>
<thead>
<tr>
<th>Rotating Stall Characteristics</th>
<th>3D numerical simulation</th>
<th>Experimental measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stall Inception Point (Q/Qₙ₀)</td>
<td>0.828</td>
<td>0.816±0.009</td>
</tr>
<tr>
<td>Number of cells</td>
<td>3 (full span)</td>
<td>1 (full span)</td>
</tr>
<tr>
<td>Cell Speed (Ω/Ωₗ)</td>
<td>0.550</td>
<td>0.400</td>
</tr>
<tr>
<td>Flow Blockage</td>
<td>144°</td>
<td>120°</td>
</tr>
<tr>
<td>Mean Mass Flow (Q/Qₙ₀)</td>
<td>0.724</td>
<td>0.816±0.009</td>
</tr>
<tr>
<td>Mean Pressure Ratio (P/P₀)</td>
<td>1.105</td>
<td>1.113</td>
</tr>
</tbody>
</table>

To conclude this section, comparisons with experimental data show that the present numerical simulation is not able to compute the good number of rotating stall cells. Some ways have been suggested to explain this fact. However, the simulation shows a realistic behavior of the stall inception phenomenon and the results may then be analyzed with specific tools. In the next parts, only the numerical stall inception process is investigated.

**V. Analysis of the numerical results**

**A. Rotating stall mechanisms**

The development of rotating stall in the compressor is explored thanks to a spectral representation. A discrete Fourier transform is performed from static pressure signals registered around the whole compressor annulus (θ=0 to
θ=2π) at the rotor/stator interface, near the casing (h/H=80%). The most interesting modes are plotted on Fig. 8 with respect to time. The spatial mode 10 is of a wide interest since it is linked to a rotating flow structure during the stall inception. This mode grows during the first three rotations of the rotor. After the third revolution, it becomes periodic in time with large amplitude variation and 10 cells of stalled flow rotate around the rotor. So, the Fourier mode 10 characterizes this first kind of rotating stall cells. This structure is called in this paper the tip leakage rotating stall since only the tip region is affected by this unsteady flow. A similar phenomenon has already been pointed out in the literature. For example, a simulation performed by März showed that a vortex develops in the tip region and is responsible of rotating instabilities. Hoying has also observed a non-linear disturbance that develops in the tip region near the leading edge of the rotor, before stall occurs.

Then the flow evolves toward a new configuration of rotating stall. During the 9th rotation, the third Fourier mode becomes non-zero. After the 11th revolution the first and the second Fourier modes are also non-zero. That means very large circumferential structures develop in the compressor. The mode 3 has the fastest growth and leads to another kind of instability with only three cells, called modal rotating stall. In this case, the cells extend on the whole compressor span. After the 13th revolution, the Fourier mode 3 becomes the dominant mode in terms of magnitude. At this time, the flow is mainly driven by the three rotating stall cells. Figure 8 shows an overview of the different flows during the simulation. These phenomena are now deeper analyzed.

![Fig. 8 Evolution of the Fourier modes during the simulation](image-url)
1. Development of the tip leakage stall

The tip leakage rotating stall is the result of an interaction between the tip leakage flow, the mean flow and a separation vortex that appears at mid-span on the suction side of the rotor blades. The relative helicity field \( \text{He}_r \), defined by the relation Eq.(3), is plotted Fig. 9.

\[
\text{He}_r = \vec{W} \cdot \vec{rot} \vec{W}
\]

Helicity is used to detect coherent 3D structures in the flow, especially vortices. High helicity structures are clearly shown near the casing upstream the rotor during the second revolution. As a result the main flow is deviated toward the hub and on the adjacent blades. Each structure extends in front of two to three channels.

The main mechanism leading to this flow phenomenon is the increase of aerodynamic blockage in the tip leakage region. At low mass flow, the direction of the tip leakage flow becomes perpendicular to the direction of the mean flow, and the flow angle at the rotor leading edge increases. The result is a separation of the suction side boundary layer near the casing of the rotor. This stalled flow interacts with the main flow and leads to an increase of the losses in the compressor. Obviously, the same process occurs in each rotor channel, but after one more rotor revolution, a coupling between the stalled flow structures is observed and only 10 cells develop. In the same time, a large vortex develops at mid-span on the suction side of each rotor blade. This vortex exists also in the middle of each rotating stall cell. Each cell moves at a partial speed of the rotor, so the associated vortex is moving with the same velocity. A similar structure has been detected by Inoue\(^{10}\) in another subsonic compressor. Figure 10 shows a surface of constant static pressure colored by entropy. The 3D structure represents the core of a separation vortex. This vortex drives a lot of losses from the suction side toward the tip region near the next blade leading edge. Then this vortex interacts with the tip leakage flow near the casing, and massive boundary layer separation occurs on the suction side of the rotor blades. At the end of the third revolution of the rotor, the rotating stall cells are fully developed. As it can be seen on Fig. 11, entropy is a good way to observe the cells. The cells extend on a part of the rotor span (PS). This phenomenon is a local instability that affects the global performance of the compressor. The axial velocity field near the casing is plotted on Fig. 12. Each cell induces a blockage with large region of reversed flow. Due to the flow incidence at the rotor/stator interface, a separation of the stator suction side boundary layer occurs. The compressor operates in these conditions during more than 6 revolutions and the flow evolves toward a new configuration of rotating stall after the 9\(^{th}\) revolution.
Fig. 9 Absolute value of the relative helicity flow field at t=2T (rotor view)

Fig. 10 Iso pressure surface colored by entropy at t=2T

Fig. 11 Instantaneous entropy flow field at t=2T (h/H=80% and rotor/stator interface)
2. **Development of the modal wave**

Correlated to the third Fourier mode, a modal wave appears in the compressor after the 10\textsuperscript{th} revolution. This structure is not linked to the interactions between the rotor and stator blade rows. The modal wave develops during five rotor revolutions and affects the system from the inlet to the outlet. This wave leads to the development of three rotating stall cells which extend on the full rotor span (FS). The instantaneous entropy flow field on Fig. 13 shows the circumferential extension of the three stall cells. In the same time, the flow reorganization induced by this phenomenon leads to the reattachment of the flow near the casing. The flow incidence at the leading edge of the rotor blades decreases outside of the three full span cells. The result is the disappearance of the tip leakage rotating stall. Thus the latter can be interpreted as a precursor of the full span rotating stall. At the end of the 14\textsuperscript{th} revolution of the rotor, the flow is completely driven by the modal wave.
3. Determination of the frequencies during rotating stall

In order to better characterize the flow during the stall inception, a Fourier transform of a static pressure time signal is performed. The signal covers the time ranging from 8 to 12 rotations. So all the different phenomena underlined in this compressor are represented (Fig. 14). The rotor passing frequency is 105 Hz. This value is used in Fig. 14 as a reference value to compare the different flow frequencies. The blade passing frequency is 3165 Hz (30 rotor passing period). The tip leakage rotating stall is characterized by a frequency of 787 Hz, which corresponds to 10 cells moving at 75% of the rotor speed. Based on the Tyler and Sofrin relation, the frequencies 1574 Hz and 2361 Hz can be interpreted as harmonics of the tip leakage rotating stall frequency. The modal rotating stall is characterized by a lower frequency (173 Hz). This value corresponds to 3 cells moving at only 55% of the rotor rotation speed.

![Figure 14: Fourier transform of a static pressure signal (evolution from 10 to 3 cells)](image)

In the present case, the fewer the number of cells is, the lower the rotating speed of the cells is. As shown by Fig. 11 and Fig. 13, these structures are responsible of a complete reorganization of the flow in the compressor. In the next part, a model is proposed to better understand the effects of the rotating stall cells on the flow.

B. Simple model for explanation of rotating stall structures

The rotating stall cells interact with the main flow. These interactions lead to the development of new flow structures and this process is very similar to the interactions between adjacent blade rows. If the flow is considered
only with a modal approach, a rotor/stator mode prediction model can be applied to rotating stall. The assumption is that the instability induces structures in the flow like a particular row for which the number of blades is equal to the number of cells. The Tyler and Sofrin model\textsuperscript{12} (TS model) describe the resulting interaction modes $m$ between one rotor and one stator with Eq.\eqref{eq:4} and give the rotation speed $\Omega_m$ of these structures (Eq.\eqref{eq:5}). This model has been generalized to the case of two rotors (Gerolymos\textsuperscript{23}).

\begin{align}
\tag{4}
m(n, b) &= nN_R + bN_S \\
\Omega_m &= \frac{nN_R}{m}\Omega_R \\
\tag{5}
n, b &\text{: relative integers}
\end{align}

It is also possible to extend this model to the case of a turbomachine with $K$ stages. Each row is simply represented by a wave that moves in the circumferential direction at a constant speed and propagates upstream and downstream. For the $k^{\text{th}}$ rotor and the $k^{\text{th}}$ stator, the waves are modeled respectively by the relations Eq.\eqref{eq:6} and Eq.\eqref{eq:7}. Thus distortions induced by the $k^{\text{th}}$ rotor and the $k^{\text{th}}$ stator in the flow are assumed to be represented by a simple harmonic wave with a spatial ($n_\theta$) and a time argument ($2\pi ft$). In the case of a non rotating blade, the time is simply zero. In other words, the flow perturbations induced by the blades are assumed to develop along a straight line in the space ($t, \theta$): this is the basic assumption of the TS model.

\begin{align}
\tag{6}
R_k &\approx e^{i(2\pi f_l t-a.N_{sl}\theta)} \\
\tag{7}
S_k &\approx e^{i(-b.N_{sl}\theta)}
\end{align}

Interactions between the different rows in a multi-stage environment are represented by the interactions between the waves. The model is built to obtain the same formalism than the TS model. One can verify that if the system is only composed of one rotor and one stator, the result is exactly the TS model. The spatial modes $m$ predicted by the model are then a linear combination of the number of rotor and stator blades given by Eq.\eqref{eq:8}. If all rotors move at the same speed $\Omega$, the rotation speed $\Omega_m$ of a spatial mode $m$ is given by the relation Eq.\eqref{eq:9}. As shown by Eq.\eqref{eq:10}, an important difference with the TS model is that frequencies linked to the interaction structures are not necessarily a
multiple of a rotor frequency. That means low frequencies can develop in the system as the result of interactions between many rotating blade rows.

\[ m = \sum_k (a_k N_{R_k} + b_k N_{S_k}) \]  
\[ \Omega_m = \sum_k a_k N_{R_k} \frac{\Omega_{rotor}}{m} \]  
\[ f_m = \sum_k a_k f_k \]

k: stage subscript  
a_k, b_k: relative integers

The model is then applied to the case of rotating stall. The aim is to understand the structures produced by interactions between the compressor and the rotating stall cells. This is not a predictive tool, as the main drawback of the model is that the rotating stall configuration has to be known to build the relations. The final configuration of the computed flow shows three rotating cells (Fig. 14). They move at 55% of the rotor speed (i.e. 173 Hz). Using Eq.(8) and Eq.(10), a model is proposed to describe the flow during rotating stall. The relations Eq.(11) and Eq.(12) are the result of a system with three waves (one for the rotor, one for the stator and one for the cells) and two main frequencies (one for the rotor, one for rotating stall).

\[ m = (a_1 N_R + b_1 N_s) + a_2 C \]  
\[ f_m = a_1 f_{BPF} + a_2 f_{RS} \]

a_1, a_2, b_1: relative integers  
f_{RS}: rotating stall frequency (173 Hz)  
f_{BPF}: blade passing frequency (3165 Hz)

In the presented case, N_R=30, N_s=40 and C=3. Interaction structures between the compressor and the cells correspond to the modes 7, 13, 17, 23, 27, 33, 37 … The first predicted frequencies for these structures are 2992 Hz and 3338 Hz. Obviously, fundamental modes, as modes 3 (cells), 30 (rotor) and 40 (stator), and fundamental
frequencies (173 Hz and 3165 Hz) are also present in the flow. Results of the model can be used to understand the data from the spectral representations.

C. Modal analysis during the stall inception

A more detailed description of the flow during stall inception is searched out. Thus the analysis focuses on the development of modes in the flow. The model presented in section V.B is used to complete the modal analysis and underlined the main characteristic of the unstable flow. The modal analysis focuses on two kinds of signals: periodic signals (signal in the circumferential direction) and non periodic signals (time signals). For periodic signals, a classical Fourier Transform is performed from 2D axial velocity signals registered around the annulus of the compressor (period is equal to 2π) at a fixed time (t=14T). Results are shown Fig. 15. This instant corresponds to the establishment of the modal wave and thus to the interaction between the rotating stall cells and the compressor. In this figure, the vertical axis indicates the value of the spatial Fourier mode and the horizontal axis represents the axial position along the machine (compressor inlet is on the left side). Modes predicted by Eq.(11) and Eq.(12) are well observed. The third Fourier mode represents the number of full span stall cells (modal rotating stall). This mode demonstrates that the structure propagates upstream and downstream the compressor and crosses the boundary of the simulation domain; this means also that this mode will disturb the whole system of the machine. As a consequence, it could be seen as an interaction mode with the system. Remembering that surge is an instability that occurs as consequence of a strong interaction between the machine and the whole system, the mode 3 can be seen as a good candidate for “a surge-like” mode. On the contrary, all other modes have a local influence close to the rotor. Fourier modes 7 and 13 are the results of the interaction between the tip leakage rotating stall (mode 10) and the modal wave (mode 3). They are backward traveling pressure modes as they propagate upstream. A slight influence for mode 7 is observed downstream the rotor as a forward traveling pressure wave.

Mode 37 is an interaction between the stator wakes (mode 40) and the cells (mode 3). The Fourier mode 37 affects the flow up to the outlet of the calculation domain. As it is an entropy mode, it does not propagate upstream. All these modes are well predicted by the model described in section V.B. The next step is the analysis of the time signals. Due to transient flow and rotating stall, periodicity for time signals does not longer exist. Thus the tool used to represent the signals cannot be a standard Fourier Transform. Some other methods exist in the literature like wavelet transform. But a simpler method to obtain a time-spectral representation is the Short Time Fourier
Transform (STFT) used in this study (Allen24). The STFT \( W \) of a signal \( s(t) \) is obtained from Eq.(13). For this application, the Hamming’s function \( w \) is chosen to decompose the signal.

\[
W(\lambda,l) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} s(t)w(t-l)e^{-i\lambda t} dt
\]

(13)

\( l, \lambda \): real parameters (time and frequency, respectively)

\[
w(t) = 0.54 - 0.46 \cos\left(\frac{2\pi t}{L}\right)
\]

(14)

\( L \): length of the window

**Fig. 15** Spectral power density of the Fourier modes upstream and downstream the compressor (axial velocity signals, \( t=14T, h/H=80\% \))

Static pressure signals are registered by a probe located at 80\% of the compressor span at the rotor/stator interface. The STFT method is performed to obtain the spectrograms (Fig. 16). A spectrogram is the absolute value of the STFT. Two resolutions for the window length are tested. The left part of Fig. 16 corresponds to a fine time resolution. In this case, the length of the window is fixed to one revolution of the rotor. In the case of a better frequency resolution (right part of Fig. 16), the length of the window is fixed to two revolutions of the rotor. The stall inception process is well pointed out by these representations. At the end of the second revolution of the rotor, the dominant frequency in the flow is the tip leakage rotating stall frequency (787 Hz) and its harmonics. This phenomenon induces the development of 10 cells in the tip leakage region. Then the modal rotating stall develops
during the 9th revolution (173 Hz). This frequency corresponds to the three full span stall cells. After the 13th, the amplitude of the tip leakage rotating stall frequency decreases and the dominant frequency becomes the modal rotating stall one.

The right part of Fig. 16 underlines the interactions between rotating stall and the compressor. As seen, the blade passing frequency (BPF) interacts with the cells and new frequencies develop (2992 Hz and 3338 Hz). These frequencies are linked to the Fourier modes shown Fig. 15 (7, 13 ...). An interaction between the part span stall cells and the full span stall cells lead to the development of others frequencies (1401 Hz and 1747 Hz). These structures are the result of the interaction between the first harmonic of the tip leakage rotating stall frequency (1574 Hz) and the modal rotating stall frequency (173 Hz).

**Fig. 16 Spectrogram from static pressure signals (rotor/stator interface, h/H=80%), left: L=1T (good resolution in time), right: L=2T (good resolution in frequency)**

This analysis combined with the model for rotating stall interactions explains, at least partially, the structure of the flow during stall inception. In the present case, the first instability which develops is observed in the tip region (tip leakage stall). The structures induced by this phenomenon are characterized by a frequency which is not linked to the rotor speed. However, these flow structures have a local influence around the blade rows, and are not observed at the calculation domain extremities. Before modal rotating stall occurs, all the Fourier modes of the flow are only a linear combination of the number of rotor and stator blades (like in standard operating conditions). Influence of the rotating stall cells is located near the compressor and the dominant Fourier mode upstream the compressor is still the fundamental mode of the rotor. The second step is the development of a modal wave. It leads
to three full span stall cells which affects the flow in the whole compressor. This structure is no longer a linear combination of the blade number and the Fourier mode linked to these cells affect the compressor from the inlet to the outlet. Unfortunately, no explanation of the selected unstable mode (mode 3) has been found. It is suggested that it is probably an influence of the whole system, as this mode propagates far upstream and downstream of the machine.

**VI. Conclusion**

Despite of a quite important CPU consumption, the complete stall inception process can be represented in the whole stage of an axial compressor with a 3D unsteady RANS simulation. The transient flow from a stable operating point to the rotating stall operating point has been computed. Unfortunately, comparisons with experiments show that the number of cells is not well predicted by the 3D calculation. The main reason is probably the dimensions of the downstream duct (too short). The present work can be summarized by the points below.

1- The stability limit is well estimated by the simulation (difference with experiment is only 1% of the nominal mass flow). With the same boundary condition (same throttle parameter) the simulation indicates that two different instabilities develop, one after the other.

2- The first phenomenon is called the tip leakage rotating stall since it is located in the tip leakage region. This local instability is the result of an interaction between the tip leakage flow, a vortex separation which occurs at mid-span on the rotor blade suction side and the inflow. The frequency linked to this instability is quite high (787 Hz, 10 cells rotating at 75% of the rotor speed). This flow is interpreted as a precursor of the second instability.

3- The second phenomenon is called the modal rotating stall. This global instability affects the whole system of compression and is induced by the development of a long length scale disturbance. The frequency of this rotating stall is low (173 Hz, 3 cells rotating at 55% of the rotor speed). This instability corresponds to the configuration of the flow at the end of the simulation (15 rotations).

4- The rotating stall cells interact strongly with the blade rows of the compressor. The result is the development of structures following the same process than for rotor/stator interaction modes. Based on this observation, a model has been proposed to explore and to analyze the interactions between the rotating stall cells and the compressor.
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