Determination of inlet impedances for acoustic eigenmode computation in combustion chambers.

Student research project of cand. aer. Michaela Myrczik realized at CERFACS CFD - Combustion under the direction of Thierry Poinsot

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Abstract

The coupling between acoustic waves and flames in industrial combustion systems can lead to high amplitude instabilities, which influence the flame motion and can provoke the destruction of the burner in extreme cases. Numerical simulation enables to predict and understand the acoustic fields of combustion chambers. The accuracy of calculation strongly depends on the quality of boundary conditions, which are generally characterized by their acoustic impedances. Eigenmode calculations of a combustion chamber using a Helmholtz solver (AVSP) have been performed by introducing different boundary conditions at the entrance of the chamber. High drifts of eigenfrequency values have been detected. These wide differences demand for the development of a method to improve the accuracy in the prediction of inlet impedances. Therefore, this research project focused on participating in the development of a new code dedicated to the computation of impedances at the inlet and the outlet of a nozzle. These impedances are computed using linearized Euler equations and form the inputs for the Helmholtz solver. The new code NOZZLE has been validated using the Navier-Stokes solver AVBP, developed by CERFACS.

Résumé

Le couplage entre des ondes acoustiques et des flammes dans un système de combustion industriel peut conduire vers des instabilités très élevées en amplitudes. Non seulement ces oscillations peuvent influencer le mouvement de la flamme, mais aussi endommager des parties du brûleur dans les cas les plus extrêmes. La simulation numérique permet de prévoir et de comprendre les champs acoustiques dans les chambres de combustion. La pertinence de calculs dépend fortement de la qualité des conditions aux limites qui sont en général caractérisées par leurs impédances acoustiques. Des calculs des modes propres dans une chambre de combustion ont été effectués en utilisant un solveur de Helmholtz (AVSP). L’entrée de la chambre a été imposée avec différentes valeurs d’impédance. Des écarts non négligeables en fréquence ont été détectés. Ces différences énormes demandent le développement d’une méthode qui permet d’améliorer l’exactitude dans la détermination des impédances à l’entrée. Donc, ce projet de fin d’études s’est focalisé sur la participation au développement d’un nouveau solveur qui calcule les impédances des tuyères en amont et à l’aval des chambres de combustion. Ces impédances sont déterminées en utilisant les équations d’Euler linearisées et constituent les entrées pour le solveur de Helmholtz. Le nouveau code NOZZLE a été validé en utilisant le code Navier-Stokes AVBP, développé par le CERFACS.
I would like to thank Thierry Poinsot, the scientific advisor of this student research project, for sharing with me his experience and for always providing me with new thought-provoking impulses during my stay at CERFACS. Many thanks also to Nicolas Lamarque, Claude Sensiau, Jacques Lavedine and Gabriel Staffelbach for their help in solving the encountered problems and for providing me with their knowledge. Finally, I would like to thank all members of the CFD team for creating such a comfortable working atmosphere.
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## Glossary

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<th>Description</th>
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<tr>
<td>$A$</td>
<td>$[m^2]$</td>
<td>section</td>
</tr>
<tr>
<td>$A_c$</td>
<td>$[m^2]$</td>
<td>critical section</td>
</tr>
<tr>
<td>$A^+$</td>
<td>[Pa]</td>
<td>acoustic wave propagating downstream the flow</td>
</tr>
<tr>
<td>$A^-$</td>
<td>[Pa]</td>
<td>acoustic wave propagating upstream the flow</td>
</tr>
<tr>
<td>$c$</td>
<td>[m/s]</td>
<td>sound speed</td>
</tr>
<tr>
<td>$c_p$</td>
<td>[J/K/kg]</td>
<td>specific heat capacity for constant pressure</td>
</tr>
<tr>
<td>$c_v$</td>
<td>[J/K/kg]</td>
<td>specific heat capacity for constant volume</td>
</tr>
<tr>
<td>$Im$</td>
<td>-</td>
<td>imaginary part</td>
</tr>
<tr>
<td>$k$</td>
<td>$[m^{-1}]$</td>
<td>wave number</td>
</tr>
<tr>
<td>$K$</td>
<td>[1/s]</td>
<td>relaxation coefficient</td>
</tr>
<tr>
<td>$L_i$</td>
<td>[Pa/s]</td>
<td>characteristic wave amplitudes</td>
</tr>
<tr>
<td>$M$</td>
<td>-</td>
<td>Mach number</td>
</tr>
<tr>
<td>$M_c$</td>
<td>-</td>
<td>critical Mach number ($M = 1$)</td>
</tr>
<tr>
<td>$p$</td>
<td>[Pa]</td>
<td>pressure</td>
</tr>
<tr>
<td>$r$</td>
<td>[J/kg/K]</td>
<td>molar gas constant</td>
</tr>
<tr>
<td>$R$</td>
<td>-</td>
<td>reflection coefficient</td>
</tr>
<tr>
<td>$Re$</td>
<td>-</td>
<td>real part</td>
</tr>
<tr>
<td>$s$</td>
<td>[J/kg/K]</td>
<td>masse entropy</td>
</tr>
<tr>
<td>$t$</td>
<td>[s]</td>
<td>temporal variable</td>
</tr>
<tr>
<td>$T$</td>
<td>[K]</td>
<td>temperature</td>
</tr>
<tr>
<td>$u$</td>
<td>[m/s]</td>
<td>speed along the x-axis</td>
</tr>
<tr>
<td>$x$</td>
<td>[m]</td>
<td>x-coordinate</td>
</tr>
<tr>
<td>$Y$</td>
<td>-</td>
<td>reduced acoustic admittance</td>
</tr>
<tr>
<td>$Z$</td>
<td>-</td>
<td>reduced acoustic impedance</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-</td>
<td>polytropic coefficient</td>
</tr>
<tr>
<td>$\delta$</td>
<td>-</td>
<td>relative volume masse fluctuation</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>-</td>
<td>relative pressure fluctuation</td>
</tr>
<tr>
<td>$\phi$</td>
<td>-</td>
<td>depahasage of a quantity</td>
</tr>
<tr>
<td>$\nu$</td>
<td>-</td>
<td>relative speed fluctuation</td>
</tr>
<tr>
<td>$\rho$</td>
<td>[kg/m$^3$]</td>
<td>volumic masse</td>
</tr>
<tr>
<td>$\omega$</td>
<td>[rad/s]</td>
<td>pulsation</td>
</tr>
</tbody>
</table>
0  mean value
1  fluctuation value
$j$ indices for the discretization along the x-axis
$t$ target value
1. Introduction

CERFACS (European Center of Research and Advanced Training in Scientific Computation)

Created in 1987, CERFACS is one of the most well known research center for scientific calculation of high performance. CERFACS is constituted of five teams in which work around 100 international researchers in the following different domains:

- parallel algorithms
- aerodynamics (CFD)
- combustion (CFD)
- climate and environment
- data assimilation
- electromagnetism

This student research project was executed in the CFD (Computational Fluid Dynamics) group, directed by experts on aerodynamics and combustion. The principal activities of the combustion group are:

- Large Eddy Simulation (LES) of mixtures, combustion instabilities, flame/ wall interaction, in gaseous and dysphasic (gas/liquid) flows.
- Direct numerical simulation (DNS) for the study of flame/turbulence interaction with complex chemistry.

CERFACS has five shareholders: CNES (the French Space Agency), EADS (France, European Aeronautic and Defence Space Company), EDF (Electricité de France), Météo-France (the French meteorological service) and SNECMA (Société Nationale d’Etude et de Construction de Moteurs d’Aviation).

Study context

This work is focused on the combustion instabilities in gas turbines. In such systems the combustion chambers are supplied with air coming from the compressor and entering in a turbine.
The present pollution and efficiency regulations force the development of new and innovative combustion chambers that are usually confronted to unexpected problems. Among these problems, combustion instabilities constitute a major source of difficulty and the reason of failure of several programs. The combustion instabilities are commonly associated to the coupling between the acoustics and the combustion in the chamber [15]. They depend not only on the type of flame, but also on the boundary conditions at the chamber entrance and exit that are generally characterized by their acoustic impedances. These instabilities constitute a radical change of topology for the flame. Figure 1.1 shows a premixed laboratory combustor, where a propane air premix is injected in a chamber via five thin slots. For stable regimes, the flow exhibits a classical structure of reactive jet (Fig. 1.2 left), but for certain regimes, the chamber enters in resonance and the jet produces long eddies (Fig. 1.2 right). The lifetime of the chamber passes from a couple of months to a few minutes, the noise from 80 to 180 dB. No chamber can be used in these conditions. Therefore, new methods are needed to understand, predict and finally control these instabilities in order to save time by avoiding expensive experimental tests.

In this context, CERFACS had developed different calculation codes. Among others AVSP, which allows the determination of the eigenmodes and the acoustic field of the combustion chambers (Benoit [1], [2], C. Martin 2006 [12]). The realization of the eigenmode calculation demands firstly an evaluation of the acoustic characteristics of the flow. Therefore, the compressible and reactive CFD code of CERFACS, AVBP, is used.

Objective of the student research project

A diffuser upstream and a distributor downstream of the chamber enclose the combustion chamber and constitute the so-called boundary conditions (BC) that can be expressed in terms of acoustic impedances as mentioned before. In order to determine them, the strategy of Marble and Candel [11] is used; the diffuser and distributor are replaced by an appropriate boundary condition instead of calculating the entire acoustic field. The AVBP code uses this method. It solves the 3D Navier-Stokes equations for compressible and reactive flows in the spatial and temporal domain. But as the impedance calculation can be delicate and requires a lot of calculation time, a faster and sufficiently exact code is required. Lamarque [8] realized this idea for the distributor of the combustion chamber during a preceding training period. He developed the code NOZZLE that calculates the impedance of nozzles by linearizing the one-dimensional Euler-equations in the frequency domain. The first assumption in this approach is, that all the geometry following the combustion chamber exit can be approximated by a nozzle. This is often true: most chambers end up in DHP (Distributeur Haute Pression), which has the shape of a nozzle and is choked.

This student research project was divided in two sections:

1. **Section 1** of the project consisted in modifying NOZZLE in order to apply it to the diffuser (upstream of the chamber, Fig. 1.3). The impedance values predicted by NOZZLE for the diffusers were then checked by performing a wave calculation in the temporal domain using an LES ¹ code (AVBP). Since the computation of an impedance with AVBP requires

¹Large Eddy Simulation
Figure 1.1.: Experimental burner of Poinsot et al. [16]: premixed propane - air turbulent flames are stabilized in a dump combustor.

\[ \dot{m}_{\text{air}} = 87 \text{ g/s} \Phi = 0.72 \]

\[ \dot{m}_{\text{air}} = 73 \text{ g/s} \Phi = 0.92 \]

Figure 1.2.: Comparison of a stable and an unstable regime in the premixed dump combustor displayed in Fig. 1.1. Schlieren views of the central jet through the quartz window of Fig. 1.1. Flow: right to left (Poinsot et al. [16]).
multiple runs (one forced case for each frequency) and is therefore expensive, another task was performed during this work: a procedure, where white noise is used instead of harmonic forcing was tested. For this procedure, the impedance at all frequencies is obtained in only one computation using a Wiener-Hopf inversion.

- **Section 2** of this student research project was dedicated to the calculation of the acoustic eigenmodes in a SNECMA combustion chamber using AVSP. The study consisted in examining the qualitative influence of the acoustic boundary conditions on the behavior of the flow in the chamber. To do this, the modified NOZZLE code was used to determine the impedances at the entrance of the chamber that constitute the inputs for the AVSP calculation (see Fig.1.3).

*Figure 1.4* gives a schema of the project organization.

Figure 1.3.: General configuration of the combustion chamber with diffuser and distributor

![Diagram of combustion chamber](image)
Navier-Stokes equations

Neglect viscous transfer

Euler equations

Linearization & harmonic analysis ($e^{j\omega t}$)

Helmholtz equation

Neglect only small scales

AVSP

Small Mach numbers in 3D

Fourier Space

NOZZLE

1D, any Mach

AVBP (Large Eddy Simulation) (LES)

Time domain

M $\sim 0$

Gives all modes in a complex combustor. Requires impedances at the boundaries

(Chapter 8)

Gives impedances for AVSP

(Chapter 4/5)

Allows to verify NOZZLE

(Chapter 5)

Pulse at one harmonic frequency or pulse with white noise (Wiener-Hopf)

Figure 1.4.: Project organization
2. Gas dynamics: the steady quasi one-dimensional flow of compressible perfect fluid

2.1. Presentation of the problem

In several flows, density can not be considered to be constant and must be handled as in a compressible flow. This is the case for combustion. But other simplifications can be made.

There exist several different modulations concerning the flow of compressible fluids in diffuser/distributor. In this chapter, the classical model, used to predict the mean flow in a converging/diverging nozzle, is presented (see Candel [4], Chassaing [5], and Thompson [19]). This model is used by NOZZLE.

2.2. Assumptions

Even though the flow in a diffuser/distributor is three dimensional, it can be treated as a quasi one-dimensional one by respecting several hypotheses:

- the quantities describing the flow only depend on the variable x
- permanent regime: $\frac{\partial}{\partial t} = 0$
- perfect fluid and gas: The behavior of real gases for not to high pressures is sufficiently well described by the ideal gas law: $p/\rho = rT$
- sufficiently slow variation of the section $A$ along $x$: $\frac{1}{A} \frac{dA}{dx} \ll 1$
- the direction change of the flow stays small compared to $A$

Thus, the flow is isentropic. Furthermore, the Euler equations, being composed of the three elementary equations of fluid mechanics, continuity, momentum and energy equation, can be written as:
\[ \rho U A = \text{cst} \] \hspace{2cm} (2.1)

\[ u \frac{du}{dx} = -\frac{1}{\rho} \frac{dp}{dx} \] \hspace{2cm} (2.2)

\[ c_p \frac{dT}{dx} = \frac{1}{\rho} \frac{dp}{dx} \] \hspace{2cm} (2.3)

By using in addition the isentropic relation,

\[ \frac{p}{\rho^\gamma} = \text{cst} \] \hspace{2cm} (2.4)

the Hugoniot equations can be found, introducing the logarithmical quantities characterizing the flow:

\[ \frac{dp}{p} = -\gamma \frac{M^2}{M^2 - 1} \frac{dA}{A} \] \hspace{2cm} (2.5)

\[ \frac{dp}{\rho} = -\frac{M^2}{M^2 - 1} \frac{dA}{A} \] \hspace{2cm} (2.6)

\[ \frac{dT}{T} = (1 - \gamma) \frac{M^2}{M^2 - 1} \frac{dA}{A} \] \hspace{2cm} (2.7)

\[ \frac{du}{u} = \frac{1}{M^2 - 1} \frac{dA}{A} \] \hspace{2cm} (2.8)

\[ \frac{dM}{M} = 2 + (\gamma - 1)M^2 \frac{dA}{M^2 - 1} \frac{A}{A} \] \hspace{2cm} (2.9)

The appearing quantity M is the Mach number, which represents the ratio of the flow speed and the sound velocity \( c = \sqrt{\gamma T} \):

\[ M = \frac{u}{c} \] \hspace{2cm} (2.10)
The “total” quantities permit to treat flows in a duct that is connected with a reservoir. They are here indicated with the index $i$ and represent state variables, which arise for flows with velocity $u = 0$. As the flow is isentropic in the duct, these values stay constant and act as references to determine the quantities that describe the flow for all $x$ as a function of the Mach number:

\[
\frac{T_i}{T} = 1 + \frac{\gamma - 1}{2} M^2 \quad (2.11)
\]

\[
\frac{p_i}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^\frac{1}{\gamma - 1} \quad (2.12)
\]

\[
\frac{\rho_i}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^\frac{1}{\gamma + 1} \quad (2.13)
\]

**Critical state variables**

The previous relations require the knowledge of the Mach number $M$ for each $x$ coordinate. In order to determine them, it is necessary to make use of a relation that permits to obtain $M$ starting from the geometry of the duct. The critical section, which is indicated with the index $*$, $A_*$, represents the section where $M = M_* = 1$. Integrating Eq. 2.9 by respecting $M = 1$ at $A_*$ leads to:

\[
\frac{A}{A_*} = \frac{1}{M} \left[\frac{2}{\gamma + 1} \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma + 1}{\gamma + 1}}\right] \quad (2.14)
\]

With Eq. 2.14, it is now possible to identify $M$ for all $x$ by knowing the critical section $A_*$ and the sections $A$ along $x$ (Remark: $x$, $A(x)$ are values that are given by the user in the input file of NOZZLE).

Once the Mach number has been determined, with the use of the equations 2.11, 2.12, 2.13 and of the isentropic law, the characteristics of the flow can be finally calculated. This is the strategy employed by NOZZLE to determine the mean flow.
3. Linear acoustics in a diffuser

3.1. Wave propagation in a constant cross section duct

This part presents the equations to describe the wave propagation in the nozzle, as illustrated in the classical textbooks on acoustics (Kinsler, Frey, Coppens, Sanders [9] or Blackstock [3]). The mean flow is the one described in chapter 2.

3.1.1. Wave equation

The study of the simplest case of plane waves is the best way to introduce the acoustic theory, and it is well known, that it describes adequately most common acoustical phenomena.

To arrive at this simple theory, it is necessary to make some hypotheses:

- perfect gas
- no volume forces
- no heat sources
- effects of viscosity are neglected (fluid is inviscid): In other words, the only significant surface force is the one due to the pressure $p$.
- flow remains isentropic
- linear acoustics: acoustic variables (index "1") are supposed to be small compared to reference quantities (index "0"): $1 \ll 0$; $1 \ll 0$; $1 \ll c_0$
- low-speed mean flow

The equations of acoustics are derived from the conservation equations for fluids, which can be written under the precedent assumptions:

Conservation of mass:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \quad (3.1)$$
Conservation of momentum:

\[ \rho \frac{\partial \vec{u}}{\partial t} + \rho \vec{u} \nabla \vec{u} = -\nabla p \]  \hspace{1cm} (3.2)

As the flow is considered to be lossless, a separate equation for energy is not needed. An equation of state, particularly useful in acoustics, is the one that relates pressure to density and entropy.

Equation of state:

\[ p = p(\rho, s) \]  \hspace{1cm} (3.3)

Under the assumption of an isentropic flow, the pressure is then a function of density alone. For gases the most frequently used isentropic equation of state is the so-called adiabatic gas law:

\[ \frac{p}{p_0} = \left( \frac{\rho}{\rho_0} \right)^\gamma \]  \hspace{1cm} (3.4)

The instantaneous quantities are decomposed into mean flow \((p_0, \vec{u}_0)\) and acoustic perturbation \((p_1, \vec{u}_1)\):

\[ p = p_0 + p_1; \hspace{0.5cm} \vec{u} = \vec{u}_0 + \vec{u}_1; \hspace{0.5cm} \rho = \rho_0 + \rho_1 \]  \hspace{1cm} (3.5)

As all incrementals are considered very small, Eq. 3.1 and 3.2 can be expressed, by conserving only first order terms and introducing \(p\) and \(\vec{u}\), as:

\[ \frac{\partial p_1}{\partial t} + \rho_0 \nabla \vec{u}_1 = 0 \]  \hspace{1cm} (3.6)

\[ \rho_0 \frac{\partial \vec{u}_1}{\partial t} + \nabla \vec{p}_1 = 0 \]  \hspace{1cm} (3.7)

With the isentropic assumption, the sound speed can now be written, linearizing Eq. 3.3:

\[ c_0^2 = \left( \frac{\partial p}{\partial \rho} \right)_{s=s_0} = \frac{p_1}{\rho_1} \]  \hspace{1cm} (3.8)

Finally, the wave equation is obtained, as it shows Poinset [15]:

\[ \nabla^2 p_1 - \frac{1}{c_0^2} \frac{\partial^2 p_1}{\partial t^2} = 0 \]  \hspace{1cm} (3.9)

### 3.1.2. Harmonic plane waves

A wave is called plane wave, if all the acoustic variables are functions of only one spatial coordinate. This is the case for acoustic waves propagating in long ducts. In this one-dimensional situation, traveling along the x-axis, the wave equation leads to:
\[ \frac{\partial^2 p_1}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 p_1}{\partial t^2} = 0 \] (3.10)

\[ p_1(x,t) = A^+ e^{i(kx-\omega t)} + A^- e^{i(-kx-\omega t)} \] (3.11)

\[ u_1(x,t) = \frac{1}{\rho_0 c} \left( A^+ e^{i(kx-\omega t)} - A^- e^{i(-kx-\omega t)} \right) \] (3.12)

Thus, pressure and velocity perturbations can be linked. For the wave \( A^+ \) (\( A^- \)) traveling in the \( x = +\infty \) (resp. \( x = -\infty \)) direction one obtains

\[ p_1 = \rho_0 c_0 u_1 \quad (resp: \quad p_1 = -\rho_0 c_0 u_1) \] (3.13)

The quantity \( \rho_0 c_0 \) is called the **characteristic impedance** of the medium. This notion of impedance allows characterizing wave transmission and reflection at a given section in a duct. At any location \( x \), the acoustic effect of all parts located downstream of this section can be measured by a reduced impedance \( Z \) defined as follows:

\[ Z = \frac{1}{\rho_0 c u_1} \] (3.14)

In some cases, it is useful to observe the inversion of \( Z \), which is called the acoustic admittance \( Y \):
Another quantity is often used in acoustics. It is the reflection coefficient $R$ that represents the ratio of the wave amplitudes at a given section $x = x_0$:

$$
R = \frac{A^+ e^{ikx}}{A^- e^{-ikx}}
$$  \quad (3.16)

With Eq. 3.11 and 3.12 one obtains:

$$
R = \frac{Z + 1}{Z - 1}
$$  \quad (3.17)

Table 3.1.2 [15] resumes the reflection coefficients $R$ and impedances $Z$ of ideal one-dimensional ducts.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Boundary condition</th>
<th>Reflection coefficient $R$</th>
<th>Impedance $Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1): Infinite duct on the right side</td>
<td>Non reflecting</td>
<td>$\infty$</td>
<td>1</td>
</tr>
<tr>
<td>(2): Infinite duct on the left side</td>
<td>Non reflecting</td>
<td>0</td>
<td>$-1$</td>
</tr>
<tr>
<td>(3): Duct terminating in large vessel</td>
<td>$p_1 = 0$</td>
<td>$-1$</td>
<td>0</td>
</tr>
<tr>
<td>(4): Duct terminating on a rigid wall</td>
<td>$u_1 = 0$</td>
<td>1</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

Table 3.1.: Reflection coefficients $R$ and impedances $Z$ of ideal one-dimensional ducts.
3.2. Wave propagation in a variable cross section duct

The previous analysis can be extended to longitudinal waves propagating in a duct of variable section \( A(x) \). This is the model implemented in NOZZLE. This model had been first studied by Tsien [20], who was interested in the transfer functions of rocket nozzles. A compressible flow in a duct with variable section \( A \) along the \( x \) axis is considered. In this case, Eq. 3.1 and 3.2 are integrated on the volume \( dV \). But as we consider only variations of quantities along \( x \), the resulting Euler equations are (Poinsot & Veyante 2005 [15]):

\[
\frac{\partial (\rho A)}{\partial t} + \frac{\partial (\rho u A)}{\partial x} = 0 \tag{3.18}
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0 \tag{3.19}
\]

\[
\left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} \right) s = 0 \tag{3.20}
\]

It is interesting to see here the appearance of \( A(x) \), that marks a difference to the model of a constant cross section duct.

In the equations 3.18, 3.19 and 3.20, \( p, u \) and \( \rho \) are mean values and can be decomposed in two parts (one stationary, the other one fluctuating), as in section 3.1.1:

\[
p(x, t) = p_0(x) + p_1(x, t); \quad u(x, t) = u_0(x) + u_1(x, t) \tag{3.21}
\]

\[
\rho(x, t) = \rho_0(x) + \rho_1(x, t); \quad s(x, t) = s_0(x) + s_1(x, t)
\]

Linearizing the system of Euler equations 3.18 to 3.20 by only retaining the first order terms and assuming isentropic flow, leads to:

\[
\frac{\partial}{\partial t} \left( \frac{\rho_1}{\rho_0} \right) + u_0 \frac{\partial}{\partial x} \left( \frac{\rho_1}{\rho_0} + \frac{u_1}{u_0} \right) = 0 \tag{3.22}
\]

\[
\frac{\partial}{\partial t} \left( \frac{u_1}{u_0} \right) + u_0 \frac{\partial}{\partial x} \left( \frac{u_1}{u_0} \right) + \left( \frac{\rho_1}{\rho_0} + 2 \frac{u_1}{u_0} \right) \frac{du_0}{dx} = \frac{p_1}{\rho_0} \frac{du_0}{dx} - \frac{p_0}{\rho_0 u_0} \frac{\partial}{\partial x} \left( \frac{p_1}{\rho_0} \right) \tag{3.23}
\]
One notices that the section $A(x)$ has disappeared from the equations. But it still arises in the calculation of the mean flow.

As one is only interested in the acoustic waves, and not in those ones of entropy and vortices, entropy fluctuations are neglected and a simple relation between $p_1$ and $p_0$ is obtained for all $x$:

$$\frac{p_1}{\gamma p_0} - \frac{p_1}{\rho_0} = \frac{s_1}{c_p} \left( t - \int_0^x \frac{d\zeta}{u} \right) = 0$$  \hspace{1cm} (3.25)

Under the assumption that all fluctuations are monochromatic, one obtains:

$$\frac{p_1}{p_0} = \varphi(x)e^{-i\omega t}; \quad \frac{u_1}{u_0} = \nu(x)e^{-i\omega t}; \quad \frac{\rho_1}{\rho_0} = \delta(x)e^{-i\omega t}$$  \hspace{1cm} (3.26)

Finally, using Eq. 3.25 and introducing the Mach number of the mean flow, the linearized one-dimensional isentropic equation system for pressure $\varphi$ and velocity $\nu$ fluctuations can be written as:

$$-i\omega \varphi + u_0 \frac{d\varphi}{dx} + \gamma u_0 \frac{d\nu}{dx} = 0$$  \hspace{1cm} (3.27)

$$\left( \frac{1}{M^2} - 1 \right) u_0 \frac{d\varphi}{dx} - \left[ (\gamma - 1) \frac{du_0}{dx} - i\omega \right] \varphi + \gamma \left[ \frac{2du_0}{dx} - i\omega \right] \nu = 0$$  \hspace{1cm} (3.28)

**Impedance calculation**

The impedance at the entrance (in) of a distributor or at the exit (out) of a diffuser can now be calculated by integrating the equation system Eq. 3.27 and 3.28. The definition of the reduced acoustic impedance for longitudinal modes is therefore:

$$Z_{\text{in/out}} = \left( \frac{1}{p_0c} \frac{p_1}{u_1} \right)_{\text{in/out}}$$  \hspace{1cm} (3.29)

Or, including the relative fluctuations:

$$Z_{\text{in/out}} = \frac{1}{\gamma M} \left( \frac{\varphi}{\nu} \right)_{\text{in/out}}$$  \hspace{1cm} (3.30)
The elaboration of this process will be given in the next chapter, which gives a short overview of the function of NOZZLE and its modifications opposite to the precedent version.
4. NOZZLE: a solver for acoustics in nozzles

4.1. Motivation

Nozzle allows the determination of the complex impedance in the Fourier domain for compressible flows in geometries, where the quasi 1D hypothesis can be applied. The diffuser or distributor are of those configuration.

4.2. Impedance of distributors

NOZZLE offers the user the impedance $Z$, the admittance $Y$ and the reflection coefficient $R$ at the inlet of a distributor as a function of frequency and gives the mean flow characteristics over the flow direction $x$. Inputs are:

- The geometry: the section $A$ as a function of $x_j$.

- The boundary characteristics: the user can choose between entering the flow rate, temperature and pressure or the Mach number, pressure and temperature at the entrance of the distributor. In addition, the code needs to be supported with the flow's thermodynamical properties (molar gas constant $r$, polytropic coefficient $\gamma$) and the range of frequency, where the impedance calculation should be performed.

4.2.1. Mean flow calculation

In a first time, the mean flow is calculated. As the Mach number at the section $A$ at the entrance is known, the critical section $A_*$ can be obtained by using Eq. 2.14. Once this had been done, the Mach number is determined with the same equation at all discretization points $x_j$. This implicit equation is resolved using either a Newton-Raphson method or a hybrid method. For more explication see Numerical Recipes [21]. The “total” quantities can then be calculated (see Eq. 2.11 to 2.13) and finally the mean flow. For the following step, the nature of the flow at the throat must be known (subsonic or supersonic). Therefore, the critical section $A_*$ is compared to the smallest one.
4.2.2. Resolution of the linearized equations for distributor impedance “seen” by the distributor inlet

The mean flow values for each \( x_j \) can now be entered into the fluctuation system, affiliated in chapter 3, Eq. 3.27 and 3.28. It can be written in discretized form as:

\[
-i \omega \varphi_j + (u_0)_j \left( \frac{d \varphi}{dx} \right)_j + \gamma (u_0)_j \left( \frac{d \nu}{dx} \right)_j = 0 \quad (4.1)
\]

\[
\left( \frac{1}{M_j^2} - 1 \right) (u_0)_j \left( \frac{d \varphi}{dx} \right)_j - \left[ (\gamma - 1) \left( \frac{du_0}{dx} \right)_j - i \omega \right] \varphi_j + \gamma \left[ 2 \left( \frac{du_0}{dx} \right)_j - i \omega \right] (\nu)_j = 0 \quad (4.2)
\]

In the case of a distributor, the discretization direction is opposite to the x axis (see Fig. 4.1); that means from the smallest section (for a supersonic flow) or rather from the exit (for subsonic flow) to the entrance.

![Figure 4.1.](image)

Figure 4.1.: The direction of discretization for the distributor/diffuser

The velocity gradient is discretized using a centered scheme of second order:

\[
\left( \frac{du_0}{dx} \right)_j = \frac{u_{j-1} - u_{j+1}}{2\Delta x} \quad (4.3)
\]

Note that \( j-1 \) comes before \( j+1 \). The gradient of the fluctuation quantities \( \varphi \) and \( \nu \) is discretized after an upwind scheme of first order; i.e. for \( \varphi \):

\[
\left( \frac{d \varphi}{dx} \right)_j = \frac{\varphi_{j-1} - \varphi_j}{\Delta x} \quad (4.4)
\]
Finally, the system’s matrix is obtained after differentiation:

\[
\begin{pmatrix}
\left(1 - \frac{1}{M_*^2}\right) \frac{u_j}{\Delta x} + i\omega + (1 - \gamma) \left( \frac{du}{dx} \right)_j \gamma \left(2 \left( \frac{du}{dx} \right)_j - i\omega \right) \\
-i\omega - \frac{u_j}{\Delta x} \\
\end{pmatrix}
\begin{pmatrix}
\varphi_j \\
\nu_j
\end{pmatrix} =
\begin{pmatrix}
\left(1 - \frac{1}{M_*^2}\right) \frac{u_j}{\Delta x} \varphi_{j-1} \\
-\frac{u_j}{\Delta x} \nu_{j-1} - \frac{\gamma u_j}{\Delta x} \nu_{j-1}
\end{pmatrix}
\]  

or:

\[
A_j \begin{pmatrix}
\varphi_j \\
\nu_j
\end{pmatrix} = \begin{pmatrix}
f(\varphi_{j-1}) \\
g(\nu_{j-1})
\end{pmatrix}
\]  

(4.5)

This system can be solved if the boundary solution for \( \varphi \) and \( \nu \) is known. For a distributor calculation, the following cases are distinguished:

**Supersonic flow**

If the flow reaches Mach = 1 at the throat, there is no need to know what happens downstream of this minimum section, as no acoustic wave is able to wander upstream (the mean flow is faster than the sound speed). Therefore, the calculation starts from the throat, where the boundary condition is obtained by using Eq. 4.1 and knowing that \( M_* = 1 \). The impedance at the throat is:

\[
Z_* = \frac{\gamma \frac{du}{dx}|_* - i\omega}{(\gamma - 1) \frac{du}{dx}|_* - i\omega}
\]

(4.7)

Combining Eq. 3.27 and 3.28, \( \varphi \) and \( \nu \) can easily be solved between the throat and the outlet (Fig. 4.3, left).

**Subsonic flow**

In this case the integration concerns the whole configuration (indeed there is an acoustic wave that goes upstream). Assuming disemboguing into the atmosphere \( (p_1 = 0) \), the impedance at the exit is zero and the initial values for \( \varphi \) and \( \nu \) determined by Eq. 3.30.
4.3. Modifications of NOZZLE

One part of this student research project was devoted to adapt NOZZLE to the calculation of diffuser configurations. The following modifications needed to be undertaken as the configuration changed (see Fig. 4.3):

- The initial values are entered from the exit, because mostly, the initial values are known directly at the entrance of the combustion chamber. The integration of the mean flow has therefore its direction from the exit to the inlet.

- One is interested now in the impedances at the exit of the diffuser. Thus, it is assumed that the impedance is known at the entrance to the domain. There are three possibilities available to describe the inlet:

  1. The entrance is assumed to disembogue into the atmosphere \((p_1 = 0); Z_0 = 0\)
  2. The entrance represents a wall \((u_1 = 0); Z_0 = \infty\)
  3. The entrance is a partially non-reflecting boundary. \(Z\) is a function of frequency and of a coefficient \(K\) that describes the degree of reflection (see annex B for more details).
The integration begins at the entrance with direction to the combustion chamber (Fig. 4.1). With this orientation of the discretization direction, the velocity and pressure gradients are written as follows:

\[
\left( \frac{du}{dx} \right)_j = \frac{u_{j+1} - u_{j-1}}{2\Delta x}; \quad \left( \frac{dp}{dx} \right)_j = \frac{\varphi_j - \varphi_{j-1}}{\Delta x}
\]

(4.8)

And this changes the equation system 8.3 to:

\[
\begin{bmatrix}
\left( \frac{1}{M_j^2} - 1 \right) \frac{u_j}{\Delta x} + i\omega + (1 - \gamma) \left( \frac{du}{dx} \right)_j \\
-i\omega + \frac{u_j}{\Delta x} \\
\end{bmatrix}
\begin{bmatrix}
\varphi_j \\
\nu_j \\
\end{bmatrix}
= \begin{bmatrix}
\left( \frac{1}{M_j^2} - 1 \right) \frac{u_j}{\Delta x} \varphi_{j-1} \\
\varphi_{j-1} + \frac{\gamma u_j}{\Delta x} \nu_{j-1} \\
\end{bmatrix} (4.9)
\]

Only subsonic flows are of interest for the reasons explained in section 4.2.

- During the validation procedure of NOZZLE (presented in the following chapter), it was observed, that for the diffuser configuration, the existing model was not appropriate for a tube with zero velocity flow. For a distributor, this test case could have been performed by replacing \( M = 0 \) with a very small Mach number, and NOZZLE has delivered good results. But for the diffuser, the real part of the admittance showed non negligible discrepancies (see Fig. 5.4 on page 28). Thus, for those cases, an other model was implemented in the code:

The hypotheses are the same as of the precedent model, but with addition that \( u_0 = 0 \) and no variation of the reference quantities along the x-axis (index “0”; for example \( \frac{\partial \rho_0}{\partial x} = 0 \)). Therefore, the Euler-equations 3.18 and 3.19 can be written (after decomposing the mean values into stationary and fluctuating part) to:

\[
\frac{\partial p_1}{\partial t} + \rho_0 \frac{\partial u_1}{\partial x} = 0
\]

(4.10)

\[
\rho_0 \frac{\partial u_1}{\partial t} + \frac{\partial p_1}{\partial x} = 0
\]

(4.11)

An adimensional expression is obtained by introducing the two variables \( \varphi \) and \( \nu \):

\[
\frac{\rho_1}{\rho_0} = \varphi(x)e^{-i\omega t}; \quad \frac{u_1}{c_0} = \nu(x)e^{-i\omega t}
\]

(4.12)
With the adiabatic gas law 3.4, the linearized one-dimensional isentropic equation system, with zero velocity, for pressure $\varphi$ and velocity $\nu$ fluctuations is obtained:

\[
\frac{-i\varphi}{\gamma} + c_0 \frac{d\nu}{dx} = 0 \quad (4.13)
\]

\[
p_0 \frac{d\varphi}{dx} - i\rho_0 c_0 \omega \nu = 0 \quad (4.14)
\]

With the corresponding discretization scheme and directions (distributor: see Eq. 4.4; diffuser: Eq. 4.8 right), the two matrix systems for the distributor and diffuser are:

**Distributor:**

\[
\begin{bmatrix}
-\frac{i\omega}{\gamma} & -\frac{c_0}{\Delta x} \\
-\frac{p_0}{\Delta x} & -i\omega\rho_0 c_0
\end{bmatrix}
\begin{bmatrix}
\varphi_j \\
\nu_j
\end{bmatrix}
= \begin{bmatrix}
-\frac{c_0}{\Delta x} \nu_{j-1} \\
-\frac{p_0}{\Delta x} \varphi_{j-1}
\end{bmatrix}
\quad (4.15)
\]

**Diffuser:**

\[
\begin{bmatrix}
-\frac{i\omega}{\gamma} & \frac{c_0}{\Delta x} \\
\frac{p_0}{\Delta x} & -i\omega\rho_0 c_0
\end{bmatrix}
\begin{bmatrix}
\varphi_j \\
\nu_j
\end{bmatrix}
= \begin{bmatrix}
\frac{c_0}{\Delta x} \nu_{j-1} \\
\frac{p_0}{\Delta x} \varphi_{j-1}
\end{bmatrix}
\quad (4.16)
\]

The impedance is finally determined as for the precedent model with Eq. 3.29. With the relative fluctuation, one obtains:

\[
Z_{\text{in/out}} = \frac{1}{\gamma} \left( \frac{\varphi}{\nu} \right)_{\text{in/out}}
\quad (4.17)
\]

In annex C, Fig. C.2 and C.1 show the validation of this model.
5. Validation

Tests of the code described in chapter 4 were performed in a constant cross section duct and in a diffuser. The validation of the modified NOZZLE code was executed by comparing the numerical solutions to analytical ones if they existed (for example in the constant cross section duct), otherwise to numerical solutions obtained by AVBP (Table 5.1). Some tests compare also AVBP and analytical solutions in order to verify that AVBP can be used as a validation tool. As described in chapter 4, there are three different boundary conditions available for the entrance of the diffuser in the NOZZLE code: $p_1 = 0$, partially reflecting boundary and $u_1 = 0$. All of them have been tested and validated. Only the validation for the last case is presented here.

<table>
<thead>
<tr>
<th>Section</th>
<th>Analytic solution</th>
<th>NOZZLE</th>
<th>AVBP harmonic</th>
<th>AVBP wiener-hopf</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant cross section duct</td>
<td>Sec. 5.2</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>diffuser A</td>
<td>Sec. 5.3</td>
<td>-</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

Table 5.1.: Different cases of validation studies

5.1. Application of the LES code AVBP to calculate the admittances

5.1.1. The code AVBP

The response of a diffuser to acoustic waves can be computed using NOZZLE in the Fourier space. It can also be calculated in the time domain, for example using a LES\textsuperscript{1} code as done here with AVBP. AVBP is a parallel CFD code that solves the three-dimensional compressible Navier-Stokes equations for laminar or turbulent, inviscid or viscous, steady or unsteady, reactive flows, multiple species on unstructured and hybrid grids for LES and RANS\textsuperscript{2}. (www.cerfacs.fr/LES.php)

The following numerical schemes are employed by AVBP:

- LW: Lax-Wendroff scheme, that uses a centered finite-volume cell-vertex space discretization (the unknown variables are stocked on the nodes); second order in space and time.

\textsuperscript{1}Large Eddy Simulation
\textsuperscript{2}Reynolds-Averaged Navier-Stokes equations
- TTGC: Taylor-Galerkin scheme, that is of a finite-element type of third order in space and time. It is a low dissipation scheme but needs a calculation time which is 2.5 times longer than LW.

Calculations during this project were performed using the Lax-Wendroff scheme.

The boundary conditions are set following the NSCBC model (see also annex B) presented by Poinset and Lele [14]. They permit to control the wave reflection at the limit of the calculation domain.

**AVBP_harmonic: Measurement of impedance using harmonic waves**

For each frequency, a single-frequency harmonic wave $A_{imp}^-$ is injected at the exit of the diffuser in the domain and the returning one $A^+$ is measured (see Fig. 5.1). With the tool *xwave_ms*, especially developed for AVBP, the waves are isolated. *Xwave_ms* works as follows:

The time variations of the local pressure $p$ and velocity $u$ at the exit are used to evaluate the wave leaving the diffuser:

$$A^+ = \frac{\partial p}{\partial t} + \rho c \frac{\partial u}{\partial t}$$  \hspace{1cm} (5.1)

While the wave entering the diffuser is obtained by:

$$A^- = \frac{\partial p}{\partial t} - \rho c \frac{\partial u}{\partial t}$$  \hspace{1cm} (5.2)

$A^-$ must match the wave imposed at the outlet ($A^- = A_{imp}^-$).

By using a Fourier transformation of the solution created by AVBP, this post-treatment tool gives the module and the phase of the reflection coefficient $R = A^+/A^-$ and the admittance $Y$ (and accordingly of the impedance $Z$) as a function of frequency.

This method gives exact results, as it will be shown later, but requires a calculation for each frequency and thus a significant CPU time.

**AVBP_wiener-hopf: Measurement of impedance using white noise and a Wiener-Hopf approximation**

A modification was introduced in the boundary condition file "OUTLET_RELAX_P_PULSE.F" (see annex B) of AVBP leading to the new file "OUTLET_RELAX_P_NOISE.F". The application of the post-treatment tool named "WIENER" allows the user to receive (in this case) the modules and phases of impedance of all frequencies with just one calculation. This tool calculates the transfer function of any "black box" by pulsating it with white noise. In the present case, the
Wiener-Hopf transformation replaces the Fourier transformation of AVBP_harmonic and is applied to $A^+$ and $A^-$. The inlet and response signal (here: velocity and pressure) are the only values to be specified. The impedance $Z$ and phase are calculated using the Wiener-Hopf relation and inversion:

\[
\begin{align*}
\gamma \ast H &= C \\
H &= INV(\gamma) \ast C \\
F &= inv - xtransf(H) \\
Z &= ABS(F)(w) \\
\text{phase} &= ANGLE(F)(w)
\end{align*}
\]

\[
\begin{align*}
\gamma & \quad \text{autocorrelation matrix of the inlet signal } u_1 \\
H & \quad \text{filter} \\
C & \quad \text{cross-correlation vector between inlet } (u_1) \text{ and outlet } (p_1/(\rho c)) \\
F(w) & \quad \text{transfer function}
\end{align*}
\]

This method is fast but not as precise as AVBP_harmonic.

\section*{5.2. Test case: Constant cross section duct}

\subsection*{5.2.1. Introduction}

In a first step, the case of a constant cross section duct has been studied. Its analytical solution is well known: this allows verifying that the solution obtained by AVBP is congruent with the
analytical solution. This was necessary to be sure, that AVBP represents an adequate validation tool to validate the NOZZLE code for a more complex geometry, such as a diffuser.

The tube and its imposed boundary conditions are shown in Fig. 5.2.

\[
R(0) = 1 + A_+ (x) - A_-(x)
\]

\[
u_1 = 0
\]

\[
A_+(L) = A_-(L)
\]

\[
A^-(L) = i p_0
\] (5.3)

In the absence of reflection at \( x = 0 \), this wave would induce a pressure fluctuation at \( x = L \) equal to \( p_0 \sin(\omega t) \). When the \( x = 0 \) boundary is reflecting, the pressure at the outlet will be composed of both ingoing and outgoing waves. The \texttt{xwave.ms} tool allows to measure both waves.

**Entrance** In this validation case, the entrance of the diffuser \((x=0)\) is an acoustically closed section. That means, that the acoustic velocity goes to zero and the wave is totally reflected \((R=1)\) while the local impedance \( Z \) is infinite in this section, so that \( u_1 = 0 \), that leads to:

\[
A^+ = A^-
\] (5.4)

**AVBP** The mesh of the constant cross section duct was generated by the HIP mesh manipulation tool, developed by CERFACS, which permits to create simple rectangular and structured meshes. Table 5.2 gives the characteristics and the boundary conditions of the duct, which are also the same for NOZZLE.

**NOZZLE** In this case, a function with a constant section had been directly implemented in the code.
<table>
<thead>
<tr>
<th>Geometry</th>
<th>Value</th>
<th>Boundary conditions</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_x$ (m)</td>
<td>0,2</td>
<td>$u_{in}^1$ (m/s)</td>
<td>0</td>
</tr>
<tr>
<td>$L_y$ (m)</td>
<td>0,001</td>
<td>$p_{in}$ (bar)</td>
<td>1,01325</td>
</tr>
<tr>
<td>$n_x$</td>
<td>401</td>
<td>$\rho_{in}$ ($\frac{kg}{m^3}$)</td>
<td>1,258</td>
</tr>
<tr>
<td>$n_y$</td>
<td>2</td>
<td>$T_{in}$ (K)</td>
<td>300,0</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1,39</td>
<td>$\rho$ ($J.kg/K$)</td>
<td>268,4</td>
</tr>
</tbody>
</table>

Table 5.2.: Characteristics of the duct and boundary conditions

### 5.2.2. Analytical solution of the impedance $Z$/ admittance $Y$

The general solution of this problem is ($k = \frac{2\pi f}{c}$):

$$p_1 = A^+ e^{i(kx-\omega t)} + A^- e^{i(-kx-\omega t)}$$
$$u_1 = \frac{1}{pc} \left( A^+ e^{i(kx-\omega t)} - A^- e^{i(-kx-\omega t)} \right)$$

(5.5)

Since the amplitudes of the waves are constant in ducts:

$$A^+(x) = A^+$$
$$A^-(x) = A^-$$

(5.6)

At $x = 0$, $u_1$ is zero so that $A^+ = A^-$. Therefore, Eq. 5.5 can be written as

$$p_1 = A^+ e^{-i\omega t}(e^{ikx} + e^{-ikx})$$
$$u_1 = \frac{A^+}{pc} e^{-i\omega t}(e^{ikx} - e^{-ikx})$$

(5.7)

So that the impedance $Z(x) = \frac{1}{pc} p_1/u_1$ becomes

$$Z = \frac{e^{ikx} + e^{-ikx}}{e^{ikx} - e^{-ikx}}$$

(5.8)

At the outlet ($x = L$), the impedance $Z(L)$ is therefore

$$Z = 1 + \frac{e^{-2ikL}}{1 - e^{-2ikL}}$$

(5.9)

As mentioned before, the impedance at the entrance is infinite. Thus, it is more adequate to observe the admittance (recall: $Y = 1/Z$) in the future. Real and imaginary parts of the admittance are:

$$Re(Y) = 0 \quad Im(Y) = \frac{isin2kL}{1+cos2kL}$$

(5.10)

And this is what NOZZLE an AVBP should find.

\footnote{For the Nozzle calculation in this chapter, it is used $M = 0.001$ as NOZZLE is only validate for non zero Mach numbers.}
5.2.3. Comparison of AVBP harmonic and analytical solution

First of all, an admittance calculation at the exit for different frequencies in the range of 250 and 3000 Hz was done using AVBP and xwave_ms. These results were compared to the analytical values. As Fig. 5.3 shows, the results of AVBP harmonic correspond sufficiently well to the analytical ones for the imaginary part of the admittance. The real part of the analytical solution is zero over the whole field, while the AVBP result differs and can reach values close to 1.5. This happens at \( f \approx 420\) Hz, which corresponds to the quarter wave made of this duct \( (\lambda = 4L, \ f = \frac{c}{\lambda}) \).

At this frequency, the forcing by the outlet is contaminated by the eigenmode of the duct leading to the discrepancy observed in Fig. 5.3.

![Graph showing comparison between AVBP and analytical solution](image)

Figure 5.3.: Comparison between AVBP and the analytical solution (real and imaginary part of admittance)

5.2.4. Comparison of NOZZLE and analytical solution

A calculation with NOZZLE was executed. Fig. 5.4 shows, that for a second time, the imaginary results correspond to the analytical solution. As for the real part, the appearance of peaks can be detected. This can be explained as follows:

The analytical solution is based on the fact that \( M = 0 \) (see table 5.2). But this validation was done with a small Mach number of 0.001, ( NOZZLE is only validate for non zero Mach numbers as Eq. 3.28 would get singular). This approximation bares certain terms in the solved equation system from simplifying themselves and thus, they arise to important values for certain frequencies.

5.2.5. Comparison of AVBP wiener-hopf and analytical solution

Figure 5.5 on page 28 shows the results by comparing the analytical solution to that one obtained by AVBP wiener-hopf.

It can be observed, that for lower frequencies, it finds the peaks of the imaginary admittance at the
same positions, but for higher ones a displacement can be detected. This imprecision in position of the peaks and in offset of values can be attributed to the input requirements of the post-treatment tool "WIENER":

1. The size of the filter is an important parameter for the inversion. It should not be too small in order to trigger large delay times, nor too high in order to minimize the size of the gamma matrix

\[(L_{\text{max}} - L_{\text{min}})^2\]

2. Frequencies are between \(\frac{0.75}{(L_{\text{max}} - L_{\text{min}})dt}\) and \(\frac{1}{4dt}\). Therefore, signals often have to be down
sampled in order to have \( L_{max} - L_{min} < 200 \) and still trigger the wanted frequencies.

3. At best, this tool should be used to get the answer of a forced system using filtered white noise.

The appearance of non zero real part values of admittance can be explained as for AVBP harmonics with the occurrence of the eigenmode of the duct.

As shown by Fig. 5.5, AVBP wiener-hopf is not an adequate validation tool for NOZZLE, as the results show to large inaccuracy. But in order to get first solutions, AVBP wiener-hopf is a time saving method, because it points out the peaks of impedance at the same positions and describes the changes in behavior of the diffuser concerning the wave frequencies.

5.3. Comparison of NOZZLE and AVBP for the diffuser A

The comparison between NOZZLE and AVBP is made for the 2D diffuser A. This geometry was used in the precedent project by Lamarque [8] to validate NOZZLE for the case of a distributor. The mesh was created with the CFD-Geom software of CAO and was also used for this validation.

The validation starts by making sure, that Nozzle provides the same values for the steady flow as AVBP. Afterwards, the acoustic impedances/admittances at the exit of the diffuser, furnished by the two codes, are compared. In table 5.3, the geometry, mesh and boundary conditions of the diffuser are described. The mesh is shown in Fig. 5.6. For further information of the geometry/mesh see also annex A.

![Figure 5.6: Mesh1 of diffuser A](image)

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Value</th>
<th>Boundary conditions</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_x ) (m)</td>
<td>0,2</td>
<td>( u_{in} ) (( \frac{m}{s} ))</td>
<td>30</td>
</tr>
<tr>
<td>( L_y ) (m)</td>
<td>0,015</td>
<td>( p_{in} ) (bar)</td>
<td>0,9292059</td>
</tr>
<tr>
<td>( n_x )</td>
<td>101</td>
<td>( \rho_{in} ) (( \frac{kg}{m^3} ))</td>
<td>1,07478</td>
</tr>
<tr>
<td>( n_y )</td>
<td>11</td>
<td>( T_{in} ) (K)</td>
<td>300,0</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>1,399</td>
<td>( r ) (J.kg/K)</td>
<td>288,19</td>
</tr>
</tbody>
</table>

In AVBP the walls are assumed to be adiabatic with zero normal velocity

Table 5.3: Characteristics of the diffuser A and boundary conditions
5.3.1. Mean flow

In order to be able to compare the results obtained by AVBP and NOZZLE, it is important to remind that NOZZLE uses a quasi-1D model in contrast to AVBP (2D). Therefore, a tool had been created by Lamarque, called `mean_q1d`, to average the values provided by the stabilized AVBP calculation for each section (here: averaging along y) to finally confront them to the values obtained with NOZZLE. Note, that the boundary conditions in table 5.3 are imposed on the averaged values, which are then fed to the NOZZLE code.

Figure 5.7 presents the compared mean flow. The values obtained with NOZZLE coincide very well with AVBP.

![Comparison between AVBP/NOZZLE for the mean flow in diffuser A](image)

Figure 5.7.: Comparison between AVBP/NOZZLE for the mean flow in diffuser A

5.3.2. Impedance calculation

As shown in the precedent section, the mean flows in AVBP and NOZZLE are the same. This section presents now the calculation of impedances with both methods.

A major question for this calculation is which boundary condition to select at the entrance of the diffuser. If the diffuser was choked, the impedance would not depend on the compressor elements located upstream of the choked section. Unfortunately, most diffusers are not choked...
and computing the diffuser outlet impedance requires to know its inlet impedance. Here, in a first step, one assumes a zero acoustic velocity at the entrance. As mentioned before, AVBP_wiener-hopf is not as precise as AVBP_harmonic. Nevertheless, calculations have been performed using AVBP_wiener-hopf to examine if it could be also a pre-dimensioning tool for more complex geometries as the diffuser.

Thus, several calculations have been realized with AVBP_harmonic. Every calculation starts from the initial solution described in section 5.3.1.

A wave \( A e^{-i\omega t} = i \rho_0 e^{-i\omega t} \)
is injected at \( t = 0 \) in the calculation domain from the exit. Once the flow is established (often a few acoustic times), the impedance calculation can be done using xwave_ms. For every calculation, the frequency of the injected wave is changed to obtain a sufficient amount of results in order to compare them to NOZZLE. (Typically, frequencies from 0 to 3000 Hz are used in AVBP_harmonic.)

Figure 5.8 shows the results obtained by the two codes. As it can be seen, they do correspond very well.

Figure 5.8.: Comparison AVBP/NOZZLE for the admittance calculation at the exit of diffuser A

AVBP_wiener-hopf

Fig. 5.9 shows the comparison between the three codes (AVBP_wiener-hopf/ AVBP_harmonic/ NOZZLE). As observed earlier in the case of the duct, and here with the diffuser configuration, AVBP_wiener-hopf finds the peaks around the same frequencies, with a phase shift for higher ones. In addition, between 1000 and \( \sim 1200 \) Hz, the results deviate significantly from the analytical solution.
As mentioned before in section 5.2.5, the quality of results, obtained by AVBP\_wiener-hopf, depends on the filter \((L_{max} - L_{min})\) used in the tool "WIENER". In the following cases, the influence of the different \(L_{max}\) (190, 170, 130) is examined by keeping \(L_{min}\) to zero in order to get also good results for lower frequencies. Fig. 5.10 shows the results.

The values obtained with \(L_{max} = 130\) differ the most from the validated NOZZLE solutions. The filter appears to be too small and hence triggers large delay times. \(L_{max} = 170\) and \(L_{max} = 190\) seem to be appropriate quantities. As it is visible in Fig. 5.10, it is not obvious which is the best one; because for one range of frequency \(L_{max} = 170\) gives better results, but for another range \(L_{max} = 190\) fits best.
Figure 5.10.: Comparison AVBP_wiener-hopf for Lmax= 190, 170 and 130 for the diffuser A (Top: Real part of admittance; Bottom: Imaginary part of admittance)
6. Influence of numerical factors

After the validation of the modified NOZZLE code, the objective of this work persisted now in examining the influence of

- the discrete scheme
- the mesh

on the acoustic impedance/admittance at the exit of the diffuser. This study was applied on the diffuser A.

6.1. Influence of discrete scheme for AVBP

For the validation of NOZZLE the classical finite volume (FV) Lax-Wendroff (LW) discrete scheme was used. Fig. 6.1 shows the influence of using the finite element (FE) Taylor-Galerkin (TTGC) scheme (in AVBP, harmonic), which is of third order in space and time. It is expected that with this scheme, one is provided with results that are less dissipative than those got with LW as the last one is only of second order in space and time.

But the diagram illustrates, that there is no visible difference between these two schemes in the range of the examined frequencies. This conclusion verifies, that it is sufficient to use the LW scheme to determine the admittance and thus to save calculation time as LW needs 2.5 times less than TTGC.

6.2. Influence of the mesh

In a second time, it was examined whether the quality of the mesh would have a significant influence on the results. Mesh 2 (Fig. 6.2 bottom) has an increased number of points from 101 to 151 in length and from 11 to 15 in heights. Apart from this, it does not exhibit a finer mesh at the throat, as in the case of the first one. A more precise mesh should provide more reliable results. But at the same time, it makes the simulation more complicated to stabilize.

Fig. 6.3 illustrates, that no big difference can be detected between the two meshes. This test justifies that the utilized mesh is sufficiently fine. Because, if this examination had shown big
differences, the mesh would not have been appropriate and thus, obtained admittances would have been wrong. As conclusion it can therefore be retained, that the used meshes should be sufficiently precise to get good results, but it is superfluous to make them too fine because the calculation could become too heavy and slow.

Figure 6.2.: top: Mesh1 \((n_x = 101; n_y = 11)\); bottom: Mesh2 \((n_x = 151; n_y = 15)\)

Remark
By studying the influence of the implementation of a finer mesh, instabilities in the mean flow field were detected: as it could have been analyzed during this project, the flow behavior is linked to the inlet boundary conditions. For an inlet "INLET\_RELAX\_UVW\_T\_Y" with a \(\textsf{relax}_{\textsf{on}_u} = \textsf{relax}_{\textsf{on}_T} = 100.000\) (see annex B) the velocity field is symmetric (see Fig. 6.4). But for higher values of this relaxation coefficient, antisymmetry, and thus instabilities appear (Fig. 6.5).
For Mesh1 it was possible to use the inlet boundary condition "INLET\_RELAX\_UVW\_T\_Y" with very high relaxation coefficients that represented at best the fully reflecting boundary condition.
that was required here. Contrary, for Mesh2, in order to avoid numerical instabilities, one is limited to use lower values.

Figure 6.4.: velocity field for $relax_{on\ u} = relax_{on\ T} = 100.000$
Figure 6.5.: velocity field for $\text{relax}_{\text{on},u} = \text{relax}_{\text{on},T} = 200.000$
7. Influence of Mach number

An interesting question at this point is to know, what really determines $Z_L$, the outlet impedance of the diffuser. To first order, $Z_L$ must depend on two things:

- the inlet impedance $Z_0$ (which depends on the whole turbine)
- the shape of the diffuser, which can be characterized by the ratio $A/A_\ast$ at the throat (or by the Mach number).

For a choked diffuser, $Z_0$ has no influence on $Z_L$. But for subsonic flows, it does. For zero Mach number and a straight duct (Fig. 7.1 left), there is a direct link between $Z_0$ and $Z_L$. Pressure and velocity fluctuations in a straight duct are:

$$
\begin{align*}
    p_1 &= A^+ e^{i(kx-\omega t)} + A^- e^{i(-kx-\omega t)} \\
    u_1 &= \frac{1}{\rho_0 c_0} \left( A^+ e^{i(kx-\omega t)} - A^- e^{i(-kx-\omega t)} \right)
\end{align*}
$$

(7.1)

If the impedance at $x = 0$ ($Z_0$) is known, then:

$$
Z_0 = \frac{p_1(x = 0)}{u_1(x = 0) \rho_0 c_0} = \frac{A^+ + A^-}{A^+ - A^-}
$$

(7.2)

Figure 7.1.: Left: straight duct; Right: Diffuser
And the impedance at \( x = L \) is

\[
Z_L = \frac{p_1(x = L)}{u_1(x = L) \rho_0 c_0} = \frac{A^+ e^{ikL} + A^- e^{-ikL}}{A^+ e^{ikL} - A^- e^{-ikL}}
\]

Eliminating the ratio \( A^+/A^- \) in 7.2 and 7.3 gives the expression of \( Z_L \):

\[
\frac{A^+}{A^-} = \frac{1 + Z_0}{-1 + Z_0} \Rightarrow Z_L = \frac{Z_0 + 1 + e^{ikL} - e^{-ikL}}{Z_0 + 1 - e^{2ikL}(Z_0 - 1)} = Z_s^L
\]

We will call \( Z_s^L \) the "shifted" impedance. It is simply the impedance at \( x = 0 \) (\( Z_0 \)) "shifted" by the duct length \( L \). The question we address now is "how does \( Z_L/Z_s^L \) change, when the diffuser shape changes?". For diffusers, which are almost straight, we expect \( Z_L/Z_s^L \) to go to 1 if the inlet Mach number is small. In such cases, knowing \( Z_L \) is equivalent to knowing \( Z_0 \), and we have already said that \( Z_0 \) is not known. In other cases, when the Mach number is large or when \( A \) goes to \( A_* \), we expect that \( Z_L \) will depend only weakly on \( Z_0 \), in which case the acoustic problem can be closed without knowing \( Z_0 \). This question is investigated here using NOZZLE. Fig. 7.2 shows how \( Z_L \) changes, when the diffuser section diminishes for an inlet impedance \( Z_0 \) and \( Z_1 \).

Both figures confirm, that \( Z_L \) is close to \( Z_s^L \), when the throat Mach is small. When the throat gets choked, \( Z_L \) and \( Z_s^L \) differ that \( Z_L \) does not depend on \( Z_0 \) any more. In conclusion, three domains can now be distinguished, as it is shown in Fig. 7.2:

1. **CHOKED NOZZLE**: \( Z_L \) does not depend on \( Z_0 \). For the calculation domain of the eigenmodes, the diffuser can therefore be assumed to be not existent. \( \Rightarrow \) The acoustic problem is closed.

2. **LOW SUBSONIC**: For Mach numbers \( Ma \leq 0.16 \), \( Z_L \) tends to \( Z_s^L \). For this area, \( Z_L \) will be only known, if \( Z_0 \) is known. That means, that it is not enough to only focus on the diffuser, but the behavior of the impedance further upstream of the diffuser inlet needs to be examined to finally obtain adequate values of \( Z_0 \) as inputs for NOZZLE.

3. **TRANSITION**: Between a Mach number of 0.16 and 1.0, \( Z_L \) differs from \( Z_s^L \) and is more influenced by the diffuser shape. It is in this zone, where NOZZLE can determine \( Z_L \) sufficiently well without knowing \( Z_0 \).

**Conclusion**

In this chapter, it has been shown, that NOZZLE is depended on further developments to predict the impedance on the diffuser inlet for Mach numbers lower than 0.16. Without this information, it is not possible to calculate the impedance at the combustion chamber inlet with precision. In the next chapter, an eigenmode calculation of a SNECMA combustion chamber will be presented.
Figure 7.2.: Top: Evolution of $Re(Z_L)$ over the ratio $A/A_*$; Bottom: Evolution of $Im(Z_L)$ over the ratio $A/A_*$. 
by coupling NOZZLE to the Helmholtz solver AVSP. The examination of the behavior of the flow in the chamber has been performed for several regimes given by SNECMA. All these regimes are situated in the area of low subsonic flow (around $Ma = 0.013$ to 0.014). $Z_L$ is therefore highly depended on $Z_0$. As just mentioned before, in those cases, NOZZLE needs to be furnished by $Z_0$ resulting of supplementary acoustic analysis that focus the geometry upstream of the diffuser. Then, NOZZLE can predict the well-imposed impedance at the entrance of the combustion chamber and an eigenmode calculation can be performed. As there do not exist any actual studies that determine $Z_0$, the project persisted in examining whether a variation of inlet impedances influences violently the eigenfrequencies and the spatial structure of the corresponding eigenmodes.
8. Coupling between AVSP and NOZZLE

8.1. The code AVSP

AVSP [1] is a numerical tool, developed at CERFACS, for acoustic analysis in three dimensional configurations. Two kinds of results are provided by this code:

1. A finite number of eigenfrequencies for a configuration (the number is fixed by the user).
2. The spatial structure of the corresponding eigenmodes, using acoustic pressure and acoustic speed.

AVSP is a code, which allows obtaining reasonable CPU times. First, as AVBP, it is a parallel code. Secondly, in most combustion devices, only the lowest frequency acoustic eigenmodes of the chamber are observed (they are interesting for a thermo-acoustic problem). This implies that relatively crude meshes can be used to perform an acoustic analysis.

8.1.1. Functionality

The code AVSP is basically a Helmholtz solver. Under the assumptions:

- Low Mach number flow
- No volume forces
- Linear acoustics (i.e. small perturbations)
- Large scale fluctuations
- Homogeneous mean pressure
- Constant polytropic coefficient $\gamma$
- neglecting of flame effects on acoustics
the general form of the Helmholtz equation is:

\[ \nabla (c_0^2 \nabla p_1) - \frac{\partial^2}{\partial t^2} p_1 = 0 \] (8.1)

The wave equation is usually solved in the frequency domain by assuming harmonic pressure variations \( p_1 = \text{Re}(p e^{-i\omega t}) \), that leads Eq. 8.1 to:

\[ \nabla (c_0^2 \nabla p) - \frac{\partial^2}{\partial t^2} p = 0 \] (8.2)

Eq. 8.2 represents an eigenvalue problem that is solved in AVSP as a matrix system:

\[ A \cdot p = -\omega^2 \cdot p = \lambda \cdot p \] (8.3)

by using the three following available boundary conditions:

- zero acoustic pressure \( p_1 = 0 \) (atmosphere)
- zero acoustic velocity \( u_1 = 0 \) (wall)
- admittance \( Y = \frac{1}{Z} \)

For the moment, the user can only define an admittance value that is complex, but not frequency dependent. In the future, the following term will be available:

\[ Y = \frac{1}{Z} = \frac{\alpha}{\omega} + \frac{1}{Z_0} + \beta \omega \] (8.4)

8.1.2. The coupling NOZZLE/AVSP

In AVSP, the resolution of the acoustic eigenmodes for a complete configuration, such as diffuser/combustion chamber/distributor, is not possible. The assumption of negligible Mach number (as for the chamber) in diffusers and distributors is not appropriate. AVBP is able to give the acoustic eigenmodes for the whole configuration, but it is reminded, that this is done only for one frequency at a time. This would require a large CPU time.

A better way is to use NOZZLE that will allow realizing an acoustic calculation of a whole configuration with AVSP. It determines in a fast way the impedances at the outlet of the diffuser and the inlet of the distributor that represent then the boundary conditions for AVSP (see Fig. 1.3 page 4).
8.2. **Industrial case**

8.2.1. **Introduction**

In respect of confidentiality, the configuration of the combustion chamber used for the following eigenmode calculation, as well as the operating characteristics will not be presented here in detail. It concerns a SNECMA combustion chamber for aeronautic engines. See its configuration in Fig. 8.1.

![Figure 8.1. Configuration of combustion chamber with bypass, plenum, foyer](image)

Previous studies analyzed the acoustic behavior of the foyer of this chamber in the configuration “plenum & foyer” only taking into consideration the distributor. For the outlet of the foyer, the following three boundary conditions have been imposed to examine their influence on the eigenfrequencies and the spatial structure of the corresponding eigenmodes:

1. atmospheric outlet \( (p_1 = 0) \)
2. wall \( (u_1 = 0) \)
3. impedance calculated with NOZZLE (taking into account the distributor)

First, it could have been pointed out, that a choked distributor (as it is the case here) behaves more like a wall than an atmospheric outlet. Secondly, regarding the non-negligible difference in eigenfrequencies for the different boundary conditions, this study showed the importance in integrating the tool NOZZLE in the determination chain of eigenmodes.

Objective of the acoustic analysis during this research project is now to focus on the configuration “plenum & foyer & bypass” (see Fig. 8.1) and to determine, whether it is essential to integrate the bypass and the diffuser in the NOZZLE/AVSP computation.

To initialize an AVSP calculation, a mean solution is demanded that was retrieved from an N3S\(^1\)

\(^1\)N3S is a code comparable to AVBP but based on the RANS model
solution, which furnishes the mean sound speed field for AVSP as well as the input conditions for NOZZLE (temperature, pressure, flow rate, \( \gamma \) and \( r \)).

**Geometry**

The geometry of the distributor corresponds to the one used during the first study without bypass. The variation of the section \( A \) as a function of the \( x \)-coordinate for the diffuser (here called: diffuser S) and the distributor is shown in Fig. 8.2 (see also annex A).

![Section variation as a function of x. left: diffuser S; right: distributor](image)

**8.2.2. Eigenmode calculation**

**Influence of boundary conditions**

For the following eigenmode examination, one regime given by SNECMA is observed. In this operating point, the distributor is choked (see Fig. 8.3 right) and thus, NOZZLE can provide AVSP with reliable admittance values. For the diffuser, as shortly mentioned in chapter 7, the calculation is situated in the area of small Mach numbers (see Fig. 8.3 left), where \( Z_0 \) must be known to obtain \( Z_L \). As there is no actual study to determine \( Z_0 \), two different cases are examined:

- \( Z_0 = 0 \): the inlet of the diffuser disembogues into the atmosphere (Fig. 8.5)
- \( Z_0 = \infty \): the inlet of the diffuser represents a wall (Fig. 8.6)

The admittance at the exit of the diffuser and the entrance of the distributor for frequencies up to 2000 Hz (range of interest for this chamber configuration) is given in Fig. 8.4 to 8.6 (It is reminded, that AVSP demands for admittance values as boundary conditions).

Results obtained with those boundary conditions are confronted with those for the condition of a wall \( (u_1 = 0) \) and a non-reflecting boundary \( (Z = -1 \rightarrow Y = -1) \) imposed at the inlet of the chamber. Thus, for those cases, the diffuser is supposed to be not existent.
Notice, that beside the diffuser outlet and distributor inlet, all the rest of the domain is delimited by walls that correspond to an acoustic velocity of $u_1 = 0$.

The calculation of the annular eigenmodes ($A$) was performed for the totality of 360 degrees of the chamber (see Fig. 8.7 left). Its mesh was generated by rotating and merging the mesh of a single sector (20 degrees). Fig. 8.7 right shows the geometry and mesh of a mono-sector used for the simulation. A precedent study (concerning only “plenum & foyer”) showed, that for capturing the annular eigenmodes a crude mesh was sufficient (90,000 nodes for the totality of 360 degrees). However, the determination of the longitudinal modes ($L$ along the x-coordinate) asked for a finer mesh in order to obtain the eigenfrequency with precision (49,000 nodes for a single sector). To avoid long calculation times, longitudinal modes were therefore computed by focusing only one sector of 20 degrees.

Table 8.1 summarizes the lowest longitudinal and annular eigenmodes in the foyer with bypass. It is pointed out, that the eigenmodes listed in table 8.1 are those exclusively appearing in the foyer and not in the bypass. This limitation is necessary to finally, in a second step, being able to compare those results to precedent studies without bypass.
Results of comparison between the boundary conditions:

\[ Z_L = Z_s^L(0) \]
\[ Z_L = Z_s^L(\infty) \]
\[ Z_L = \text{condition of type wall: } u_1 = 0 \]
\[ Z_L = \text{non reflecting condition: } Y = -1 \]

For a non reflecting inlet condition \( (Z = -1 \rightarrow Y = -1) \), the low eigenfrequencies (in the range of interest of 700 Hz to 1440 Hz) seem unable to develop. The first longitudinal eigenmode appears at 2800 Hz and concerns exclusively the bypass.

AVSP finds for the other three conditions the first three longitudinal eigenmodes and the first annular eigenmode around the same frequencies. Their spatial structure is of same type: for the lowest longitudinal eigenfrequency for example, a quarter wave appears in the foyer. Here, the absolute variation of frequency between the conditions \( Z_s^L(0) \) and \( Z_L = Z_s^L(\infty) \) rises up to
<table>
<thead>
<tr>
<th>Mode</th>
<th>$Z_0 = 0 \rightarrow Z_L = Z_L^s(0)$</th>
<th>$Z_0 = \infty \rightarrow Z_L = Z_L^s(\infty)$</th>
<th>Type wall: $u_1 = 0$</th>
<th>Non refl. $Y = -1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1L</td>
<td>767 Hz</td>
<td>662 Hz</td>
<td>699 Hz</td>
<td>-</td>
</tr>
<tr>
<td>2L</td>
<td>1344 Hz</td>
<td>1347 Hz</td>
<td>1340 Hz</td>
<td>-</td>
</tr>
<tr>
<td>3L</td>
<td>1435 Hz</td>
<td>1441 Hz</td>
<td>1425 Hz</td>
<td>-</td>
</tr>
<tr>
<td>1A</td>
<td>349 Hz</td>
<td>307 Hz</td>
<td>311 Hz</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 8.1: Longitudinal eigenmodes of a 20 degrees sector and annular eigenmodes of the whole chamber with bypass.
16 %. However, compared to \( u_1 = 0 \), this difference decreases to 10 % for \( Z_L^1(0) \) and 5 % for \( Z_L = Z_L^1(\infty) \). Same tendency shows the first annular mode. For the higher eigenmodes, variations of less than 1 % could have been detected. The second longitudinal eigenmode (around 1344 Hz) is superposed by a transversal mode, as it shows table 8.1.

Computation with two other boundary conditions have been effectuated, which are not listed here. First, at the diffuser outlet, the reflecting boundary condition \( p_1 = 0 \) (type disemboguing into the atmosphere) was imposed, and secondly, at the diffuser inlet, a partially non-reflecting condition with the relaxation coefficient \( K = 1000 \) and \( K = 500 \) (NOZZLE calculation, see annex B). Same tendencies as shown in table 8.1 were found: same range of eigenfrequencies and mode structure with higher variations for the first longitudinal mode.

In a first step, this study points out, that the diffuser acts like a reflecting or partially reflecting boundary condition and not like a non-reflecting one. For the smallest eigenfrequency, there are indeed relatively high variations, which confirm the assumption, that it is important to choose well imposed boundary conditions and thus to integrate NOZZLE in the eigenmode determination chain. However, it is not clearly visible, which one of the examined conditions (apart from the non-reflecting one) represents more precisely the reality. It is reminded, that the observed operating point is situated in the “LOW SUBSONIC” area, where the calculated impedance value at the outlet of the diffuser \( Z_L \) is strongly dependent on \( Z_0 \) (the imposed inlet impedance). In
order to impose the proper inlet impedance for AVSP, it seems to be necessary to include the
gometry further upstream of the diffuser in the acoustic analysis to finally deliver NOZZLE with
the appropriate value of $Z_0$.

**Influence of the bypass**

*Table 8.2.* shows the eigenmode comparison for the configuration “plenum & foyer” and “plenum
& foyer & bypass”.

<table>
<thead>
<tr>
<th>Mode</th>
<th>1L</th>
<th>2L</th>
<th>1A</th>
<th>2A</th>
</tr>
</thead>
<tbody>
<tr>
<td>With bypass: $Z_L^+(0)$</td>
<td>767 Hz</td>
<td>1344 Hz</td>
<td>349 Hz</td>
<td>500 Hz</td>
</tr>
<tr>
<td>With bypass: $Z_L^+(\infty)$</td>
<td>662 Hz</td>
<td>1347 Hz</td>
<td>307 Hz</td>
<td>500 Hz</td>
</tr>
<tr>
<td>With bypass: $u_1 = 0$</td>
<td>699 Hz</td>
<td>1340 Hz</td>
<td>311 Hz</td>
<td>500 Hz</td>
</tr>
<tr>
<td>Without bypass: $u_1 = 0$</td>
<td>837 Hz</td>
<td>-</td>
<td>344 Hz</td>
<td>686 Hz</td>
</tr>
</tbody>
</table>

*Table 8.2.*: Comparison of eigenmodes between chamber with and without bypass

The influence of the consideration of the plenum in the AVSP computation is clearly visible: the
first longitudinal eigenfrequency is between 21 % for the $Z_L^+(\infty)$ condition and 8 % for $Z_L^+(0)$ lower
than the one with bypass. Same tendency appears for the first annular eigenmode (~ 12% variation)
and the difference points out to be even more important for the second annular eigenmode (up to
37 %).

Therefore, as a second result it can be retained, that the bypass decreases the eigenmodes in the
foyer to lower frequencies. In order to get the eigenfrequencies in precision, it is indispensable to
observe the whole configuration such as “plenum & foyer & bypass”.

**Comparison of the acoustic modes for 3 operating regimes**

The calculation chain NOZZLE/AVSP is now applied to two other operating regimes. The eigen-
frequencies, corresponding to the different regimes, calculated with AVSP are presented in *table
8.2.2.*. *Fig. 8.9* drafts the evolution of these frequencies as a function of the temperature at the
entrance of the chamber.

At the inlet and outlet of the domain, one imposes the impedance as boundary condition, which
considers the diffuser and distributor. This impedance (or admittance) is dependent on the fre-
quency and the regime.

Increasing the regime of the chamber causes an augmentation of the eigenfrequency values. It
should be mentioned here, that the difference between the temperatures of regime 1 and regime 2
is about 200 K, however between regime 2 and regime 3 only 10 K. Therefore, the augmentation
can be considered as quasi linear. This behavior is due to the fact, that by increasing the regime, the inlet temperature of the gases and thus the mean temperature in the chamber increases as well as the sound speed, which leads to the increasing of the eigenfrequencies of the acoustic modes. It can be seen, that the frequency changes with the regime (more than 100 Hz for certain modes).

<table>
<thead>
<tr>
<th>Mode</th>
<th>1A</th>
<th>1L</th>
<th>2L</th>
<th>3L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime 1</td>
<td>$Z_1^L(0)$</td>
<td>349 Hz</td>
<td>767 Hz</td>
<td>1344 Hz</td>
</tr>
<tr>
<td></td>
<td>$Z_1^L(\infty)$</td>
<td>307 Hz</td>
<td>662 Hz</td>
<td>1347 Hz</td>
</tr>
<tr>
<td>Regime 2</td>
<td>$Z_1^L(0)$</td>
<td>395 Hz</td>
<td>914 Hz</td>
<td>1569 Hz</td>
</tr>
<tr>
<td></td>
<td>$Z_1^L(\infty)$</td>
<td>347 Hz</td>
<td>791 Hz</td>
<td>1571 Hz</td>
</tr>
<tr>
<td>Regime 3</td>
<td>$Z_1^L(0)$</td>
<td>412 Hz</td>
<td>922 Hz</td>
<td>1611 Hz</td>
</tr>
<tr>
<td></td>
<td>$Z_1^L(\infty)$</td>
<td>362 Hz</td>
<td>797 Hz</td>
<td>1615 Hz</td>
</tr>
</tbody>
</table>

Table 8.3.: Eigenfrequencies with impedance at inlet and outlet

Figure 8.9.: Evolution of the eigenfrequencies as a function of the inlet temperature for the three regimes.
Conclusion and perspectives

During this student research project, two codes were used to determine the impedance at the exit of a diffuser. First, the multidimensional spatio-temporal code AVBP, developed at CERFACS, and secondly NOZZLE, a quasi-one-dimensional code that works in the frequency domain, written by Lamarque [8] during his trainee period at CERFACS and modified during this student research project.

The global objective of this student research project was divided in two parts:

1. **Adaptation of the existing NOZZLE code**, which was limited to the calculation of entrance impedances of distributors, on diffuser configurations. This gave the opportunity to obtain a more profound knowledge in acoustics and to make use of the programming language Fortran. The following validation of NOZZLE allowed learning the application of AVBP and its functionality.

2. **Eigenmode calculation of a SNECMA combustion chamber with bypass** using the modified NOZZLE code to impose adequate impedances at the boundaries (inlet and outlet) of the chamber. Therefore, AVSP was employed, a Helmholtz solver that determines the acoustic eigenmodes.

A principal question accompanied this project: What determines $Z_L$, the impedance at the exit of the diffuser? As answer, three operational sectors were found dependent on the Mach number (see Fig. 7.2):

1. **CHOKED NOZZLE**: For Mach numbers $\geq 1$, the outlet impedance of the diffuser $Z_L$ is completely independent of the inlet impedance $Z_0$. The existence of the diffuser has thus no influence on the behavior of the flow in the chamber and an eigenmode calculation can be performed in an acoustically closed domain.

2. **TRANSITION**: In the area of $0.16 \leq Ma \leq 1$, NOZZLE is an adequate numerical tool for impedance calculation at the outlet of diffusers. There, $Z_L$ is influenced by $Z_0$, but still differs largely from $Z'_L$ (the impedance at $x = 0$ ($Z_0$) “shifted” by the duct length $L$).
3. **LOW SUBSONIC:** For Mach numbers $\leq 0.16$, $Z_L$ goes to $Z_L^*$ and adapts its value for small Mach numbers. Thus, if $Z_0$ is not known, neither is $Z_L$. There, the limit of NOZZLE is reached. Without any information of what happens further upstream of the diffuser, NOZZLE is not able to deliver appropriate impedances for the eigenmode calculation.

The AVSP computation showed the importance of including sufficient geometry details in the computational domain upstream (diffuser), downstream (distributor) as well as around the foyer (bypass) in order to resolve reasonably the thermo acoustic instability problems (appropriate boundary conditions concerning the acoustics are required).

Furthermore, the eigenmode study turned out to be situated operationally in the so called “LOW SUBSONIC” area. For aeronautical engine configurations, those low Mach numbers ($\sim 0.013$ to 0.014) seem to be usual. It is therefore indispensable to undertake further studies regarding the acoustic upstream of the diffuser to finally provide NOZZLE with an appropriate value of $Z_0$. 

Bibliography


A. The Geometries

A.1. DIFFUSER A

The geometry and mesh of the diffuser A had been taken over from Lamarque (see [8] Annexe C, Tuyère A), who defined the geometry with CFD-Geom starting with 7 points and creating the wall by an interpolating line. The variation of its section along x is shown in Fig. A.1.

![Section variation of the diffuser A as a function of x](image)

Figure A.1.: Section variation of the diffuser A as a function of x

A.2. DIFFUSER S

Its geometry was simplified starting from a CAO given by SNECMA with the tool CFD-Geom. To obtain the variation of the diffuser section as a function of the x coordinate, several cuts had to be made perpendicularly to the x axis through the geometry to finally get the sections at forty points. Tracing an interpolation line through these points, gave the 401 sections that were introduced in “nozzle.choices” (geometry input file of NOZZLE). See the section variation of the diffuser S in Fig. 8.2 left, chapter 8 page 45.
B. Boundary Conditions

B.1. The NSCBC Model

In the AVBP code, the boundary condition method derived by Poinsot and Lele [14], called Navier-Stokes characteristic boundary conditions (NSCBC), is implemented. This method allows a control of different waves crossing the boundary. The principle of the NSCBC method is to take conditions that correspond to Euler conditions (the ESCBC conditions), an extensively studied method (Kreiss [7], Hirsch [6], Engquist and Majda [10]), and to add supplementary relations, called “viscous” conditions (= viscous dissipation + thermal diffusion + species diffusion). To impose these physical boundary conditions is not enough to solve the problem numerically, so a second class of boundary conditions must be introduced: the numerical. It allows computing the variables at the boundary that are not imposed by physical boundary conditions. That means, outgoing waves can be computed with the information inside of the domain, but incoming waves must be specified by the boundary condition. In the NSCBC method, the incoming wave amplitude is determined using the Local One Dimensional Inviscid (LODI) relations. The LODI relations link the wave amplitude \((L_i)\) and the temporal evolution of Navier-Stokes variables \((\rho, u, v, w, p)\):

\[
\frac{\partial \rho}{\partial t} + \frac{1}{c^2} \left[ L_2 + 1/2 \left( L_5 + L_1 \right) \right] = 0 \tag{B.1}
\]

\[
\frac{\partial u}{\partial t} + \frac{1}{2\rho c} (L_5 - L_1) = 0 \tag{B.2}
\]

\[
\frac{\partial v}{\partial t} + L_3 = 0 \tag{B.3}
\]

\[
\frac{\partial w}{\partial t} + L_4 = 0 \tag{B.4}
\]

\[
\frac{\partial p}{\partial t} + 1/2 (L_5 + L_1) = 0 \tag{B.5}
\]
Where \( L_1 \) is the left traveling acoustic wave, \( L_2 \) the entropy wave, \( L_3 \) the first vorticity wave, \( L_4 \) the second vorticity wave and \( L_5 \) the right traveling acoustic wave:

\[
L_1 = (u_1 - c) \left( \frac{\partial p}{\partial x_1} - \rho c \frac{\partial u_1}{\partial x_1} \right) \tag{B.6}
\]

\[
L_2 = u_1 \left( c^2 \frac{\partial p}{\partial x_1} - \frac{\partial p}{\partial x_1} \right) \tag{B.7}
\]

\[
L_3 = u_1 \frac{\partial u_2}{\partial x_1} \tag{B.8}
\]

\[
L_4 = u_1 \frac{\partial u_3}{\partial x_1} \tag{B.9}
\]

\[
L_5 = (u_1 + c) \left( \frac{\partial p}{\partial x_1} + \rho c \frac{\partial u_1}{\partial x_1} \right) \tag{B.10}
\]

Equations B.1 to B.5 provide a simple method to choose the incoming wave amplitudes to be imposed at a boundary.

### B.2. Boundary Conditions used in AVBP

The boundary conditions used in the calculations presented in this report are shortly described here. For further information and physical background, the reader is referred to the AVBP Handbook [17].

**INLET_RELAX_UWW_TW_Y**

This represents an inlet with relaxation towards a reference velocity, temperature and species. This means, that the ingoing waves are taken proportional to the difference between the actual state at the boundary nodes and the reference velocity and temperature. For high values of the relaxation coefficients relax\_on\_U\_n (normal velocity), relax\_on\_U\_t (tangential velocity) and relax\_on\_T, the partially non reflecting boundary condition passes over to a fully reflecting one.
OUTLET_RELAX_P_PULSE

It is an outlet with relaxation towards a reference pressure that means, that the ingoing wave is taken proportional to the difference between the actual state at the boundary nodes and the reference static pressure. Simultaneously, it allows introducing an acoustic excitation entering the domain especially from the back.

WALL_NOSLIP_ADIAB

It defines an adiabatic wall with zero velocity

B.3. Partially non-reflecting inlet of the diffuser implemented in NOZZLE

By studying the influence of numerical factors on the impedance calculation, it seemed also interesting to examine the impedance evolution by imposing a partially non reflecting boundary condition at the diffuser inlet. Not only calculations with AVBP have been performed, but also with NOZZLE after having implemented the possibility to enter a complex impedance as a function of the relaxation coefficient $K$ and $\omega$.

In this section, the NSCBC model is used and thus the LODI equations. This demands the introduction of the link between the acoustic wave amplitude $A$, which is used in this report, and the NSCBC characteristic wave amplitude $L$.

\[ L_5 = \bar{x} \frac{\partial A^+}{\partial t} \quad \text{(B.11)} \]

\[ L_1 = -\bar{x} \frac{\partial A^-}{\partial t} \quad \text{(B.12)} \]

For a non-reflecting inlet condition, the simplest method is to set the amplitude of the incoming wave $L_5$ to zero. But various authors [14], [18] indicate, that this choice may lead to ill-posed problems. One solution is the linear relaxation method, LRM [14], [18], [13], where the amplitude of $L_5$ is set as proportional to the velocity difference.

\[ L_5 = 2K \rho c (u^t - u) \quad \text{(B.13)} \]

Where $u$ is the predicted velocity at the entrance and $u^t$ the target value, $K$ represents the relaxation coefficient. To receive a partially non-reflecting inlet, $K$ is set to higher values. This method is used here to simulate a partial non-reflecting boundary.
Together with the expression of the complex wave that is introduced at the outlet (see section 5.2.1 Eq. 5.3)

\[ L_1 = -2p_0 i \omega e^{-i \omega t} \]  

(B.14)

and the LODI relations B.1 and B.2, one obtains

\[
\frac{\partial u}{\partial t} + \frac{1}{2 \rho c} \left[ 2K \rho c \left( u^t - u \right) + 2p_0 i \omega e^{-i \omega t} \right] = 0 
\]

(B.15)

\[
\frac{\partial p}{\partial t} + \frac{1}{2} \left[ 2K \rho c \left( u^t - u \right) - 2p_0 i \omega e^{-i \omega t} \right] = 0 
\]

(B.16)

Equation B.15 only involves \( u \) and can be easily solved. The solution for \( u \) is

\[
u(t) = u^t + A_0 e^{-Kt} + \frac{p_0 i \omega / (\rho c)}{K - i \omega} e^{-i \omega t}.\]  

(B.17)

With reference to the article of Selle, Nicoud and Poinsot [13], the transient term \( A_0 e^{-Kt} \) can be omitted and finally the reflection coefficient at the inlet is constructed:

\[
R_{in} = \frac{L_0}{L_1} = \frac{K^2 + i \omega K}{K^2 + \omega^2} 
\]

(B.18)

The magnitude \( ||R|| \) is

\[
||R|| = \frac{1}{\sqrt{1 + \frac{\omega^2}{K^2}}} 
\]

(B.19)

The asymptotic behavior of \( ||R|| \) is illustrated in Fig. B.1 for the analytical and AVBP solution. Obviously, non zero values of \( K \) lead to partially reflecting boundaries. However, when \( K \) goes to infinite \( ||R|| \) goes to 1, making the boundary fully reflecting. Equation B.19 shows, that for a fixed value of \( K \), the boundary condition (BC) is reflecting for low frequencies \( (f < f_c = \frac{K}{2\pi}) \) and non-reflecting for high frequencies.

Consequently, the admittance at the entrance and thus at the exit of the diffuser is a function of \( K \) and \( \omega \).

In Fig. B.2, the evolution of the admittance at the exit for the fully and partially reflecting boundary condition is compared. These results were obtained with AVBP

...
condition "INLET_RELAX_UVW_T_Y" (see annex B), and on the other hand with NOZZLE by introducing the complex impedance value at the entrance:

\[ Z_{in} = \frac{-\omega - i2K}{\omega} . \quad (B.20) \]

This expression was obtained from the basic definition \( R_{in} = \frac{Z_{in} + 1}{Z_{in} - 1} \).

The amplitudes of both, real and imaginary part decrease and outstanding peaks for several frequencies disappear. Once more it is visible, that modifying the boundary conditions strongly influences the evolution of the admittance over the frequency.
Figure B.1.: Modulus of reflection coefficient vs. frequency for $K=\infty, 10000, 3000$

Figure B.2.: Imaginary and real part of the admittance at the exit of the diffuser for $K=\infty \equiv u_1 = 0, K = 3000$
C. Validation of the model for zero flow rate

The validation of this model, which had been implemented in the code to calculate nozzles with zero velocity flow (see chapter 4), was performed by comparing the results obtained with NOZZLE to the analytical ones. See the analytical solution in chapter 5, Eq. 5.10 page 26. The characteristics of the duct and boundary conditions can be seen in table 5.2, with the difference, that now \( u_0 = 0 \) is used instead of a very small Mach number as in chapter 5.

In the two figures C.1 and C.2, the absence of the outstanding points of the real part of the admittance can be noticed. They were originally present in the initial model (compare to Fig. 5.4 on page 28).

Figure C.1.: Comparison between NOZZLE and the analytical solution (real and imaginary part of admittance \( Y \)) for the distributor.
Figure C.2.: Comparison between NOZZLE and the analytical solution (real and imaginary part of admittance $Y$) for the diffuser.