Uncertainty quantification of the far-field noise from a rod-airfoil configuration

By J-C. Giret†, S. Moreau‡ and J-F. Boussuge¶

A Large-Eddy Simulation (LES) solver together with a Ffowcs-Williams and Hawking (FW-H) solver are coupled with a non-intrusive stochastic collocation method to propagate the geometric uncertainty observed in a rod-airfoil experiment. The LES are performed for several displacements of the rod with respect to its nominal position in the cross-wise direction. The positions are chosen to match the Clenshaw-Curtis (CC) points. The acoustic fields are obtained using an FW-H analogy on aerodynamic results sampled over the time on a porous surface embedded in the mesh. An uncertainty quantification is then performed on the mean and fluctuating root-mean-squared (rms) velocity at several transverse profiles, on the mean wall-pressure coefficient $C_p$ on the airfoil and on both the acoustic spectra and the directivity. The uncertainty of the rod position only partially recovers the discrepancies seen in the experimental data. An uncertainty in the position of the rod of ±0.004 m leads to an uncertainty of 0.5-1 dB in the Overall Sound Pressure Levels (OASPL).

1. Motivations and objectives

The understanding and the prediction of airframe noise is important nowadays to meet future noise specifications (ACARE 2020 goals). Indeed, as both turbofan and jet noise have been greatly reduced through the use of high bypass ratios, the design of new aircraft requires the prediction and if needed the reduction of airframe noise coming from the deployed landing-gear and high-lift devices, which have become a major source of noise in approach conditions.

In addition to wind tunnel experiments, computational aeroacoustics provides a promising path to better understand sound generation mechanisms and to speed up the design process of quiet aircrafts. Among the several numerical methods developed in this way, the Large-Eddy Simulation (LES) offers a good compromise between direct numerical simulations and unsteady RANS simulations and can provide accurate broadband noise predictions over the large frequency range that is required for airframe noise.

A requirement for such simulations is the ability to handle complex geometries. Most of the numerical aeroacoustic studies have been performed on structured meshes, as they enable the use of high-order schemes (Lele 1992; Tam & Webb 1993). However, meshing a complex geometry with a block-structured mesh can be at least difficult and at worst impossible. In order to overcome such limitations, overset grids (Berland et al. 2011) or unstructured meshes (Sanjose et al. 2011) can be used. This study focuses on the use of unstructured meshes with a special attention to accuracy issues. The rod-airfoil test case performed by Jacob et al. (2005) at Ecole Centrale de Lyon has been selected for such an assessment as it is now considered a canonical benchmark for landing-gear noise. Indeed, this experiment mimics the simplest geometrical elements found in landing-gears and provides the two acoustical features observed in the noise of landing-gears: a quasi-tonal noise due to the periodic shedding of vortices at the rod and a broadband noise component caused by the impingement of the developed turbulent wake on the airfoil. This test case has already been used in numerous numerical studies (Jacob et al. 2008; Berland et al. 2010; Eltaweel & Wang 2011) to validate their numerical methods for airframe noise predictions. However, the experimental setup showed a bias of -2 mm in the cross-stream direction for the position of the rod (Jacob et al. 2005).

† CERFACS, 42 Avenue Gaspard Coriolis, 31057 Toulouse Cedex 01, France
‡ Université de Sherbrooke, GAUS, 2500 bd. de l’université, Sherbrooke, QC, J1K2R1, Canada
¶ CERFACS, 42 Avenue Gaspard Coriolis, 31057 Toulouse Cedex 01, France
Nevertheless, all the numerical simulations performed so far on this configuration do not account for this bias, and assume a negligible effect of the rod displacement on the far-field noise. A preliminary assessment was achieved for the reported shift of position by Giret et al. (2012) that showed some effect.

This study aims at performing a systematic sensitivity analysis on the rod position in the rod-airfoil configuration shown in Figure 1, which could also represent assembly tolerances in actual landing-gear configurations, and at investigating the validity of the assumption made in the numerical simulations of the acoustic field. Uncertainty analysis is conducted on the rod position in the cross-wise direction to represent a typical bias found in the experiment.

2. Methodology

2.1. Uncertainty quantification

The approach to uncertainty quantification for the rod-airfoil configuration is illustrated in Figure 2. The crosswise displacement of the rod with respect to its nominal position is chosen as an uncertain parameter. This parameter is associated with a uniform probability density function (PDF) over \( \Delta y = \pm 0.004 \text{ m} \) representative of a typical bias found in the experiment. Such a PDF has been chosen as each position is equally likely to occur. A LES chained with an FW-H analogy is used to propagate the uncertainty in the far-field acoustic results.

As existing solvers are used, a non-intrusive stochastic method is preferred to an intrusive method. The stochastic collocation method (SC) is chosen as it has proved its efficiency for flow simulations in various regimes (Babuška et al. 2007; Christophe et al. 2008). The quantities of interest are sampled at particular points in the parameter space. Integral statistics such as mean and standard deviation are obtained using quadrature rules. As LES still remains expensive, the Clenshaw-Curtis (CC) abscissas have been used in this study as they allow the reuse of the abscissae in quadrature rules of higher order. They are defined as the extrema of the Chebyshev polynomials in the intervals \([-1, 1]\). Nine CC points have been used in this study.

2.2. Grid topology

Based on the initial study (Giret et al. 2012), the computational domain for each mesh extends from \(-6C\) to \(7C\) in the streamwise direction and from \(-3C\) to \(3C\) in the crosswise direction. In the spanwise direction, a length of \(30d\) would lead to a huge mesh and to an unaffordable numerical simulation. However, as shown by Jacob et al. (2005), the spanwise pressure correlation length on
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the rod is $6.5d$ for the present configuration. The acoustic levels are then corrected using Kato’s formula.

The computational domain is meshed with CentaurSoft using tetrahedral elements only. The mesh is refined in the region close to the rod and the airfoil to capture the vortex shedding well. The rod has been discretized in the chordwise direction with 52 points and in the spanwise direction with 156 points. It is then slightly coarsened but remains fine enough to propagate the acoustic waves correctly to the FW-H surface. The mesh density is designed in this zone to have a cut-off Strouhal number of $St = fD/U = 7$. Then, it is stretched in both cross-stream and downstream directions. The stretching downstream with the use of a sponge zone is designed to dissipate the vortical structures and to prevent any residual acoustic reflections that may not have been fully damped by the non-reflective boundary condition (Poinsot & Lele 1992).

2.3. Numerical aeroacoustics

The simulations are carried out with AVBP (Schønfeld & Rudgyard 1999), which is a massively parallel, unstructured compressible LES solver, jointly developed by CERFACS and IFP Energies Nouvelles, dedicated to both reactive and non-reactive flows. AVBP has been mainly dedicated to the study of combustion instabilities in aero-engines chambers, and has only been recently successfully applied to aeroacoustics problems such as jet noise by Sanjosé et al. (2011). The Two-Step Taylor-Galerkin TTGC scheme developed by Colin & Rudgyard (2000) has been used in this work due to its low dissipative character compared to the other schemes of its family. A selective viscosity operator of 2nd and 4th order developed by Colin et al. (2000) is also applied to dampen the numerical instabilities at high frequencies. The dynamic Smagorinsky subgrid-scale model of Germano et al. (1991) is used to compute the subgrid eddy viscosity $\nu_t$.

The LES have been carried out until the statistics are converged after the transient state. They run over $1.2 \times 10^{6}$ iterations with a constant time-step of $3 \times 10^{-7}$ s corresponding to a Courant-Friedrichs-Lewy number of 0.7. It leads to a total physical time of 0.3 s to ensure the convergence of the aerodynamic and of the acoustic results. Averaged solutions have been calculated using 1 solution each 10 iterations and the convergence has been improved by averaging over the spanwise dimension. The far-field acoustics is obtained from the near-field unsteady conservative variables by a classical FW-H analogy over a porous surface. An advanced-time formulation (or source-time-dominant algorithm) of the FW-H analogy (Ffowcs-Williams & Hawkings 1969) developed by Francescantonio (1997) is used. Providing that the volumic sources are neglected or that the control surface encloses all the noise sources, the advanced-time FW-H analogy reads:

$$
4\pi p'(x,t_{adv}) = \frac{\partial}{\partial r_i} \int_{f=0} \left[ L_i \frac{t}{r(1-M_r)} \right]_t dS + \frac{\partial}{\partial t} \int_{f=0} \left[ Q \frac{1}{r(1-M_r)} \right]_t dS,
$$

where the porous integration surface is defined by the equation $f(y,t) = 0$. $r = |y-x|$ denotes the distance between the source and the observer and $M_r = M_i r_1$ denotes the Mach number of the observer in the radiation direction. $L_i$ and $Q$ denote the loading noise component and the thickness noise component respectively. $t_{adv} = t + \mathcal{T}$ denotes the advanced time, with $\mathcal{T} = |x(t+\mathcal{T}) - y(t)|/c$ the time needed for a disturbance emitted at a source element $y$ at the time $t$ to reach the observer $x$.

3. Stochastic aerodynamic results

3.1. Baseline case flow topology

Figure 3 exhibits the vortex shedding behind the rod for the baseline case. The transition to turbulence of the wake occurs in the shear layer just after the separation. The turbulent wake develops leading to a wide range of turbulent structures. No reflections are noticeable from the computational domain boundaries. The porous FW-H surface is placed outside the hydrodynamic region. It is open downstream to avoid the crossing of the wake through the integration surface and to avoid the generation of spurious waves as shown by Casper et al. (2004).
Figure 3. Snapshot of the dilatation field (in grayscale) in the central plane and isovalue of the Q-criterion field ($Q = 10^8 \text{ s}^{-1}$) for the baseline case.

Figure 4. UQ results for converged results for the mean streamwise velocity (a) and for the rms streamwise velocity (b).

3.2. Mean and rms velocity profiles

Figures 4 (a) and (b) show the UQ results obtained for a transverse profile at 0.5C in front of the airfoil (see Figure 1) of the mean streamwise velocity $u$ and the root-mean-squared (rms) streamwise velocity $u_{rms}$ respectively. The stochastic mean of $u$ corresponds to the wake deficit of the baseline case and the stochastic mean of $u_{rms}$ to the baseline case turbulent wake. The stochastic mean of $u$ shows a symmetric pattern with respect the $y$–axis and its minimum is located at $y = 0$, as expected from the symmetrical distribution of the CC points with respect to the baseline geometry. The stochastic mean of $u_{rms}$ has two symmetric maxima with respect to the $y$–axis at the positions $y = \pm 0.005 \text{ m}$.

The standard deviation of both velocities are represented by the error bars. The zone dominated by the uncertainties is located in the region where $|y| < 0.04 \text{ m}$, which corresponds to ten times the range of the uncertain parameter. Outside the displacement does not affect the flow. The maximum of the uncertainty magnitude is located at the maximum of the stochastic mean slope as the uncertainty mainly shifts the velocity profile in this case. The uncertainties are less important on the airfoil axis ($y = 0 \text{ m}$) but still remain significant as the slope of both the mean and rms velocities are smoother. Similar results were found at other axial positions in front of the airfoil.
The standard deviation are not symmetric with respect to the $y$–axis for both results, which is discussed in Section 5.

3.3. Wall-pressure coefficient and root-mean-squared pressure on the airfoil

Figures 5 (a) and (b) show respectively the UQ results obtained on the top side of the airfoil for the mean pressure coefficient $-C_p$ and the rms wall-pressure $P_{rms}$. The stochastic mean of $-C_p$ is negative at the leading-edge. It grows to a maximum at 20% of the chord. It then decreases steadily to the airfoil trailing-edge. A local maximum of the stochastic mean of $-C_p$ is noticeable at 1.5% of the chord length. The stochastic mean of $P_{rms}$ shows high values at the leading-edge caused by the vortex-airfoil interaction, growing to a maximum at 1.5% of the chord length that causes the dip in the mean pressure coefficient. It then drops first steeply up to 30% of the chord and then slowly to the trailing-edge of the airfoil. Therefore, both the stochastic means of $-C_p$ and $P_{rms}$ show a maximum at 1.5% of the chord that corresponds to an airfoil half-thickness of 0.005 m. The vertical positions match the maxima locations found for the stochastic mean of $u_{rms}$ in Section 3.2, which corresponds to the impingement of the most turbulent part of the wake. These locations could then contribute the most to the acoustic far-field and provide mostly dipolar noise sources (Amiet 1975). Finally, most uncertainties are mainly present in the first 60% of the chord length for $-C_p$, and in the first 20% of the chord length for $P_{rms}$, where the vortex-airfoil interaction is strong.

4. Stochastic acoustic results

Figures 6 (a), (b) and (c) show the UQ results for the Power Spectral Density (PSD) of the far-field acoustic pressure at the angular positions 60°, 90° and 120° respectively (see Figure 1), and at 1.85 m from the airfoil leading-edge. The stochastic mean of the PSD agree well with the experimental results. The shedding frequency at $St = 0.19$ is well captured. Yet its amplitude is slightly underestimated, increasing from 1dB to 3dB with the angular position. The first harmonic of the shedding frequency is also resolved at $St = 0.38$. The broadband component of the stochastic mean PSD agrees fairly well with the experimental results over the whole frequency range. Most of the uncertainties are present in the broadband part of the spectra and at the first harmonic of the shedding frequency. Except at 120°, the uncertainty remains smaller at the shedding frequency at $St = 0.19$ even though the noise produced contributes the most to the Overall Sound Pressure Level (OASPL).

Figure 6(d) shows the UQ results for the acoustic directivity at 1.85 m. The stochastic mean OASPL underpredicts the experimental results by 1 dB to 3 dB. The largest underpredictions are found at high angles above 110°. This agrees with the trend found in the prediction of the PSD tonal peak intensity. The uncertainty bars on the OASPL have a range between 0.1 and 1 dB and are larger at 120°. The uncertainty at the peak of radiation at 75° is only 0.5 dB. The uncertainty
in the position of the rod in the $y$–direction then only partially recovers the discrepancies seen with the experimental data. Moreover, these results stress that the sensitivity of the directivity to a perturbation of the position of the rod in the $y$–direction is the most important at the grazing angles.

5. Statistic convergence and mesh influence

5.1. Influence of the statistic convergence

The influence of the statistic convergence on the UQ results is assessed in this section using two sets of data. The first one was done using 0.15 s of physical time for both the averaging of the aerodynamic field and the FW-H calculation for all simulations, which was found to be enough for the nominal condition (Giret et al. 2012). Yet, certain CC simulations were not statistically converged. The second one was obtained using 0.3 s of physical time and was statistically converged. Figures 7 (a) and (b) compare the UQ results obtained on a transverse profile at 0.5C in front of the airfoil for the streamwise mean and rms velocities for the two sets of data respectively. The non-converged mean velocity underpredicts the velocity deficit compared with the converged results. The non-converged velocity deficit at $y = 0$ shows an error of 2% with the converged results. Moreover, the converged results of the mean velocity show smaller uncertainties especially at $y = 0$. The same trend occurs to a lesser extent for the streamwise rms velocity. It can then be inferred that the lack of convergence of the results adds spurious uncertainties to the UQ results. The same conclusion can be drawn from Figures 8 (a) and (b). Both the non-converged stochastic mean and rms of $C_p$ and $P_{rms}$ are only larger than the converged results at the leading-edge of the airfoil where the vortex interaction is strong. Figure 9 shows the comparison of the acoustic results for the converged and the non-converged results. The stochastic mean OASPL and the uncertainty
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Figure 7. UQ results for converged results (solid line, small error bars) and for non-converged results (dashed line, large error bars) for the mean streamwise velocity (a) and for the rms streamwise velocity (b).

Figure 8. UQ results for converged results (solid line, small error bars) and for non-converged results (dashed line, large error bars) for $-C_p$ (a) and for the rms wall-pressure (b).

Figure 9. Comparison of UQ results from converged results (solid line, small error bars) with non-converged results (dashed line, large error bars) for the directivity.

bars overlap fairly well on the range between $50^\circ$ and $110^\circ$. The non-converged OASPL tends to over-predict the converged results by 0.1dB at angles higher than $110^\circ$. The uncertainty bars are also over-predicted in that range.

5.2. Influence of the mesh

The CC points are symmetric with respect to the $x-$axis yielding symmetrical computational domains for positive and negative vertical displacement of the cylinder. However, as an automatic
unstructured meshing software is used, the generated meshes are not strictly symmetric. Solutions obtained from the LES may then not be symmetric and the uncertainty propagation could lead to asymmetric results as seen in Figure 4 where the uncertainty bars are not symmetric with respect to the $y$-axis. The UQ results obtained with the full set of CC points are therefore compared with the results obtained with half of the CC points in the lower side of the $y$-axis. The results for the other half of the discarded CC points are obtained by symmetry. Figures 10 (a) and (b) show that the full and the reduced sets of CC points yield the same stochastic mean streamwise
velocity but slight differences on the rms streamwise velocities at \( y = +0.005 \, \text{m} \). The uncertainty bars remain nearly the same for \( y < 0 \). However, they show smaller amplitudes on both the mean and rms streamwise velocities for \( y > 0 \) to recover the symmetry with respect to the \( y \)-axis. The stochastic \(-C_p\) and \( P_{rms} \) (Figures 11 (a) and (b)) only exhibit a mesh influence at the leading-edge, where strong vortex-airfoil interactions occur. The OASPL for the two UQ studies are shown in Figure 12. Even though the stochastic mean yields slightly lower levels for the reduced set of CC points for the angular positions between \( 65^\circ \) and \( 95^\circ \) and higher levels for angular positions higher than \( 115^\circ \), the differences remain smaller than 0.1 dB. Most of the uncertainty bars have the same amplitude between the full and reduced set of CC points. The latter only yields smaller uncertainty bars at the angular positions below 50\(^\circ\) with 100\% of relative error and higher uncertainty bars at the angular positions after 115\(^\circ\) with 35\% of relative error. The mesh asymmetry then shows a significant influence on both the stochastic mean and rms OASPL results mainly at the the grazing angles.

6. Conclusions

An uncertainty quantification study has been carried out on a rod-airfoil configuration. Uncertainty is introduced in the rod position observed in the experiment. The random variable is the vertical position of the rod and has a uniform distribution. A deterministic compressible LES solver has been chained with a FW-H code to propagate the geometric uncertainty using a stochastic collocation method with Clenshaw-Curtis abscissa. The results show a sensitivity on both the aerodynamic and acoustic fields. Similar results were found at other axial positions in front of the airfoil. Most of the uncertainties obtained on transverse profiles in front of the airfoil are located in the region where \( y < 0.04 \, \text{m} \) for both the mean and rms streamwise velocities. Larger uncertainties are located where both the mean and rms velocities have sharp variations. The results of \(-C_p\) and \( P_{rms} \) show the largest uncertainties in the first 20\% of the chord length. Especially, the maxima of \( P_{rms} \) at 1.5\% of the chord length correspond to the maxima of the rms streamwise velocity in front of the airfoil. Most of the uncertainties obtained for the PSD of the far-field acoustic pressure at different angular positions are located in the broadband part of the spectra. Except at angular positions higher than \( 100^\circ \), the uncertainties remain smaller at the shedding frequency. Consequently, as the tonal peak contributes mostly to the OASPL, uncertainties remain small (0.5 dB) at angular positions below \( 100^\circ \), but become significant (1 dB) for the angular positions above \( 110^\circ \). This result stresses the sensitivity of the OASPL at the grazing angles to the considered input uncertainty. Yet the latter does not fully recover the differences seen with the experimental acoustic data. The remaining differences may be attributed to the flow modeling or additional experimental uncertainties (spurious noise sources from the installation).

The lack of convergence in the LES results is shown to yield some differences in the stochastic results on both the aerodynamic and acoustic fields. The influence of the mesh asymmetry is also assessed by only using half of the CC points for \( y < 0 \), which recovers the symmetry of the aerodynamic stochastic results. In both studies, most of the uncertainty differences are located in the shear layers developing from the cylinder, at the leading-edge of the airfoil and at the grazing angles for the OASPL that are the most sensitive to the present input uncertainlty. These discrepancies stress the importance of the statistical convergence of the results, of the solver and of the mesh qualities to perform a UQ study using LES. Yet their magnitude remain very small in the present study (\( \sim 0.1 \, \text{dB} \)), much smaller than the uncertainty caused by the rod displacement.

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REFERENCES


