Subsonic jet noise simulations using both structured
and unstructured grids

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For the last ten years, large eddy simulations have become a major tool for investigating jet noise sources because of their intrinsic ability to capture broadband turbulent features. However, many challenges still arise when dealing with complex geometries in terms of method accuracy and computational costs. Two different approaches to compute jet noise in an industrial context are here validated and compared. Both approaches are based on a hybrid methodology combining Large Eddy Simulation of jet flows for sources computations and a Ffowcs Williams and Hawkings’ analogy for far field noise prediction but they differ on their grid topologies. The first approach uses classical block structured grids. The numerical scheme is a low dispersive, low dissipative finite volume compact scheme. The second approach uses fully unstructured tetrahedral grids with a low dispersive, low dissipative Taylor-Galerkin finite-element scheme. Both approaches are used to compute a 0.9 Mach, cold jet at moderate Reynolds number $4 \times 10^5$ without accounting for the nozzle geometry. Comparisons between simulations and experimental measurements highlight the need to correctly capture the initial turbulent development of the mixing layer at the nozzle exit. In the present simulations, since the nozzle geometry is not discretized, the turbulent transition is done by injecting perturbations as vortex ring modes. Results obtained

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on this benchmark test case demonstrate the capability of both methods to correctly simulate and predict jet noise. The validation of the approach using fully tetrahedral grids provides a promising way to account for complex noise reduction devices such as chevrons, realistic dual-stream nozzles or lobed mixers.

Nomenclature

Roman symbols

\( c_\infty \) = Ambient sound speed

\( D \) = Jet diameter

\( g = \frac{u_{i}^{n+1}}{u_{i}^{n}} \) numerical factor introduced by the numerical scheme

\( h \) = Mesh size

\( k = \frac{2\pi}{\lambda} \), wave number

\( k' \) = Modified wave number obtained through the numerical scheme

\( L_c \) = Jet potential core length

\( M_\infty = \frac{U_j}{c_\infty} \), jet Mach number

\( r_0 = \frac{D}{2} \), jet radius

\( Re_D = \frac{\rho U_j D}{\mu} \), jet exit Reynolds number

\( Str = \frac{f D}{U_j} \), Strouhal number

\( T_j \) = Jet temperature

\( T^* = \frac{D}{U_j} \), characteristic time scale

\( U_j \) = Jet exit velocity
Greek symbols

\[ \alpha = \text{Amplitude of the vortex-ring perturbation} \]
\[ \delta_\theta = \text{Jet mixing layer momentum thickness} \]
\[ \Delta_0 = \text{Size of the vortex-ring perturbation} \]
\[ \epsilon_i, \phi_i = \text{Amplitude and phase of the } i\text{-th mode of the vortex ring perturbation} \]
\[ \lambda = \text{Wavelength} \]
\[ \mu = \text{Molecular dynamic viscosity} \]
\[ \rho = \text{Density} \]

I. Introduction

During the last ten years, an important effort has been done to achieve realistic unsteady jet simulations in order to better understand the sources of the broadband jet noise. A large agreement is now established on the methodology to numerically predict noise from low and moderate Reynolds number jet noise. Indeed, all authors resort to a hybrid methodology. In a first step, either a Direct Numerical Simulation (DNS) method (for low Reynolds numbers) [1, 2] or a Large Eddy Simulation (LES) method is used to simulate the jet flow and acoustic sources [3–10]. The prediction of the far-field noise is done in a second step using either an acoustic analogy like the Ffowcs Williams & Hawkings’ analogy (FWH) for instance, or some acoustic propagator such as the Linearized Euler Equations (LEE) with data collected in the previous step. However, most of these works do not provide a sufficient description of the shear layer development that can greatly influence the generated noise [11]. Indeed, a good correspondence between some particular mean flow statistics and global acoustics do not insure that acoustic sources are correctly described. Moreover, many authors have highlighted the importance of the nozzle exit conditions on the development of the mixing layer and the transition region [12, 13] of the jet that are known to contain the main sources of the jet noise [14, 15]. It is true that the comparisons with experimental data were generally incomplete because acoustic measurements were not always associated with detailed aerodynamic measurements, especially at the nozzle exit. An important effort has been done to circumvent that in recent published experiments [15–17]. However, it has been very difficult to find a clear under-
standing of the elements to take into account to numerically simulate a correct jet aerodynamics and get a proper acoustics prediction. Bogey et al. [18–22] have highlighted how the nozzle exit conditions, especially the initial thickness of the mixing layer, the initial turbulent level and the Reynolds number, influence the jet development and the noise produced. They showed numerical evidence of noise mainly caused by vortex pairing when initially laminar jets or nominally turbulent jets at moderate Reynolds number are simulated. Therefore, a special care must be taken to avoid this phenomenon and then study the actual turbulent jet noise sources.

This paper brings together two different aerodynamics simulation methodologies. The first methodology uses more classical structured grids. The latter are known to be well suited for simulating shear flows. Since jet noise simulation imposes stringent constraints in terms of precision, dispersion and dissipation [23], the present structured approach uses a recently developed sixth-order compact Finite Volume scheme [24]. This compact scheme is based on an implicit interpolation formula derived in the computational space. This scheme matches with the sixth-order Finite Difference scheme of Lele [25] on uniformly distributed cartesian grids. This scheme presents high precision and low dispersion, low dissipation properties compared to the explicit schemes with the same stencil. Moreover, this scheme derivation accounts for curvilinear meshes in a better way than the standard scheme of Lele [25]. However, structured approaches become limited when dealing with complex geometries. The second methodology, based on unstructured grids, becomes in that case very interesting. In this second approach, fully tetrahedral grids have been used, which significantly differs from what is usually done to simulate jet noise. Indeed, some authors have used unstructured solvers to simulate jet noise but most of them are using hexahedral elements at least in jet shear layers [7–9]. Khalighi et al [26, 27] have been using similar unstructured grid topologies mostly for supersonic jets. This approach is therefore original since a fully tetrahedral grid is used here to capture the mixing layer of a subsonic jet. It has been tested for the first time by Sanjose et al. [28, 29] and showed promising results on configurations with or without nozzle geometry. A two-step Taylor-Galerkin finite-element scheme called TTG4A is used [30]. This scheme is third and fourth order accurate in space and time respectively and shows low dispersion and low dissipation properties. The present study focuses on computations done without considering the
nozzle geometry. Indeed, its main objective is to demonstrate that the numerical tools used in both methodologies allow for the correct simulation of the jet aerodynamics and far-field acoustics. The different advantages and drawbacks of each approach are also discussed.

Section II describes the jet configuration and the experimental data used for comparison. Then the two approaches (structured and unstructured) are detailed in section III. The grids are described in section IV and the inflow conditions in section V. The latter also describes the perturbation injection used to trigger the transition from laminar to turbulent of the initial mixing layer. The acoustic prediction tools are then briefly presented in section VI. The aerodynamic and acoustic results are presented in section VII. Finally, a summarized comparative analysis is done in section VIII.

II. Jet configuration and experimental data

The jet configuration considered for this study is a 0.9 Mach, isothermal jet. This configuration corresponds to the set point SP07 experimentally investigated by Tanna [31]. Both DNS [1, 2] and LES [5–7, 32–34] have been done at different Reynolds numbers for this configuration. It is an excellent benchmark case for jet noise predictions as it has been used in numerous previous numerical studies. Several experimental data are also available although few of them provide a complete aerodynamic and acoustic set of data.

The Reynolds number based on the jet diameter \(D = 2r_0\) is \(4 \times 10^5\). The ambient temperature and pressure values are respectively 300 K and \(1.013 \times 10^5\) Pa. The initial momentum thickness \(\delta_\theta(0)\) is set to 0.03D. None of the presently reported simulations include any nozzle geometry.

The results obtained in these simulations are compared with different experimental data, which could be classified into initially laminar or nominally turbulent jets. To obtain nominally turbulent jets, the nozzle boundary layer must generally be tripped. Some of the experiments correspond to tripped jets [13, 16, 35–38] while other to initially laminar jets [39, 40]. Most of these experiments provide only aerodynamic data [13, 35–40] while few others provide acoustic data [31, 41–43]. Only the study of Fleury et al. [15, 16] provides a complete description (from aerodynamic to acoustic) of an initially turbulent jet flow. Although these experiments generally show smaller initial momentum thickness \((\sim 10^{-3}D)\) and greater Reynolds number than in the presented simulations, they are a
reliable source to evaluate the evolution of the jet half-width or of the momentum thickness, for example.

III. Numerical flow solvers

A. Approach based on structured grids

The first approach is quite common since it is based on the so-called multi-block structured grids for which elements can be indexed using a triplet \((i, j, k)\) in each block. These multi-blocks structured topologies are the most used for jet noise studies [4–7]. Indeed, most of numerical studies of subsonic jet noise has been done using high-order, less dispersive and less dissipative Finite Difference schemes designed for structured grids. Two classes of schemes are generally used. The first class consists in the so-called DRP (Dispersion Relation Preserving) schemes originally developed by Tam and Webb [44] and used in all works of Bogey et al. [32]. The second class refers to compact or Padé schemes [25] used by almost all other authors [5, 6, 28]. However, in the present case, a compact sixth-order accurate Finite Volume formulation is used [24]. This approach has been implemented in the industrial code elsA [45] and has been used to compute turbulent jets with and without nozzle geometry at different Mach numbers [46–48].

1. Numerical scheme

The Finite Volume formulation used is a low-dispersive low-dissipative scheme based on a compact (implicit) interpolation formula approximating interface-averaged values using cell-averaged values. This scheme exactly matches with the Finite Difference sixth-order compact scheme of Lele on regularly spaced grid, and it could reach the fifth-order accuracy on smoothly varying curvilinear meshes [24]. This scheme correctly resolves a wavelength discretized on only 5 points.

To eliminate poorly resolved waves, the mesh is combined with a high-order compact filtering operator that is also used as an implicit subgrid-scale model in the present LES [14, 49].

For time advancement, a DRP six-steps low storage Runge-Kutta scheme [32] is used. Combined with the spatial compact scheme (without considering wall boundary closure), this scheme allows
reaching a CFL of 1.1 while keeping stability and limited dispersion and dissipation effects. In the present LES a CFL of 0.7 is used.

2. Boundary conditions

Boundary conditions are of crucial importance in aeroacoustic simulations since very small reflections from boundaries could overwhelm the computed acoustic radiation. To avoid such problems, non-reflecting boundary conditions are generally combined with sponge zones on outlets that strong vorticity waves could reach. These sponge zones could also be used to maintain a prescribed inlet flow. For this approach, the inlet flow is prescribed using a Tam and Webb radiative condition [44] combined with a sponge zone to prescribed the inlet flow. The outlet at the end of the domain is prescribed using a Navier-Stokes boundary condition of Poinsot and Lele [50] and a sponge zone to dissipate strong vorticity waves. The outlet on the lateral sides is prescribed using a Tam and Webb radiative condition [44]. The reader could find details on implementation and study of the application range of each of these boundary conditions methods in Fosso et al. [51].

B. Approach based on unstructured grids

This second approach is based on an unstructured CFD code called AVBP that is a compressible Navier-Stokes solver primarily developed for LES of reacting and non-reacting flows on unstructured grids (with hexahedral and tetrahedral elements) [52]. AVBP has been dedicated to the study of acoustic instabilities in combustion chamber of aero-engine [53], and can thus handle complex geometries while providing enough accuracy to propagate acoustic waves precisely. Although previous attempts have been done to compute jet noise using unstructured hybrid grids [8, 9, 26], the approach used here is original for jet simulation since a fully tetrahedral mesh is used, which makes the mesh generation easier and more automatic. This methodology was introduced by Sanjose et al. [28, 51] and the results presented here are a continuation of these initial studies.

1. Numerical scheme

A two-step Taylor-Galerkin finite-element scheme called TTG4A is used [30]. This scheme generalizes the Taylor-Galerkin method proposed by Donea [54]. The temporal derivatives that appear
in the Taylor expansion are replaced by spatial derivatives according to the advection equation. The finite-element approximation is obtained using piecewise linear functions. TTG4A is third and fourth order accurate in space and time respectively. An artificial viscosity operator of 2\(^{nd}\) and 4\(^{th}\) order is also applied according to a sensor that is activated in areas of strong gradient non-linearities [55, 56].

A comparative study of dispersion and dissipation properties of the TTG4A scheme and the sixth-order compact scheme (COMP6) combined with the DRP six-step low-storage Runge-Kutta scheme of Bogey and Bailly [32] (RK6 OPT) could be performed using the standard 1D linear convective equation. The gain when computing the next iteration value for a point of the domain is defined as:

\[
g = \frac{u_i^{n+1}}{u_i^n}.
\]

It could be shown that the phase of the gain \(\arg(g)\) defines the dispersion properties of the spatial-temporal scheme while its amplitude \(|g|\) defines its dissipation properties. Naturally, \(g\) depends both on the CFL number and spatial wavenumber, in other words on the number of points per wavelength. Fig. 1 shows the dispersive and dissipation properties of TTG4A for the fixed CFL number of 0.5 compared with those of COMP6+RK6 OPT for the fixed CFL number of 0.7. With 8 points per wavelength, both schemes provide a dispersion relative error less than \(10^{-4}\) and a dissipation error less than \(5 \times 10^{-3}\). These dispersion and dissipation errors are sufficient to obtain relatively low errors on the acoustic sources. Yet as expected, with these 8 points per wavelength, the multi-block structured scheme COMP6+RK6 OPT is less dissipative and less dispersive than the TTG4A scheme.

The subgrid-scale model is the WALE (Wall-Adapting Local Eddy viscosity) model [57]. This model is not a dynamic model but it accounts for the effect of the strain and the rotation on the smallest resolved eddies and has the correct asymptotic behavior at the walls. It is less dissipative than a classical Smagorinsky model and is therefore preferred.
Fig. 1 Dispersion and dissipation properties of the sixth-order compact scheme associated with the six-step Runge-Kutta algorithm of Bogey and Bailly [32] and the TTG4A scheme. \( \phi = \arg(g)/\text{CFL} \) defines the normalized modified wavenumber of the spatio-temporal scheme.

2. Boundary conditions

The outlet boundary conditions are treated by a three-dimensional extension of the classical Local One-Dimensional Inviscid (LODI) Navier-Stokes Characteristic Boundary Conditions (NSCBC) method [50, 58]. The relaxation coefficients have been selected according to the helpful conclusions from Selle et al. [59]. A sponge layer is added to damp the vortices and perturbations to prevent spurious effects on outlet boundary conditions.

IV. Grids

Two grids have been setup one for the multi-block structured solver and another for the unstructured solver. For both grids, the physical domain that needs to be well resolved is defined by \( 0 < x < 20D \) and \( 0 < r < 3D \) near the nozzle exit and \( 0 < r < 5D \) at the end of the physical domain. The other parts of the domain are needed as sponge zones and are adapted to the specific need of each solver.

The multi-block structured grid (STR-OH) has an O-H configuration as shown in Fig. 2(a) to avoid the jet axis singularity that is not handled by the code. The sponge zone of the domain extends to \( 35D \) in the downstream direction. In the radial direction, the domain extends to \( 10D \)
at the domain inlet and to $12D$ at the outlet. The unstructured grid (UNSTR) is fully tetrahedral. The whole computational domain has a cylindrical shape of radius $14D$, and extends to $40D$ in the axial direction. To ensure some symmetry of the node distribution, the mesh is generated on a fourth of the computational domain and duplicated by rotation around the $x$ axis as shown in Fig. 2(b). Both STR-OH and UNSTR grids are designed to have 4 points to define the mixing layer momentum thickness $\delta_0 = 0.03D$. Since 4 points are not sufficient to properly resolve eddies in the mixing layer, the aim is not to fully compute the mixing layer structures, but to capture the large scale effects of the flow gradient in the developing mixing layer on the whole jet dynamics. Wavelengths in the main noise source region (the physical domain up to $x = 10D$) are resolved up to a Strouhal number of 3 if a discretization of 8 points per wavelength is considered. The STR-OH grid uses 128 points in the azimuthal direction. The cell size distribution along the centerline, the lipline and the radial line at $x = 2D$ and $x = 20D$ are shown in Figs 3(a), 3(b), 3(c) and 3(d) respectively. In each figure, both the directional length ($\Delta x$ or $\Delta r$) and the characteristic length ($V^{1/3}$) are presented for the multi-block structured grid. This allows a fair comparison with the fully tetrahedral unstructured grid that uses more isotropic cells. It is also worth noting that the multi-block structured grids have more constraints when dealing with stretching. Indeed, because of the wider scheme stencil, a high stretching ratio could cause important errors. Based on the dual
cells, the TTG4A scheme allows more flexibility in stretching cells. Therefore, some differences in the cell size distribution are caused by these intrinsic constraints: isotropy for the unstructured grid and low stretching ratio for the multi-block structured grid. However, it can be observed that the cell sizes of the UNSTR grid are generally lying between the directional size and the mean sizes of the STR-OH grid. Therefore, fair comparisons between computations on both grids are done in the present work. Finally, STR-OH grid contains 26.2 millions of nodes and 24.5 millions of cells while UNSTR grid contains 18 millions of nodes and 105.6 millions of cells.
V. Inflow forcing

The main jet stream is initialized through a hyperbolic-tangent profile for the axial velocity similarly to previous studies [4, 28, 60]:

\[
u(r) = \frac{U_j}{2} \left[ 1 + \tanh \left( \frac{r - D/2}{2 \delta(0)} \right) \right]
\]  

(2)

where \(U_j = M_{\infty} c_{\infty}\) is the inflow jet velocity. The inlet temperature profile is computed from the Crocco-Buseman relation, assuming constant pressure:

\[
\frac{T}{T_j} = \frac{T_{\infty}}{T_j} + \left( 1 - \frac{T_{\infty}}{T_j} + \frac{\gamma - 1}{2} M_j^2 \left( 1 - \frac{\alpha u(r)}{U_j} \right) \right) \frac{u(r)}{U_j}
\]

(3)

where \(T_j\), and \(T_{\infty}\) are the jet center-line and ambient temperatures respectively, \(M_j\) is the Mach based on the local speed of sound and \(\sigma = \sqrt{\Pr} \) is the recovery factor, a correction factor to take into account non unity Prandtl number [61]. For the present isothermal case, \(T_j = T_{\infty}\) and \(M_j = M_{\infty}\).

For both solvers a random forcing is required to destabilize the initial cylindrical laminar jet. Small random velocity disturbances are added to the instantaneous field in the shear layer of the jet following the vortex ring excitation suggested by Bogey & Bailly [4]. The injected perturbations are vortex ring modes given by the following formula:

\[
\begin{align*}
\{ u'_x \} & = \alpha U_j \frac{2r_0}{r \Delta_0} \exp \left( -\log(2) \frac{(x - x_0)^2 + (r - r_0)^2}{\Delta_0^2} \right) \left\{ \begin{array}{c} r - r_0 \\ x - x_0 \\ \end{array} \right\} \sum_{i=r_0}^{n} \epsilon_i \cos(i\theta + \phi_i),
\end{align*}
\]

(4)

where \(r = \sqrt{y^2 + z^2}\), \(U_j\) is the jet center velocity, \(r_0 = D/2\) is the jet radius, \(x_0\) is the axial position of the perturbation, \(\Delta_0\) is its characteristic size, \(r_0\) and \(n\) are the first and the last azimuthal modes introduced. \(\epsilon_i\) and \(\phi_i\) are respectively the amplitude and the phase of the \(i\)-th mode. They are chosen randomly between -1 and 1 and between 0 and \(2\pi\) respectively. \(\alpha\) is the amplitude of the perturbation.

Only the modes from 4 to 15 are used because the first modes (0 to 3) create an excess noise as shown in previous studies [4, 6]. The amplitude of the perturbation highly depends on the numerical dissipation introduced by the grid and the solver. It is selected by trial and error to initiate the laminar to turbulent transition of the mixing layer without spoiling the acoustic fields. The parameters used in the present study for the two methodologies are given in Tab. 1.
Table 1 Perturbation parameters for the simulations: $\alpha$ and $\Delta_0$ refer to the amplitude and characteristic size of the perturbation respectively.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Mesh</th>
<th>Perturbation</th>
<th>$\alpha$</th>
<th>$\Delta_0$</th>
<th>Modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>STRUCT</td>
<td>STR-OH</td>
<td>YES</td>
<td>2e-3</td>
<td>0.025 $D$</td>
<td>4,...,15</td>
</tr>
<tr>
<td>UNSTRUCT</td>
<td>UNSTR</td>
<td>YES</td>
<td>8e-3</td>
<td>0.025 $D$</td>
<td>4,...,15</td>
</tr>
</tbody>
</table>

VI. Acoustic prediction

The acoustic prediction has been performed using a porous FWH formulation using only the surface integrals terms (formulation 1A of Farassat [62]) and an advanced time approach [63] to predict the far-field sound pressure history from near-field acoustic field. Acoustic surfaces are placed on topological surfaces to prevent any interpolation error [64]. They are in the well resolved zone to insure that waves are still well resolved by the numerical schemes and they are also located to surround all non-linear sources. In the STR-OH grid, the surface is located along a topological surface. The surface begins at the radial position of $3D$ at $x = 0$ and ends at the radial position of $5D$ at $x = 20D$. In the UNSTR grid, the radial position is $1.37D$ for $0 < x < 2.5D$, then it evolves linearly to reach $7.5D$ at $x = 30.5D$, and it stays constant up to the end of the domain. For both computations, the FWH surfaces are topological surfaces, therefore, it was difficult to have exactly the same FWH surface. It is also worth noting that the STRUCT FWH surface is shorter and located further from the jet axis than the UNSTRUCT one. In both grids, surfaces are not closed to avoid parasitic noise coming from the turbulent jet flow crossing the downstream surfaces. Even though the STR-OH FWH surface is further from the axis than the UNSTR surface, the numerical scheme and the mesh distribution allow predicting the same cutoff Strouhal number for the UNSTR grid.

For the structured approach, the FWH code used is embedded in the CFD software to prevent huge data storage. For the unstructured approach, the acoustic far field is computed using an external in-house code. Both FWH solvers are based on the same formulation and use the same time derivatives approximations.
VII. Results

The present work compares two simulations based on the grids and methodology presented in the previous sections. According to Shur et al. [7] and Bodony & Lele [49], they have been run for a sufficiently long time reported in Tab. 2 to converge both the turbulent statistics of the first and second order, and the acoustic field. The results are compared with the experimental data presented in Section II. It is worth recalling that most of these experimental data are from nominally turbulent jets (moderate Reynolds number and high turbulent rate in the initial mixing layer). It has already been stressed that the turbulent development, and by consequence, the acoustic radiation is very different when the jet is nominally laminar or nominally turbulent at the nozzle exit. Jet simulations that do not include nozzle geometry and no inlet perturbation generally reproduce jets that are nominally laminar flow at the nozzle exit (provided no numerical dissipation at the inlet). However the present simulations including a sufficient perturbation show results that are close to data measured in nominally turbulent jets as shown below.

Table 2 Simulation physical time step and time duration given in terms of $T^* = D/U_j$ the characteristic time scale of the flow. Different phases of computations are distinguished: $T_{\text{trans}}$ is the duration of the transient phase, $T_{\text{stats}}$ is the time during which statistics and data for acoustic prediction are recorded. $\Delta t$ is the time step of the LES simulation and $\Delta t_{\text{ac}}$ is the time step of the acoustic prediction.

<table>
<thead>
<tr>
<th>Name</th>
<th>$T_{\text{trans}}$</th>
<th>$T_{\text{stats}}$</th>
<th>$\Delta t$</th>
<th>$\Delta t_{\text{ac}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>STRUCT</td>
<td>146</td>
<td>288</td>
<td>$2.34 \times 10^{-3}$</td>
<td>$8.89 \times 10^{-2}$</td>
</tr>
<tr>
<td>UNSTRUCT</td>
<td>88</td>
<td>490</td>
<td>$6.8 \times 10^{-4}$</td>
<td>$6.8 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

A. Instantaneous snapshots

Figs. 4(a) and 4(b) show instantaneous views of the vorticity magnitude isocontours and the pressure grayscales in the near field for the multi-block structured and the unstructured codes respectively. For the STRUCT simulation, the vorticity contours show that there is an important turbulent production around the position $x = 1D$, where the laminar to turbulent transition goes off. The pressure contours show that the acoustic waves propagate through the whole region properly.
Fig. 4 Instantaneous snapshots: pressure grayscales (fluctuations around $P_0$ between -100Pa and 100Pa), and vorticity magnitude isocontours (between 0 and $13.5D/U_j$). Only a part of the computational domain is shown.

The capability of the numerical scheme and the non-reflecting property of the boundary conditions is therefore clearly highlighted. For the unstructured code, the laminar to turbulent transition of the jet mixing layer is occurring almost at the perturbation point, $0.5D$ from the inlet. This clearly indicates that the perturbation initiates the transition. Furthermore, computations without the perturbation (not shown) show a delayed transition with strong vortex pairing development as in simulations using state-of-art high-order, low dispersive and low dissipative schemes on structured grids [20, 28, 48]. Pressure grayscales show low frequency radiations in downstream direction and high frequency radiations along the mixing layer in the radial direction. Very localized high frequency spurious noise around the perturbation point is noticed propagating in the $60^\circ$ direction and rapidly attenuated. This is most likely caused by the perturbation. Some visualization artefacts could be seen along the line of transition between the refined source region and the coarse sponge region in the UNSTRUCT snapshot. The FWH surface is located 1D before this rapid coarsening. No reflections from boundaries can be observed again highlighting the non-reflecting property of the boundary conditions.
Fig. 5 Evolution of the jet center-line axial mean velocity.

B. Turbulent field statistics

Fig. 5 shows the evolution of the mean axial velocity along the jet center-line. The differences between potential core lengths \(L_c\) could be seen in Fig. 5(a) while the differences in the axial mean velocity decay along the center-line could be observed in Figs. 5(b) for both structured and unstructured computations. The potential core length is about 5.8\(D\) for STRUCT and 6.9\(D\) for UNSTRUCT. Therefore, the STRUCT computation presents slightly shorter potential core as visible in Fig. 4. However, the decay for both simulations is in good agreement with experimental data. Fig. 6(a) shows the jet half-width along the jet axis for both computations. It is in good agreement with experimental data. In the STRUCT simulation, the jet expands slightly more than in experiments after 6\(D\) (the end of the potential core). This is related to the larger structures observed in the instantaneous snapshot in Fig. 4. Such structures are characteristic of an insufficient perturbation leading to a delayed and more intense laminar to turbulent transition. However, a good agreement with experiments is achieved for both simulations in the main noise source region \(0 < x < 5D\).

Fig. 6(b) shows the variation of the momentum thickness \(\delta(x)\) for the first six diameters. Results of the both computations are very close but the predicted momentum thickness is greater than in experiments. This difference is expected since an initially larger momentum thickness of
$\delta_\theta(0) = 0.03D$ has been set. For both simulations, at the perturbation point, the initial momentum thickness ($0.036D$) is slightly higher than the prescribed value because of the perturbation width. In UNSTRUCT, $\delta_\theta(x)$ grows linearly at a slightly weaker rate as in most recent experiments [16]. For STRUCT, there is some slight variations of the $\delta_\theta(x)$ growth rate. Up to $x = 1.5D$, $\delta_\theta(x)$ evolves linearly at the same rate as in the experiments, then it grows slower.

Fig. 7(a) shows the evolution of the root mean square (rms) of the axial velocity along the jet center-line ($r = 0$). The axial position is scaled by the corresponding potential core for each simulation to compare the development of the turbulent intensities with respect to the jet center-line velocity decay. The positions of the turbulent intensity maxima are in good agreement with experimental data. They are found around $1.5L_c$ with $L_c$ the potential core length as observed in experimental data [35].

The evolution of the rms axial velocity along the jet lip-line is shown in Fig. 7(b). In STRUCT, the turbulence levels increase rapidly to reach a peak level of about 20% around $x = D$. After the peak position, the levels decrease rapidly until $x = 3D$ where experimental data values around 16% are reached, then they still decrease but at a lower rate. For UNSTRUCT, the injected perturbation allows the turbulence levels to reach a maximum level of about 15.5% near $x = D$. Then, the turbulent levels decay slowly as in the experiment. The results in the main noise sources
Fig. 7 Evolution of the rms axial velocity.

(up to \(x = 8D\)) are consistent. The presence of the peak in the STRUCT profile is an additional evidence of a delayed and more intense laminar to turbulent transition than in the UNSTRUCT simulation [11, 17, 22].

Fig. 8 presents similarity profiles in the developing mixing layer for mean and rms axial velocity for structured and unstructured computations compared with the experimental data of Fleury et al. [16]. It is worth noting that the half-width and the momentum thickness used to compute the similarity variable \(\eta\) depend on the axial position \(x\). Similarity profiles allow to evaluate the development of the turbulent mixing layer independently of the momentum thickness. Similarity of mean and rms radial profiles for the STRUCT simulation is only achieved in a short axial range from \(x = 2D\) to \(x = 6D\), at the end of the potential core. The profiles at \(x = 1D\) are more typical of laminar inflow profiles. After the potential core, in the transition region, there is no more a similarity law that holds since the mixing has reached the full jet width. For UNSTRUCT, from \(x = 1D\) to \(x = 7D\), the mean and rms flow similarities are in very good agreement with experimental data. After the potential cone at \(x = 8D\) the similarity does not hold anymore.

Tab. 3 summarizes some key values of the mean and rms statistics for the two simulations. In this table, \(\delta_\theta(0)/D\) is the effective momentum thickness at the perturbation point; \(u_{\text{rms},i}/U_j\) is the turbulent level at the perturbation point; \(x_{p,c}\) and \(u_{\text{rms},p,c}/U_j\) are the position and the value of the
Fig. 8 Similarity profiles in the mixing layer.

turbulence peak level on the center-line respectively; \( x_{p,1} \) and \( u_{\text{rms,p,1}}/U_j \) are the position and the value of the turbulence peak level on the lip-line respectively. Maximum turbulent levels on the center-line are found at the same position relatively to the end of the potential core (1.5\(L_c\)). Because of a slightly insufficient perturbation, the STRUCT simulation shows higher peak turbulent level in the mixing layer and then a shorter potential cone. It is worth noting that the effective initial momentum thickness is greater than the prescribed value because of the perturbation width.
Table 3 Mean and rms characteristic values for the different computations and the experiments of Fleury [16].

<table>
<thead>
<tr>
<th>Name</th>
<th>$L_c/D$</th>
<th>$\delta(0)/D$</th>
<th>$u_{rms,c}/U_j$</th>
<th>$x_{p,c}/L_c$</th>
<th>$u_{rms,p,c}/U_j$</th>
<th>$x_{p,1}/D$</th>
<th>$u_{rms,p,1}/U_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fleury et al. [16]</td>
<td>6.9</td>
<td>0.003</td>
<td>-</td>
<td>1.5</td>
<td>0.135</td>
<td>-</td>
<td>0.158</td>
</tr>
<tr>
<td>STRUCT</td>
<td>5.8</td>
<td>0.036</td>
<td>0.19</td>
<td>1.5</td>
<td>0.107</td>
<td>0.80</td>
<td>0.203</td>
</tr>
<tr>
<td>UNSTRUCT</td>
<td>6.9</td>
<td>0.036</td>
<td>0.05</td>
<td>1.5</td>
<td>0.125</td>
<td>1.06</td>
<td>0.155</td>
</tr>
</tbody>
</table>

C. Turbulent spectral contents

In this section, the turbulent spectral content in both jet simulations is analyzed by considering power spectrum density (PSD) of the turbulent axial velocity at different locations in the jet.

Fig. 9 shows the PSD of the turbulent axial velocity at different points on the lip-line ($r = 0.5D$), at a distance of $1D$, $2D$, $3D$ and $5D$ from the perturbation point. All PSD show a broadband behavior for all positions. For positions $x = 3D$ and $x = 5D$, the -5/3 power law, characteristic of the inertial range of turbulence, is very well retrieved from the spectrum maximum located between Strouhal numbers 0.2 and 0.3 to the LES cutoff Strouhal number. For positions closer to the inlet $x = 1D$ and $x = 2D$, the -5/3 power law is not very well achieved since the turbulence is not yet fully developed at these positions. It can be seen that STRUCT presents greater fluctuations levels at $x = 1D$ than UNSTRUCT. Moreover, some peaks could be observed at different Strouhal numbers: 0.26, 0.44, 0.68 and 1.45. These peaks are the traces of the most unstable modes developing in the mixing layer during the laminar to turbulent transition that occurs around $x = 1D$.

Fig. 10 shows the PSD of the axial velocity fluctuations at two locations on the center-line ($r = 0$, $x = 8D$ and $x = 10D$ from the perturbation point). The cutoff Strouhal number is clearly identified at maximum 3 as expected when designing the grid. Therefore, this is the maximal Strouhal number for which acoustic results could be properly resolved since the main acoustic sources are found in the region $0 \leq x \leq 10D$. At these center-line positions, the -5/3 slope is again recovered from $Str = 0.3$ and extends over a limited increasing range as the jet is not fully developed.
Fig. 9 PSD of the axial velocity fluctuation at different points on the lip-line ($r = 0.5D$): $x_0$ is the perturbation location. The vertical line indicates the position of the targeted LES cutoff Strouhal number.

D. Acoustic far field

The power spectral density of the acoustic pressure at 30°, 60° and 90° from the jet axis, 100$D$ from the jet origin, are shown in Figs. 11(a), 11(b) and 11(c). A 30° angle indicates the downstream direction. The estimated cutoff Strouhal number of 3 based on the mesh size around the FWH surfaces is indicated by a vertical line. The lowest resolved Strouhal number is 0.1 for STRUCT simulation and 0.035 for UNSTRUCT because of the different simulation durations reported in Tab. 2.
Both simulations predict the correct shape of the noise spectra recorded in the experiments at low angles as shown in Fig. 11(a), especially the position of the maximum is correctly captured. However, low frequencies are underestimated by up to 2dB for Strouhal numbers less than 0.2 in UNSTRUCT while they are underestimated by up to 4dB for Strouhal numbers less than 0.5 in STRUCT. Similar underestimations at low frequency are found in Bodony & Lele [49] and in Uzun & Hussaini [65].

Indeed FWH surfaces are open and may fail to capture all the low frequencies in the downstream direction. The underestimation is greater for STRUCT because the FWH surface is shorter than in UNSTRUCT. For mid frequencies (0.5 Strouhal number up to 3, STRUCT overestimates acoustic pressure levels by a maximum of 4dB around the Strouhal number of 1.5 while UNSTRUCT simulation is in really good agreement with experimental data. After the Strouhal number of 3, the spectrum decreases rapidly for STRUCT and the offset from experimental data is increasing for UNSTRUCT.

At 60° angle, the underestimation of low frequencies is still present in both simulations. For STRUCT, low frequencies up to the Strouhal number of 0.5 are underestimated and from 0.5 to the Strouhal number of 3, the levels are also overestimated by a maximum of 4dB around the Strouhal
number of 1.5. The 90° angle PSD shows the same behaviour. It is possible that this overestimation of noise in STRUCT results for Strouhal numbers between 3 and 5 is related to the peaks observed in the PSD of the turbulent velocity on the lip line at $x = 1D$ (see Fig. 9(a)). These peaks may correspond to the destabilization modes developed during the initial laminar to turbulent transition. For UNSTRUCT, the low frequencies are similarly underestimated up to the Strouhal number of 0.5, then from 0.5 up to the Strouhal number of 3, the levels are in very good agreement with experimental data. After, the cutoff frequency, a slight bump is seen between Strouhal numbers 3
and 7. This bump is most likely related to the spurious noise from the perturbation that was seen in Fig. 4(b). As observed in Fig. 4(b), this spurious radiates mainly in side directions. Indeed, the 90° angle PSD is quite similar to the 60° one with the same high frequency bump.

The OASPL directivity shown in Fig. 11(d) is computed from the whole spectra obtained from the acoustic solver. Then these spectra have been filtered around the well resolved range of Strouhal: 0.1 to 3 for the STRUCT simulation and 0.05 to 3 for the UNSTRUCT results. The OASPL computed from the filtered pressure is provided in Fig. 12. In Fig. 11(d) both simulations capture the proper directivity, especially the maximum position at low angles. For STRUCT, the overestimation of SPL levels between the Strouhal numbers 0.5 and 3 induces an overestimation of OASPL for angles between 45° and 100°. This overestimation is still present in the OASPL computed from filtered pressure in Fig. 12. In UNSTRUCT, the spurious noise observed in spectra Figs. 11(b) and 11(c), increases the OASPL levels by a maximum of 1dB since when suppressed by the filtering (see Fig. 12), the levels are underestimated by 1dB over the high angle range above 50°. This 1dB underestimate accounts for the low frequency underestimation observed in Figs. 11(b) and 11(c), and the frequencies that were not resolved in the spectra. Therefore the underestimation in the spectra is acceptable as low frequencies are not the major contribution. At low angles, all simulations underestimate OASPL (maximum of 3dB for UNSTRUCT and 4dB for STRUCT) because of the underestimation of low frequencies that are the main contribution in the OASPL in the downstream direction.

**VIII. Comparative analysis**

The results of each methodology presented in the previous sections help assess some advantages and drawbacks that are discussed in the present section.

In terms of grid design, both methodologies present different challenges. For structured grids, the main advantage is the capability to handle shear layers in their preferred direction allowing to better resolve the shear stresses while keeping a reasonable number of points. However, the refinement in the direction orthogonal to the shear layer is consequently propagated up to the end of the domain. Therefore in the transition region, the grid is not isotropic. The lack of isotropy,
especially in the azimuthal direction alters the initial mixing layer development as it has been shown by Bogey et al [21]. For the full tetrahedral unstructured grids, the natural isotropy of elements leads to a rapid increase of the number of cells in shear regions. However, full tetrahedral unstructured grids counterbalance this drawback with the flexibility of the refinement. Moreover, this isotropy allows the turbulent structures to develop in all directions with the same accuracy as stressed by the excellent self-similarity in the shear-layers. This also improves the efficiency of the perturbation, yielding a rapid and smooth transition to turbulence in the UNSTRUCT simulation compared to the STRUCT simulation.

In terms of numerical schemes, the structured methodology has the advantage that high-order, low dispersive and low dissipative methods are easier to derive than in the unstructured methodology. Therefore, more points are generally needed in the unstructured simulation to reach the same accuracy as the structured simulation. Considering this aspect, it is clear that the study of the acoustic near field is more relevant using the structured methodology than using the unstructured one. However, using the sixth-order compact scheme requires to avoid high stretching ratios that would yield important dispersion errors while the present TTG4A scheme handles larger stretching ratio without any problem other than the loss of accuracy in coarse regions and visualization artefacts as shown in Fig. 4(b). No parasitic reflections are however generated in these coarsening zones. In
particular, the important stretching 1D after the FWH surface do not alter or modify the expected cutoff frequency that is still captured in acoustic results. The present TTG4A scheme could also be considered satisfactory since, in the present results, the turbulent flow is clearly better captured in UNSTRUCT than in STRUCT. The difference between both computations could be caused by a different influence of the perturbation method. Indeed, the multi-block structured approach is theoretically less dissipative than the unstructured approach. Therefore, the perturbation might not sufficiently destabilize the STRUCT simulation leading to a delayed then stronger laminar to turbulent transition that alters the turbulent statistics compared to fully turbulent experiments.

One major problem for both methodologies is the tuning of the perturbation. Indeed, both solvers do not behave in a similar way when using the same perturbation parameters. Indeed the numerical error induced by each scheme and the artificial numerical dissipation are clearly not the same and may induce artefacts at the perturbation location. However, the influence of the perturbation on the jet development is evident and has been highlighted in rms profiles on the lipline in Fig. 7(b) and in the turbulent spectra in Fig. 9.

In terms of computational resources, Tab. 4 presents the computational resources used for each simulation reported on the same cluster equipped with dual-processors quad cores Intel Xeon Nehalem 2.66GHz. The total wall-clock time required to simulate the physical time $T_{simu}$ reported in Tab. 2 correspond to $N_{it}$ iterations on $N_n$ cluster nodes.

The time to solution $E_c$ is defined as the wall-clock time on a single cluster node to compute one characteristic time $T^* (T^* = D/U_j)$ for each methodology. The reduced efficiency is defined as the cpu time required by the solver to perform one iteration on a single cell.

The unstructured simulation is 6 times longer than the structured simulation, but each iteration calculation is three times faster than in the structured simulation. The over-cost of the unstructured simulation is related to the larger number of cells and the smaller time step. Taking into account parallel performances of the solvers, the simulation, the response time would be of the same order (2.5 weeks on the above mentioned platform) with both methodologies.
Table 4 CPU performances for each simulation: $N_n$ is the number of nodes, $N_i t$ is the number of iterations, $T_{simu}$ is the simulated physical time, the wall-clock time is the number of hours the simulation has run, $E_c$ is the wall-clock time required to simulate one period $T^*$ of physical time and RE is the reduced efficiency.

<table>
<thead>
<tr>
<th>Name</th>
<th>$N_n$</th>
<th>$N_{it}$</th>
<th>$T_{simu}$</th>
<th>Wall-clock time</th>
<th>$E_c (hrs/T^*)$</th>
<th>RE (s/iter/cell)</th>
</tr>
</thead>
<tbody>
<tr>
<td>STRUCT</td>
<td>8</td>
<td>200000</td>
<td>434 $T^*$</td>
<td>231 hrs</td>
<td>4.26</td>
<td>$2.16 \times 10^{-5}$</td>
</tr>
<tr>
<td>UNSTRUCT</td>
<td>52</td>
<td>850000</td>
<td>578 $T^*$</td>
<td>287 hrs</td>
<td>25.82</td>
<td>$9.58 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

IX. Conclusions and perspectives

Subsonic jet noise simulations using two numerical methodologies, one based on structured multi-block grid and another on unstructured fully tetrahedral grid, have been successfully performed. The capabilities of both methods to accurately predict turbulent and acoustic quantities of a single jet case of Mach number of 0.9 at a moderate Reynolds number of $4 \times 10^5$, have been demonstrated. The latter test case has become the benchmark for jet noise since the original DNS by Freund at low Reynolds number [1]. Both methodologies are using high-order, accurate, low dispersion and low dissipation numerical schemes and non-reflecting boundary conditions. The used grids have been designed to obtain a similar spectral resolution. Both simulations provide a satisfactory description of a jet that presents initial fully turbulent mixing layers, and show a good agreement with corresponding experimental data and published simulations. The results from the unstructured simulation compare particularly well with the most recent and detailed aeroacoustic measurements of Fleury et al. [16], both on the centerline and lip-line for the turbulent field, and in all directions for the far-field acoustic pressure. The remaining discrepancies could be related to well identified issues. In the structured simulation, the insufficient amount of the perturbation is most likely responsible for the stronger laminar to turbulent transition, and the development of additional modes during this transition. By contrast, in the unstructured simulation, the perturbation generates a weak spurious noise only at frequencies higher than the cut-off frequency. Finally the open FWH surfaces yield the slight underestimate of sound pressure levels at low angles and low frequencies. Despite these issues stressing the difficulty and the cost of injecting a proper disturbance, the present results could be considered at the same level as already published works in
the field, demonstrating the capabilities of both industrial solvers.

The comparative analysis of these computations also leads to the following conclusions. First, multi-block structured grids, because of the easiest derivation of high-order, low dispersive and low dissipative schemes are still the best option for direct noise computations on canonical configurations. Indeed, the propagation up to the limits of the near field could be achieved at a reasonable computational cost. Secondly, unstructured grids using non hexahedral elements with surface-based FWH acoustic analogy could be a good alternative for actual industrial applications. Indeed, the mesh flexibility and the properties of the numerical scheme (TTG4A in that case) allow refining the grid only where it is needed without creating important spurious noise in the well resolved wavelength range. Moreover introducing the nozzle and some geometrical tripping that can be easily dealt with an unstructured grid will avoid the difficult perturbation tuning that is needed to trigger turbulence transition. Finally, both methodologies show that they are able to provide reliable simulations of single jet noise in order to better understand jet noise source mechanisms at a reasonable cost, considering the size of the used grids and modern computer resources. This is therefore a promising avenue for more complex geometries that can be more easily handled with a fully unstructured tetrahedral topology. On-going work is already accounting for different nozzle geometries [60, 66]. The next step will be to assess jet noise reduction devices and to tackle actual in-flight turbo-engine configurations.

Acknowledgments

The work was partly supported by NSERC Canada. The authors acknowledge the RQCHP/Compute Canada and Airbus for providing technical support and the necessary computational resources. The authors want to thanks the CERFACS, which develops and provides us with AVBP for academic research. They also thank Nicolas Lamarque for fruitful discussions on AVBP numerical schemes.

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