RESEARCH ARTICLE

Time-Domain Harmonic Balance Method for Aerodynamics and Aeroelastic Simulations of Turbomachinery Flows

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(Received 00 Month 200x; final version received 00 Month 200x)

A time-domain Harmonic Balance method is applied to simulate the blade row interactions and vibrations of state-of-the-art industrial turbomachinery configurations. The present harmonic balance approach is a time-integration scheme that turns a periodic or almost-periodic flow problem into the coupled resolution of several steady computations at different time samples of the period of interest. The coupling is performed by a spectral time-derivative operator that appears as a source term of all steady problems. These are converged simultaneously making the method parallel in time. In this paper, a non-uniform time sampling is used to improve the robustness and accuracy regardless of the considered frequency set. Blade row interactions are studied within a 3.5-stage high-pressure axial compressor representative of the high-pressure core of modern turbofan engines. Comparisons with reference time-accurate computations show that four frequencies allow a fair match of the compressor performance, with a reduction of the computational time up to a factor 30. Finally, an aeroelastic study is performed for a counter-rotating fan stage, where the rear blade is submitted to a forced harmonic vibration along its first torsion mode. The aerodynamic damping is analysed, showing possible flutter.

\textbf{Keywords}: harmonic balance, non-uniform sampling, compressor, counter-rotating fan, aeroelasticity

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1. Introduction

Computational Fluid Dynamics (CFD) is now a key element of the modern design of turbomachinery blades. Most of the time, the design relies on steady analyses but new performance-driven aggressive design choices have increased the importance of unsteady phenomena, such as: blade interactions in compact turboengines, separated flows at or close to stable operability limits, aeroelastic coupling, to name the most important ones. Tools and methods are thus needed to account for these effects as early as possible in the design cycle but unsteady computations are only entering industrial practise. Indeed, the associated restitution time remains an obstacle for design cycle process.

To reduce the cost of periodic flow simulations, Fourier-based time-integration algorithms have undergone major developments in the last decade (He 2010). The present paper focuses on a time-domain Fourier-based method, namely the Harmonic Balance (HB) method, which transforms an unsteady time-marching problem into the coupled resolution of several steady computations at different time samples of the period of interest. Initially developed for single frequency problems (Hall et al. 2002, Gopinath and Jameson 2005), the HB method has been extended to account for multiple frequencies (van der Weide et al. 2005, Ekici and Hall 2007, 2008). The present approach relies on the non-uniform time-sampling approach proposed by Guédéney et al. (2011).

The HB method has been extensively applied to turbomachinery configurations; some of the most relevant achievements are: 3D RANS computations of subsonic (Sicot et al. 2012) and transonic (Ekici et al. 2010) single compressor stages, 2D RANS computation of a 1.5-stage compressor cascade (Ekici and Hall 2007); forced motion of 3D isolated fan and compressors (Sicot et al. 2011), 2D wake/rotor interaction and blade vibration simulations (Ekici and Hall 2007, 2008). In this context, the present contribution aims at demonstrating the efficiency of the HB method for state-of-the-art industrial configurations using 3D RANS computations. First, blade row interactions are studied within a 3.5-stage high-pressure axial compressor designed by Snecma, and representative of the high-pressure core of modern turbofan engines. Then, an aeroelastic study is performed for a counter-rotating fan stage, where the rear blade is submitted to a forced harmonic vibration along its first torsion mode.

The paper is organised as follows: First, the multi-frequential Harmonic Balance with non-uniform time sampling is presented. Then, phase-lagged boundary conditions for row interactions and complex aeroelastic modes are discussed. Finally, the results obtained for the two industrial cases are presented and analysed.

2. Time-domain harmonic balance technique

By integrating the unsteady Reynolds-averaged Navier-Stokes (U-RANS) equations over a control volume, the following semi-discrete finite-volume form is obtained:

\[ V \frac{\partial \mathbf{W}}{\partial t} + \mathbf{R} (\mathbf{W}) = 0, \]  \hspace{1cm} (1)

where \( \mathbf{W} \) is the vector of the conservative variables comprising the fluid density, momentum, total energy and an arbitrary number of turbulent variables depending on the model, \( V \) is the volume of the cell and \( \mathbf{R} \) are the residuals resulting from the spatial discretisation of the inviscid and viscous terms complemented with possible source terms.
2.1. **Almost-periodic flows**

2.1.1. Mapping on an arbitrary set of frequencies

A flow is said to be almost-periodic if it is driven by a discrete set of frequencies \( f_k \), \( i.e. \) its spectrum has high-energy discrete-frequency modes (Besicovitch 1932). The U-RANS equations are therefore mapped on a set of complex exponentials with angular frequencies \( \omega_k = 2\pi f_k \). The conservative variables and the residuals can be approximated by

\[
W(t) \approx \sum_{k=-N}^{N} \hat{W}_k e^{i\omega_k t}, \quad R(t) \approx \sum_{k=-N}^{N} \hat{R}_k e^{i\omega_k t},
\]

(2)

where \( \hat{W}_k \) and \( \hat{R}_k \) are the coefficients of the almost-periodic Fourier series corresponding to the frequency \( f_k \). By setting \( f_k = kf, k \in \mathbb{Z} \), the regular Fourier series of a \( f \)-periodic flow is retrieved. Inserting Eqs. (2) in Eq. (1) yields:

\[
\sum_{k=-N}^{N} (iV\omega_k \hat{W}_k + \hat{R}_k) e^{i\omega_k t} = 0.
\]

(3)

By cancelling out the term inside the parenthesis, \( 2N + 1 \) equations need to be solved in the frequency domain. This set of problems is then cast back to the time domain by an almost-periodic inverse discrete Fourier transform:

\[
A^{-1} \cdot \left( iVP\hat{W}^* + \hat{R}^* \right) = 0,
\]

(4)

with

\[
P = \text{diag}(-\omega_N, \ldots, \omega_0, \ldots, \omega_N),
\]

\[
\hat{W}^* = \begin{bmatrix} \hat{W}_N \ldots \hat{W}_0 \ldots \hat{W}_N \end{bmatrix}^T, \quad \hat{R}^* = \begin{bmatrix} \hat{R}_N \ldots \hat{R}_0 \ldots \hat{R}_N \end{bmatrix}^T,
\]

(5)

and the inverse almost-periodic discrete Fourier transform matrix reads:

\[
A^{-1} = \begin{bmatrix}
\exp(i\omega_N t_0) & \ldots & \exp(i\omega_0 t_0) & \ldots & \exp(i\omega_N t_0) \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\exp(i\omega_N t_k) & \ldots & \exp(i\omega_0 t_k) & \ldots & \exp(i\omega_N t_k) \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\exp(i\omega_N t_{2N}) & \ldots & \exp(i\omega_0 t_{2N}) & \ldots & \exp(i\omega_N t_{2N})
\end{bmatrix}, \quad \omega_0 = t_0 = 0.
\]

(6)

This creates a set of \( 2N + 1 \) equations in the time domain, each equation corresponding to a time instant \( t_n \). The almost-periodic Fourier coefficients can therefore be computed by

\[
\begin{align*}
\hat{W}^* &= AW^*, \quad \text{with } W^* = [W(t_0), \ldots, W(t_i), \ldots, W(t_{2N})]^T, \\
\hat{R}^* &= AR^*, \quad \text{with } R^* = [R(t_0), \ldots, R(t_i), \ldots, R(t_{2N})]^T.
\end{align*}
\]

(7)
And Finally, Eq. (4) becomes:

$$iV A^{-1} PAW^* + R^* = 0,$$

where the term \( D_{t} = iA^{-1}PA \), appears as a source term that represents a high-order formulation of the initial time derivative in Eq. (1). A pseudo-time derivative is added to Eqs. (8) in order to time march the equations to the steady-state solutions of all the \( 2N + 1 \) instants. In the periodic case \( \omega_k = k\omega \), an analytical derivation can be derived as in the Time Spectral Method (Gopinath and Jameson 2005). For stability reasons, the computation of the local time step is modified as in van der Weide et al. (2005) to take into account this additional source term. The present HB method is implemented in the elsA solver (Cambier and Veuillot 2008) which uses the block-Jacobi algorithms derived in Sicot et al. (2008) to improve robustness and efficiency.

Since the matrix \( A \) can not be derived analytically in an easy way, the source term \( D_{t} \) has to be numerically computed. Kundert et al. (1988) showed that the condition number of \( A, \kappa = \| A \| \| A^{-1} \| \), and thus \( A^{-1} \), has a salient role in the convergence of a Harmonic Balance computation. If the matrix \( A^{-1} \) is not well conditioned, \( A \) will be not be computed accurately, thus impairing the high order accuracy of this method.

2.1.2. Non-Uniform time sampling

Having a well-conditioned \( A^{-1} \) matrix means that every column that compose the matrix are close to be orthogonal to each others. As the set of frequencies is defined by the user, the only degrees of freedom that can be used to improve the orthogonality of \( A^{-1} \) are the time instants. Unlike the periodic case, it is usually impossible to choose an evenly spaced set of time instants over which the vectors \( \exp(i\omega_k t_n) \) are orthogonal. Actually, it is common for uniformly sampled sinusoids at two or more frequencies to be nearly linearly dependent which causes the severe ill-conditioning problems encountered in practise.

Based on the work done by Kundert et al. (1988) in electronics, Guédeney et al. (2011) implemented an algorithm to automatically choose the best time levels ensuring the orthogonality of the columns. The Almost Periodic Fourier Transform (APFT) algorithm is based on the Gram-Schmidt orthogonalisation procedure. First, the greatest time period (which corresponds to the smallest frequency) is oversampled with \( M \) equally-spaced time levels, \( M \gg 2N + 1 \) being chosen by the user. Using the formula \( A_{k,n}^{-1} = \exp(i\omega_n t_k) \), these time levels allow to build a \( M \times (2N + 1) \) matrix. The first column vector \( V_0 \) (which corresponds to \( t = 0 \)) is arbitrarily chosen as the first time level, and any component in the direction of \( V_0 \) is removed from the remaining vectors using the Gram-Schmidt formula:

$$V_s = V_s - \frac{V_0^\top \cdot V_s}{V_0^\top \cdot V_0} V_0, \quad s = 2, \ldots, M.$$  

The remaining vectors are now orthogonal to \( V_0 \). Since the vectors had initially the same Euclidean norm, the largest remaining vector is the most orthogonal to \( V_0 \). It is then named \( V_1 \). The previous operations are performed on \( M - 2 \) remaining vectors using \( V_1 \) as \( V_0 \). This process is repeated until the required \( 2N + 1 \) vectors have been chosen. As a time instant is assigned to a vector, it yields \( 2N + 1 \) time levels, which allows the construction the matrix \( A^{-1} \).
2.2. Boundary conditions for turbomachinery sector reduction

The HB method allows to contain the problem size by reducing the time span over which the solution is sought. In turbomachinery flows, the spatial periodicity of blade rows can be taken into account so that the spatial domain can be reduced to a single blade passage per row. Under the assumption that all unsteady phenomena in a blade row and during stable operation, are periodic and can be correlated with the rotation rate $\Omega$ of the shaft, the dominant frequencies are those created by the passage of the neighbouring blades. In a multistage compressor, each blade row is sandwiched between the upstream and the downstream blade rows and is subjected to their wakes and potential effects. In practical turbomachines, the blade counts of the rows are generally different and coprime. Consequently the sandwiched blade rows resolve various combinations of the frequencies (addition and subtraction of multiples of the blade passing frequencies). According to Tyler and Sofrin (1962), the $k^{th}$ frequency resolved in the blade row $j$ is given by:

$$\omega_{row}^{trow,j} = \sum_{i=1}^{n_{Rows}} n_{k,i} B_i (\Omega_i - \Omega_j).$$  \(10\)

Here, $B_i$ and $\Omega_i$ are respectively the blade count and the rotation rate of the $i^{th}$ blade row. The $n_{k,i}$ are $k$ sets of integers that drive the frequency combinations. It must be noted that only the blade rows that are mobile relative to the considered blade row $i$ contribute to its temporal frequencies. On a practical point of view, the spectrum is not known $a$ priori and, arbitrarily, a row only solves for the harmonics of the adjacent rows’ BPF:

$$\omega_{row}^{trow,j} = n_{k,j\pm1} B_{j\pm1} (\Omega_{j\pm1} - \Omega_j).$$  \(11\)

2.2.1. Phase-lagged periodic boundary conditions

In a single blade passage configuration, the phase-lag condition (Erdos et al. 1977) needs to be used to take the space-time periodicity into account. It was adapted to time-domain HB by Gopinath and Jameson (2005). It states that the flow in one blade passage is the same as in the next blade passage but at another time $t + \delta t$:

$$W (\theta + \Delta \theta, t) = W (\theta, t + \delta t),$$  \(12\)

where $\Delta \theta = 2\pi / B$ is the pitch of the considered row. The Fourier transform of Eq. (12) yields:

$$\sum_{k=-N}^{N} \tilde{W}_k (\theta + \Delta \theta, t) e^{i\omega_k t} = \sum_{k=-N}^{N} \tilde{W}_k (\theta, t) e^{i\omega_k \delta t} e^{i\omega_k t}.$$  \(13\)

Assuming that every temporal lag is associated to a rotating wave of rotational speed $\omega_k$ and an inter-blade phase angle (IBPA) $\beta_k$, the constant time lag can be expressed as a generalisation of the work of Gerolymos et al. (2002):

$$\delta t = \frac{\beta_k}{\omega_k}, \quad \beta_k = 2\pi \text{sgn}(\omega_k) \left( 1 - \frac{1}{B} \sum_{i=1}^{N} n_{k,i} B_i \right), \quad \forall k,$$  \(14\)
Table 1. Blade number of the compressor rows.

<table>
<thead>
<tr>
<th>Row</th>
<th>IGV</th>
<th>R1</th>
<th>S1</th>
<th>R2</th>
<th>S2</th>
<th>R3</th>
<th>S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of blades per row</td>
<td>32</td>
<td>64</td>
<td>96</td>
<td>80</td>
<td>112</td>
<td>80</td>
<td>128</td>
</tr>
<tr>
<td>Number of blades for $2\pi/16$</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td>7</td>
<td>5</td>
<td>8</td>
</tr>
</tbody>
</table>

where the $n_{k,i}$ are the integer coefficients specified for the calculation of the frequencies in Eq. (10) and $B$ the number of blades in the current row. Finally, the spectrum of the flow in a blade passage is equal to the neighbouring passage modulated by a complex exponential depending on the IBPA:

$$\hat{W}_k(\theta + \Delta \theta, t) = \hat{W}_k(\theta, t) e^{i\beta_k}. \quad (15)$$

Using the same notation as previously, the following matrix formulation is obtained:

$$W^*(\theta + \Delta \theta) = A^{-1}MAW^*(\theta), \quad M = \text{diag}(-\beta_N, \ldots, \beta_0, \ldots, \beta_N). \quad (16)$$

This new periodic boundary condition is applied to the upper and lower azimuthal boundaries of a single blade passage.

2.2.2. Stage coupling

It must be emphasised that every blade row solves its own set of frequencies and thus its own set of time levels. It implies that the $n^{th}$ time level in the $j^{th}$ and $j+1^{th}$ rows do not necessarily match the same physical time. Consequently, at the interface between adjacent blade rows, the flow field on the donor row needs to be generated for all the time levels of the receiver row using a spectral interpolation. A non-abutting join interface is used to perform the spatial communication between the two rows (Lerat and Wu 1996). In order to account for the pitch difference and relative motion, a duplication of the flow is performed in the azimuthal direction taking into account the phase-lag periodicity. Moreover, as described in Sicot et al. (2012) the time levels at the interface are oversampled and filtered to prevent aliasing.

3. Aerodynamics of a 3.5-stage compressor

The first test case is a research multistage compressor dedicated to aerothermal and aerodynamic studies. This 3.5-stage axial compressor, named CREATE (Compresseur de Recherche pour l’Etude des effets Aerodynamiques et TEchnologiques) has been designed and built by SNECMA and is hosted at Laboratoire de Mecanique des Fluides et Acoustique (LMFA) in Lyon (France). Its geometry and its rotation speed are representative of a high pressure compressor median-rear block of modern turbojet engine. At design operating point, the rotation rate is 11,543 RPM leading to a tip speed of 313 m.s$^{-1}$. The number of stages was chosen in order to have a magnitude of the secondary flow effects similar to a real compressor, and to be within the rig torque power limitation. The circumferential periodicity of the whole machine (obviously $2\pi$ in general case with primary blade numbers) has been reduced to $2\pi/16$, choosing the number of blades of each rotor and stator (Inlet Guide Vane -IGV- included) as a multiple of 16 (see Tab. 1). The axial view of the compressor is presented Fig. 1.

The HB simulations are performed on the single blade passage mesh shown Fig. 2
comprising about 6 millions grid points. The Roe scheme (Roe 1981) with Van Leer slope limiter (Van Leer 1974) was used. The turbulence is modelled by the two transport equations \( k-\omega \) of Wilcox (1998). The walls are non-slipping and adiabatic. A non uniform inlet injection allows to take into account the wakes of the IGV without meshing it. At the outlet, a radial equilibrium condition coupled with a throttle relaxation enables to change the outlet static pressure and thus the mass flow rate.

A set of four frequencies is used in each row consisting in the BPF of adjacent rows as listed Tab. 2. Therefore, the IGV and the last stator have a single-frequency set up to the 4th harmonic. It is also the case of the second stator which is sandwiched by rows having the same number of blades. The first rotor captures up to the 4th harmonic of IGV’s BPF and as \( 3B_{IGV} = B_{S1}, R1 \) also captures the BPF of the downstream row by \( B_{PF_{S1}} = 3 B_{PF_{IGV}} \).

The convergence of the residuals and mass flow rate for the nominal and chocked operating points (O. P.) are shown Fig. 3. The nominal operating point shows a faster convergence rate in term of residuals but reach the mass flow rate plateau later than the chocked one.

The pressure ratio and efficiency maps are shown Fig. 4 and 5. The HB computations are close to the reference computation performed with the DTS scheme (Jameson et al. 1981) on a \( 2\pi/16 \) sector.

Figures 6, 7 and 8 show the instantaneous entropy field at respectively 10 \%, 50 \% and
90% of the blade span for two operating points: nominal and choked. The HB method is able to capture the unsteady wake pattern across the stages at different accuracy levels: S1 for instance captures up to the 2nd harmonic of R1’s BPF. Some numerical wiggles can be observed downstream the R1/S1 interface and the wakes are slightly thickened. On the other hand the second stator S2 is sandwiched between rows having the same number of blades and captures up to the 4th harmonic of R2’s BPF. The interface is therefore better resolved with much lower wiggles and no increase of the wake thickness.

The HB can correctly captures the two operating point flows: for instance R3 has a detached flow at mid-chord at 10% blade height. The tip leakage flow is also well captured as observed at the 90% snapshots. The wakes can be convected downstream through several rows when the number of blades are integer multiples: this is, for instance, the case of R1’s wakes impinging S1 and R2.

The CPU time gains observed range from 25 to 30 times faster than the DTS computations depending on the operating point.
4. Aeroelasticity of a counter-rotating fan

The second application concerns the aeroelastic stability of a counter-rotating fan using a weak coupling approach (Rougeault-Sens and Dugeai 2003): first, a modal identification $\Phi$ of the structure is carried out by mean of a Finite Element model. The equation of structure dynamics under aerodynamic loads $F_A$ reads:

$$M\ddot{q} + D\dot{q} + Kq = \Phi^T F_A(t), \quad x = \Phi q,$$

where $M$, $D$ and $K$ are the modal mass, damping and stiffness matrices. The structure displacement $x$ projected on the modal basis is called generalised coordinates $q$. The fluid
response to the harmonic forced motion of the structure modes is simulated allowing to compute $F_A(t)$. The harmonic motion of the geometry is ensured by a mesh deformation technique, based on a structural analogy method implementing linear elastic elements. The weak coupling approach assumes the linearity of the response of the fluid with respect to the displacement of the structure. Therefore small displacements are assumed and the so-called Generalised Aerodynamic Forces (GAF) are linearised, which adds aerodynamic stiffness $K_A$ and damping $D_A$:

$$
\Phi^\top F_A(t) = D_A \dot{q} + K_A q.
$$

A stability analysis is then performed in the frequency domain:

$$
q = \hat{q} e^{pt} \Rightarrow \left( p^2 M + p(D - D_A) + (K - K_A) \right) \hat{q} = 0,
$$

where the Laplace variable $p$ is of the form $p = i \omega (1 + i \alpha)$. Finally, considering only weakly damped or amplified modes (i.e., $|\alpha| \ll 1$), the damping of the fluid/structure coupled system reads $\alpha = -\Re(p)/\Im(p)$.

As the blade row is rotating, the stiffness of the blades is increased and gyroscopic terms are added. The disk being flexible, the blades do not vibrate independently of each other. The cyclic symmetry leads to complex vibration modes, which can be seen as rotating waves travelling at an integer multiple $n_d$ of the rotation speed, where $n_d$ is termed a nodal diameter (Lane 1956). Opposite nodal diameters have the same vibration mode propagating in opposite directions. Therefore their respective modes are complex conjugate.

Until now, this kind of studies were performed on an isolated blade row (e.g. Srivastava et al. (2003)). In this case, the unsteadiness is only due to the blade vibration. Single-frequency HB methods are perfectly suited for such harmonic forced-motion simulations and were successfully applied on applications such as fan configurations (Sicot et al. 2011). The present multi-frequency HB method enables to perform aeroelastic studies in stage when the vibrating blade is exposed to the unsteady upstream wakes. It has been adapted to an arbitrary Lagrangian/Eulerian formulation by Dufour et al. (2010): each HB time level has its own deformed mesh and the deformation speed is approximated by applying the HB time discretisation operator $D^t$ on these meshes. This operator is exact when the signal is purely harmonic which is the case here.

The HB computations are performed on a single blade passage of the two fans complemented with the presence of the bypass separator. The downstream fan is subjected to a forced harmonic vibration along its first torsion mode (cf. Fig. 9), which can lead to dramatic variations in blade relative angle of attack. It captures the upstream fan's BPF and the vibration frequency. In this case, the frequency ratio between the BPFs and vibration frequency is higher than the BPF ratios met in multistage turbomachinery such as CREATE. The phase-lag condition must take into account the de-phasing coming from the unsteady interactions (Eq. (15)) with the upstream fan as well as the nodal diameter of the blade forced vibration: $\beta = 2\pi n_d / B$. In the upstream fan, the effect of the downstream blade vibration is neglected and therefore only its BPF is captured. Previous studies (Sicot et al. 2011) showed that only one harmonic of vibration is sufficient to correctly predict the aerodynamic damping. Finally, the frequencies captured for $N = 4$ are summarised Tab. 3.

The mesh comprises about 4.5 million points (cf. Fig. 10) The present simulations are performed using the same numerical schemes and boundary conditions as in the CREATE
Table 3. Computed frequencies in each fan.

<table>
<thead>
<tr>
<th>Row</th>
<th>Frequencies (N = 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fan 1:</td>
<td>BPF₂ 2 BPF₂ 3 BPF₂ 4 BPF₂</td>
</tr>
<tr>
<td>Fan 2:</td>
<td>BPF₁ 2 BPF₁ 3 BPF₁ f_vib</td>
</tr>
</tbody>
</table>

Figure 9. Downstream fan torsion mode (normalised).

Figure 10. Contra-fan mesh (one out every two points).

Figure 11. Relative Mach number of the contra-fan configuration at 85 % of the blade span.

compressor except that the inlet injection is uniform in the azimuthal direction and the turbulence model of Spalart and Allmaras (1992) is used.

The relative Mach number is shown Fig. 11 at 85 % of the blade span. The shock structure in the downstream fan is affected by the upstream wakes.

Figure 12 shows the aerodynamic damping for four nodal diameters at two rotation speeds. Some diameters have a negative damping meaning that the fluid provides energy to the torsion mode of the structure. This could possibly lead to flutter but not necessarily as other modes (especially bending modes) require to be investigated. The two regimes have the same critical diameter.

Classical DTS computations on a sector are more difficult: even on an isolated row, Cinnella et al. (2004) employ a multipassage computational domain, using a number of blade passages, \( n_p \), given that

\[
n_p = \frac{2\pi z}{|\beta_{nd}|}, \quad n_d \neq 0, \quad n_p > 1
\]  

(20)

where \( z \) is the minimum integer which leads to an integer value for \( n_p \). One drawback is that the domain change for every nodal diameter. The stage configuration is even more challenging as a common domain must be found that also match the phase-lag due to
the blade row interaction. The domain is therefore often the full annulus and such a HB method can again present significant CPU savings.

5. Conclusion

This paper presented a multi-frequency time-domain harmonic balance dedicated to capture the flow in turbomachinery. An algorithm to find the best non-uniform time sampling was derived as well as specific boundary conditions to reduce the spatial extent to a single blade passage. A 3.5 stage compressor was simulated at different operating points to show the ability of the method to capture the aerodynamics of multi-stage turbomachinery. It was then extended to aeroelasticity stability study of a contra-fan at several rotating speeds and at different nodal diameters. The present HB method shows important CPU savings as well as its ability to capture real flow physics.

Acknowledgements

The present Harmonic Balance formulation was developed thanks to the support of the Direction des Programmes Aéronautiques Civils (french civil aviation agency) and of the Aerospace Valley (Midi-Pyrénées and Aquitaine world competitiveness cluster). The authors would also like to thank Snecma from the Safran Group for their kind permission to publish this study. Finally, Prof. Li He is gratefully acknowledge for the opportunity to publish in this special issue.

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