Prediction of the sound generated by a rod-airfoil configuration using a compressible unstructured LES solver and a FW-H analogy

Jean-Christophe Giret* and Alois Sengissen†

Airbus Operations SAS, Toulouse, France

Stéphane Moreau‡ and Marlène Sanjosé §

GAUS, Université de Sherbrooke, Sherbrooke, QC, Canada

Jean-Christophe Jouhaud¶

CFD Team, CERFACS, Toulouse, France

This paper aims at predicting the noise generated by flows interacting with airframe elements using unstructured LES coupled with a FW-H technique. The rod-airfoil canonical geometry has been selected as a benchmark representative of such phenomena. The detailed experimental database and several numerical simulations available enable an extensive validation of the proposed methodology. Similar or improved results are obtained both in the near-field (velocity profiles) and in the acoustic far-field (power spectral densities obtained with both porous and solid surfaces) compared with the best, most recent simulations. The impact of two important numerical parameters is also assessed: the spanwise dimension of the computational domain, and the sensitivity of the rod/airfoil alignment. The former improves the low frequency content of the simulated acoustic pressure, the latter only the simulated near-field in the cylinder wake. Finally best practices are drawn from this test case for future airframe industrial configurations.

I. Introduction

Noise prediction capability is a key enabler to the design of future commercial aircraft to ensure that current and future regulations are met. The turbofan noise of modern aircraft has been significantly reduced through the integration of high bypass ratio engines and airframe noise (landing gear and high lift devices) has become a major noise source at approach conditions. As a consequence, the design of new aircraft requires the prediction and the reduction of the airframe noise.

Besides wind tunnel experiments, computational aeroacoustics is a promising path to better understand the sound generation mechanisms and to speed up the design process of an aircraft. Among the several numerical methods, the Large-Eddy Simulation (LES) offers a good compromise between direct numerical simulations and unsteady RANS simulations and can provide accurate broadband noise predictions over a large frequency range that is required for airframe noise.

A requirement for such simulations is the ability to handle complex geometries. Most of the numerical aeroacoustics studies have been performed on structured meshes as it enables the use of high-order schemes. However, meshing a complex geometry with a block-structured mesh is at least difficult and at

*Ph.D. student, Airbus Operations SAS, PO Box M0112/4, 316 route de Bayonne, 31060 Toulouse cedex 03, France
†Research Engineer, Airbus Operations SAS, PO Box M0112/4, 316 route de Bayonne, 31060 Toulouse cedex 03, France
‡Professor, Mechanical Engineering, Université de Sherbrooke, 2500 Blvd de l’Université, QC, J1K 2R1, Canada; AIAA Lifetime Member.
§Post-doctorate Student, Mechanical Engineering, Université de Sherbrooke, 2500 Blvd de l’Université, QC, J1K 2R1, Canada; AIAA Student Member
¶Senior researcher, CERFACS, 42 Avenue Gaspard Coriolis, 31057 Toulouse Cedex 01, France
In order to overcome such limitations, overset grids\[^3\] or unstructured meshes\[^4\] can be used. This study focuses on the use of unstructured meshes with a special attention to accuracy issues.

The rod-airfoil test case performed by Jacob et al.\[^5\] at ECL is now considered as a canonical benchmark for landing-gear noise. Indeed, this experiment mimics the simplest geometrical elements found in a landing gear and provides the two acoustical features observed in the noise of a landing-gear: a quasi-tonal noise caused by the periodic shedding of vortices at the rod and a broadband noise component caused by the impingement of the developed turbulent wake on the airfoil. From a numerical point of view, this test case allows assessing the ability of a CFD code to predict the correct shedding frequency of a bluff body and to model the decay of the turbulent structures in the wake correctly. It has then been extensively used to validate numerical methods\[^3,6–11\] for airframe noise predictions.

Moreover, the experimental set-up showed a slight bias of -2 mm in the cross-stream direction for the position of the rod. Nevertheless, almost all numerical simulations performed so far on this configuration do not account for this bias, and assume a negligible effect of the rod displacement on the far-field noise. They also involve a small spanwise extent far below the known pressure spanwise coherence along the cylinder.

The goals of this study are therefore twofold. Firstly, the capability of the high-order unstructured LES solver AVBP coupled with a Ffowcs-Williams and Hawkings (FW-H) code to calculate the far-field noise is assessed by comparing the results with the experiment and the other numerical data available in the literature. Secondly, the relevance of the assumption to neglect the impact of the bias on the radiated noise and the effect of the spanwise dimension of the computational domain is investigated.

II. Experimental benchmark

The experiment reported by Jacob et al.\[^5\] was carried out in the large anechoic wind tunnel at Ecole Centrale de Lyon.\[^6\] The aim of this experiment was to build an experimental database against which CFD/CAA methods could be compared with. A sketch of the configuration is shown in Fig. 1. A NACA0012 airfoil of chord \(C = 10^{-1}\) m is placed one chord downstream of a cylindrical rod with a diameter of \(d = 0.1C\). This set is placed in an uniform air flow with \(U_{in} = 72 m/s\), \(T_{in} = 293 K\) and \(\rho_{in} = 1.2\). The Reynolds numbers based on the rod diameter and the airfoil chord are \(Re_d = 4.8 \times 10^4\) and \(Re_C = 4.8 \times 10^5\) respectively. The spanwise length is \(30d\). The rod was slightly misaligned with the airfoil by about 2 mm in the cross-wise direction.

![Figure 1. Set-up of the rod-airfoil experiment and location of measurements of interest.](image)

The physical mechanism observed by Jacob et al.\[^5\] is summarized as follows: a von Kármán vortex street is shed by the rod at a Strouhal number of \(St = 0.19\). For this flow regime, the boundary layer on the rod is laminar and the transition to turbulence occurs in the wake. Instability waves occur in the shear layers and interact with the two-dimensional structures. As shown by Kourta et al.,\[^12\] this leads to the transition of the wake to turbulence.

The resulting turbulent wake impinges on the airfoil leading-edge one chord downstream. As shown by Casalino,\[^13\] the basic principle of a direct and a non-direct vortex-airfoil interaction can be studied by the
means of a discrete vortex-method study. Large vortices impinging directly the leading-edge split and the two smaller resulting eddies pass above and below the airfoil. Smaller vortices or vortices which do not directly impinge the airfoil are not split and only undergo a distortion. These interactions result on unsteady loads on the airfoil and in a modification of the acoustic spectrum. The sound pressure level for the tonal peak is higher for the rod-airfoil configuration than for the rod only configuration. The tonal peak is also broadened due to the turbulence development in the wake and to non-linear effects during the impingement of large vortices on the airfoil leading-edge. The small vortices of a small size compared to the airfoil maximum thickness add a true broadband component to the far-field pressure spectrum.

Hot wire anemometry measurements have been carried out to determine mean velocity magnitude and velocity fluctuations. These measurements were carried out using with an IFA 100 anemometer. They were performed at several axial positions including the [A], [B] and [C] profiles and the point P shown in Fig. 1. Far-field pressure measurements have been performed at a distance \( R = 1.85 \) m from the airfoil center in the mid-pan plane \((z = 0)\). Brüel & Kjær type 4191 microphones with Brüel & Kjær type 2669 preamplifiers were used for these measurements. Particle Image Velocimetry (PIV) and unsteady wall pressure measurements were also done but are not used in this study.

### III. Numerical background

<table>
<thead>
<tr>
<th>Name</th>
<th>Year</th>
<th>Method</th>
<th>Mesh</th>
<th>DoF</th>
<th>( \Delta t ) [s]</th>
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<td>2003</td>
<td>Compressible U-RANS + FWH</td>
<td>2D Structured</td>
<td>54,640</td>
<td>(3.15 \times 10^{-2})</td>
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<td>Magagnato(^{14})</td>
<td>2003</td>
<td>Compressible LES + FWH</td>
<td>3D Structured</td>
<td>N/A</td>
<td>N/A</td>
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<tr>
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<td>2005</td>
<td>Compressible LES + FWH</td>
<td>3D Structured</td>
<td>67,000</td>
<td>(6 \times 10^{-3})</td>
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<tr>
<td>Greschner(^{15})</td>
<td>2004</td>
<td>Compressible LES + FWH</td>
<td>3D Structured</td>
<td>(1.9 \times 10^{5})</td>
<td>(5 \times 10^{-2})</td>
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<tr>
<td>Boudet(^7)</td>
<td>2005</td>
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<td>3D Structured</td>
<td>(2.4 \times 10^{6})</td>
<td>(1.3 \times 10^{-2})</td>
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<tr>
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<td>2006</td>
<td>Compressible LES + LPCE</td>
<td>3D Structured</td>
<td>(2.4 \times 10^{6})</td>
<td>(3.6 \times 10^{-2})</td>
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<tr>
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<td>Compressible LES + FWH</td>
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<td>2010</td>
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<td>(20 \times 10^{6})</td>
<td>(7.3 \times 10^{-1})</td>
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<td>(10 \times 10^{6})</td>
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<td>Eltaweel(^{11})</td>
<td>2011</td>
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<td>Present work</td>
<td>—</td>
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<td>3D Unstructured</td>
<td>(4.25 \times 10^{6})</td>
<td>(1.5 \times 10^{-1})</td>
</tr>
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Table 1. Previous numerical work on the rod.

The current section aims to draw from the literature the best practices on aeroacoustics computations that could be applicable on industrial configurations. All the numerical studies performed on the rod-airfoil configuration are summarized in Tab. 2.

The first numerical attempt to predict the far-field noise of this configuration has been carried out by Casalino \textit{et al.}\(^6\) using unsteady compressible RANS simulations. U-RANS simulations were performed on 2D meshes to reduce the computational effort needed. The three dimensional effects on the far-field noise were modeled by the means of a spanwise statistical model. However, the model needed experimental data to be calibrated. The near-field results and the flow statistics such as the mean velocity and the turbulent intensity qualitatively agreed with the experimental data. However, the simulations over-predicted the shedding frequency by 25\%. Moreover, as the turbulent breakdown is not resolved in these U-RANS simulations, the broadband component of the wall pressure spectra of the rod could not be captured.

To tackle the broadband component, LES was then extensively used to resolve all the turbulent scales in the wake impinging the airfoil. Magagnato \textit{et al.}\(^{14}\) and Boudet \textit{et al.}\(^7\) performed the first LES computations on the rod-airfoil configuration. Boudet’s LES was performed with the finite-volume compressible multi-block structured code Turb’Flow originally developed for turbomachinery applications. The grid consisted of 2.4 million cells. The acoustic propagation in the far-field was carried out using a porous FW-H analogy.\(^{18}\) Although the near-field results rather agree with the experiment, the statistics of the far-field acoustic

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results are poorly converged due to a too small simulation time. Nevertheless, the shedding frequency is well determined.

Peth et al.\textsuperscript{16} performed a compressible implicit LES on a structured mesh of 2.4 million cells and the radiated acoustic field was determined using the Linearized Perturbed Compressible Equations (LPCE). The near-field results show a reasonable agreement with the experiment. The shedding frequency is slightly underestimated on the far-field pressure Power Spectral Density (PSD) and the peak is underestimated by 5 to 7 dB. One should highlight that the influence of a 1 mm shift in the \( y \)-direction has also been addressed in that work. A negligible effect has been observed on both the near-field behind the cylinder and on the far-field acoustics.

Berland et al.\textsuperscript{3,10} carried out a compressible LES using a high-order scheme on a mesh of 20 million cells and with an overset grid strategy to cope with the limitations imposed by a block-structured mesh. A direct noise computation is made to propagate the sound waves and aims at studying the coupling between the hydrodynamic and the acoustic part of the flow. Although a fairly good agreement can be noticed in the near-field, the far-field noise shows discrepancies by over-predicting a large part of the sound pressure spectrum. This may be due to a wrong transmission of the sound waves between the overlapping grids.

Eltaweel and Wang\textsuperscript{11} used the CDP incompressible LES solver together with a Boundary Element Method (BEM) code to get the far-field noise. An unstructured mesh of 22.3 million cells was used. This method is only applicable to low-mach number flows as it approximates the quadrupolar sources from the incompressible flow variables. Nevertheless, it shows the ability to distinguish the direct and scattered components of a sound signal. However, due to the Boundary Element discretization, this method is more demanding in computational resources in comparison with acoustic analogies. The methodology applied on the rod-airfoil experiment shows a very good agreement with the experiment for both the near-field and the acoustic results. The shedding frequency is accurately predicted, but the peak is slightly under-predicted on the far-field acoustic PSD.

All these LES studies show globally a good agreement with the experiment in the near-field and in the far-field. The broadened tone related to the fundamental shedding frequency and the impingement of the vortical structures on the leading edge of the airfoil are well reproduced. The broadband component of the spectra is also well reproduced, contrary to the U-RANS simulations. In all cases, longer time series lead to better converged statistics and smoother spectra. The use of finer meshes obviously leads to a better resolution of the turbulent wake and the impinging turbulence on the leading edge is then better resolved. Finer meshes also resolve higher frequencies on the sound pressure spectra.

As the limiting factor in a LES is the necessity to resolve the boundary layers at the walls, some Detached-Eddy Simulations (DES) studies have been performed by Greschner et al.\textsuperscript{8,15} The spanwise extent of the domain was 3\( d \). The block-structured mesh consisted of 2.3 million cells. The Spalart-Allmaras (SA-DES), \( k - \epsilon \ (k - \epsilon-\text{DES}) \) and Explicit Algebraic Stress Model (EASM-DES) RANS models were used. The results globally match the experimental ones for the three RANS models, but the results are highly dependent on the turbulence model in the RANS-modeled zone. Among the three models, the EASM-DES yielded the best results. Its mean and rms velocity profiles match the best with the experimental results and predict the shedding frequency. These studies also investigated the role of the quadrupolar sound sources. They concluded that quadrupolar contributions are significant at the high frequencies but remain negligible for low and mid frequency ranges.

Lately, Galdéano et al.\textsuperscript{17} performed a compressible DES for the first time with an unstructured solver. The Spalart-Allmaras model was used for the RANS-modeled zone. Spanwise lengths of 3\( d \), 30\( d \) and 45\( d \) were investigated. They correspond to meshes of 3.5 million, 10 million and 15 million points respectively. Near-field velocity fluctuations PSD and far-field acoustic pressure PSD were compared with the experimental data. The three configurations recover the Strouhal peak well. Although the far-field pressure PSD of the 30\( d \) and 45\( d \) configurations match fairly well with the experimental ones, the far-field pressure of the 3\( d \) configuration undershoots the experimental results by about 6 dB. Moreover, the spanwise coherence length was not used to correct the PSD. It is then underestimated by 3.36 dB.

Although DES shows good results against LES and U-RANS, it handles the boundary layers with a turbulence model. Depending on the flow regime around a bluff-body, the boundary layer transition to turbulence and the boundary layer detachment can greatly affect the shedding frequency and the unsteady loads. The DES method may be too dependent on the model used to resolve the boundary layers around a bluff-body for a wide range of flow regimes.

\textsuperscript{16} Peth et al.

\textsuperscript{3} Berland et al.

\textsuperscript{4} Peth et al.

\textsuperscript{10} Berland et al.

\textsuperscript{11} Eltaweel and Wang

\textsuperscript{17} Galdéano et al.
Consequently, the selected approach for the prediction of airframe noise in an industrial context is based on an unstructured compressible LES solver coupled with a FW-H analogy. Indeed, the past studies show that LES is the best method to predict accurately the acoustic sources in a turbulent flow at an affordable cost. It allows to resolve the tonal and the broadband components of the acoustic pressure spectrum. Moreover, a compressible solver permits the calculation of the acoustic interactions such as reflexion and diffraction in the region of interest, while the FW-H analogy enables the efficient propagation of the pressure in the acoustic far-field without the numerical dissipation or dispersion of a numerical scheme. Finally, the unstructured meshing strategy offers the possibility to deal with the complex geometry of an airplane, such as landing-gears or high-lift devices.

IV. Numerical methods

A. Large-Eddy simulations

The simulations are carried out with the AVBP code, which is a massively parallel, unstructured compressible LES solver jointly developed by CERFACS and IFP Energies Nouvelles dedicated to both reactive and non-reactive flows. AVBP has been mainly dedicated to the study of combustion instabilities in chamber of aero-engines, and has only been recently successfully applied to aeroacoustics problems such as jet noise.\textsuperscript{4} The dynamic Smagorinsky subgrid-scale model of Germano et al.\textsuperscript{19,20} is used to compute the subgrid eddy viscosity $\nu_t$. The TTGC\textsuperscript{21} scheme has been used in this work due to its low dissipative character compared to the other scheme of its family. A selective viscosity operator of 2\textsuperscript{nd} and 4\textsuperscript{th} order developed by Colin et al.\textsuperscript{22} is applied to damp the numerical instabilities at high frequencies.

B. Acoustic propagation with an advanced-time approach

The far-field acoustics is obtained from the near-field unsteady conservative variables by a classical FW-H analogy. The KIM\textsuperscript{23,24} code from ONERA is used. It uses an advanced-time formulation (or source-time-dominant algorithm) of the FW-H analogy\textsuperscript{25} developed by di Francescantonio.\textsuperscript{18} It enables to perform on-the-fly acoustic calculations. On the contrary, a retarded-time approach would need the complete sampling of the solution to perform the acoustic extrapolation. Results from a solid surface source and from a porous surface source are compared.

Providing that the volumic sources are neglected or that the control surface encloses all the noise sources, the advanced-time FW-H analogy reads

$$4\pi p'(\mathbf{x}, t_{adv}) = \frac{\partial}{\partial x_i} \int_{f=0} \left[ \frac{L_i}{r(1 - M_r)} \right]_t dS + \frac{\partial}{\partial t} \int_{f=0} \left[ \frac{Q}{r(1 - M_r)} \right]_t dS,$$

where the solid or porous integration surface is defined by the equation $f(y, t) = 0$. $r = |y - x|$ denotes the distance between the source and the observer and $M_r = M_i r_i$ denotes the Mach number of the observer in the radiation direction. $L_i$ and $Q$ denote respectively the loading noise component and the thickness noise component. $t_{adv} = t + \mathcal{T}$ denotes the advanced time, with $\mathcal{T} = |x(t + \mathcal{T}) - y(t)|/c$ the time needed for a disturbance emitted at a source element $y$ at the time $t$ to reach the observer $x$.

C. Sound pressure level corrections

As the set-up spanwise length $L_{Sim}$ in the numerical simulations is shorter than the experimental spanwise length $L_{exp}$, the sound pressure levels obtained from the numerical simulations have to be corrected. Corrections have been developed by Kato et al.,\textsuperscript{26} Seo & Moon,\textsuperscript{27} Ewert et al.\textsuperscript{28} and Boudet et al.\textsuperscript{29} involving the pressure coherence function in the spanwise direction.

Let us consider a rod with a spanwise extent of $L$. The Fourier transform of the pressure in the far-field $P_{ac}(f)$ can be expressed as

$$P_{ac}(f) = \int_0^L P_s(z, f) dz$$  \hspace{1cm} (1)
where $P_a(z, f)$ stands for the sound contribution in the far-field at the span $z$ of the solid surface. It leads to

$$P_{ac}(f) P_{ac}^*(f) = \int_0^L \int_0^L P_a(z_1, f) P_a^*(z_2, f) dz_1 dz_2. \quad (2)$$

Under the assumption that $E(|P_s^2(z, f)|)$ is independent of $z$ and that the phase lag between two contributions in the far-field is equal to the phase lag of their sources on the rod, it is possible to write

$$|E(P_a(z_1, f) P_a^*(z_2, f))| = E(|P_s^2(f)|) \times \Gamma(\Delta z, f), \quad (3)$$

where $\Gamma(\Delta z, f)$ stands for the pressure coherence function on the rod, $\Delta z = |z_2 - z_1|$ and $E(...)$ is the mathematical expectation. Equation 2 then reads

$$E(P_a(z_1, f) P_a^*(z_2, f)) = \int_0^L \int_0^L E(|P_s^2(f)|) \times \Gamma(\Delta z, f) dz_1 dz_2, \quad (4)$$

and it yields

$$\frac{(E(P_{ac}(f) P_{ac}^*(f)))_{exp}}{(E(P_{ac}(f) P_{ac}^*(f)))_{sim}} = \frac{\int_0^{L_{exp}} \int_0^{L_{exp}} \Gamma(\Delta z, f) dz_1 dz_2}{\int_0^{L_{sim}} \int_0^{L_{sim}} \Gamma(\Delta z, f) dz_1 dz_2}. \quad (6)$$

Defining the PSD in logarithmic scale as

$$(S_{pp}(f))_{exp} = 10 \log \left( \frac{(E(P_{ac}(f) P_{ac}^*(f)))_{exp}}{T} \right), \quad (7)$$

where $T$ is the sampling time, the correction on the power spectral density of the pressure in the far-field can be expressed as

$$(S_{pp}(f))_{exp} = (S_{pp}(f))_{sim} + 10 \log \left( \frac{\int_0^{L_{exp}} \int_0^{L_{exp}} \Gamma(|z_2 - z_1|, f) dz_1 dz_2}{\int_0^{L_{sim}} \int_0^{L_{sim}} \Gamma(|z_2 - z_1|, f) dz_1 dz_2} \right). \quad (8)$$

The coherence length $L_c$ is defined as

$$L_c = \int_{z_1}^{+\infty} \Gamma(|z_2 - z_1|, f) dz_2. \quad (9)$$

Simplifications of Eq. 8 can be made by inferring the mathematical form of the coherence function. The most famous simplification is Kato's formula, which assume that the coherence function has the form of a boxcar function. Precisely, by setting $\Gamma(|z_2 - z_1|, f) = 1$ for $|z_2 - z_1| < L_c$ and $\Gamma(|z_2 - z_1|, f) = 0$ for $|z_2 - z_1| > L_c$, Eq. 8 leads to

If $L_c < L_{sim}$, \hspace{1cm} $(S_{pp}(f))_{exp} = (S_{pp}(f))_{sim} + 10 \log \frac{L_{exp}}{L_{sim}}, \quad (10)$

If $L_{sim} < L_c < L_{exp}$, \hspace{1cm} $(S_{pp}(f))_{exp} = (S_{pp}(f))_{sim} + 20 \log \frac{L_c}{L_{sim}} + 10 \log \frac{L_{exp}}{L_c}, \quad (11)$

If $L_{exp} < L_c$, \hspace{1cm} $(S_{pp}(f))_{exp} = (S_{pp}(f))_{sim} + 20 \log \frac{L_{exp}}{L_{sim}}. \quad (12)$

It should be noted that more elaborated corrections can be found in the literature by choosing a less crude approximation for the coherence function, such a gaussian function or an exponential function as done by Seo and Moon.\cite{12}
Table 2. Summary of the simulated cases and varying parameters.

<table>
<thead>
<tr>
<th>Case name</th>
<th>#Nodes</th>
<th>Spanwise length</th>
<th>Num. scheme</th>
<th>Rod bias</th>
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<tr>
<td>CANONICAL_TTGC_35</td>
<td>$4.25 \times 10^6$</td>
<td>$3.5d$</td>
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<td>$y = 0$ mm</td>
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<td>TTGC</td>
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V. Computational cases and meshes

A. Computational domain and meshes

The computational domain (Fig. 2) for each mesh extends from $-6C$ to $7C$ in the streamwise direction and from $-3C$ to $3C$ in the crosswise direction. In the spanwise direction, a length of $30d$ would lead to a huge mesh and to an unaffordable numerical simulation. However, as shown by Jacob et al.,\textsuperscript{5} the spanwise pressure correlation length on the rod is $6.5d$ for the present configuration. The acoustic levels are then corrected using the method described in Sec. IV C.

![Figure 2. Sketch of the computational domain and location of FW-H porous and solid surfaces.](image_url)

The domain is meshed with the CentaurSoft software using tetrahedral elements. The mesh is refined in the region close to the rod and the airfoil in order to capture the vortex shedding well. Mesh convergence around the rod has been studied but will not be presented in the current paper. The rod has been discretized in the chordwise direction with 52 points and in the spanwise direction with 156 points. It is then slightly coarsened but remains fine enough to propagate the acoustic waves correctly to the FW-H surface shown in Fig. 4(a). The mesh density is designed in this zone to have a cut-off Strouhal of $St = fD/U = 7$. Then, it is stretched in both cross-stream and downstream directions. The stretching downstream with the use of a sponge zone is designed to dissipate the vortical structures and prevent any residual acoustic reflections that may not have been fully damped by the non-reflective boundary condition shown in Sec. V B. It leads to a mesh of 4.25 million nodes for the CANONICAL_TTGC_35 setup.
B. Boundary conditions

Navier-Stokes Characteristics Boundary Conditions\textsuperscript{20,30} (NSCBC) have been used at the computational domain boundaries (shown in Fig. 2) in addition to the mesh stretching to avoid any spurious acoustic reflections. Such boundary conditions have been successfully compared\textsuperscript{31} with Tam & Dong\textsuperscript{32} boundary conditions and have been used in aeroacoustic computations.\textsuperscript{4} Periodicity boundary conditions have been used in the spanwise directions. Adiabatic no-slip boundary conditions have been used on the rod and on the airfoil.

C. Numerical parameters

The simulations were run after a convergence of the flow statistics for 600,000 iterations with a constant time step of $\Delta t = 2.5 \times 10^{-7}$ s, which correspond to a physical time of $T = 0.15$ s (transient time excluded). The simulations were performed on 1024 Intel Nehalem processors during two days. All the simulations were run with the TTGC scheme. The data sampling on the FW-H surfaces were done every 20 time step providing a $F_{\text{max}} = 6$ Hz.

The acoustic post-processing with the code KIM was used to obtain the acoustic pressure field from the solid ($S_{\text{Solid}}$) and porous ($S_{\text{Porous}}$) surfaces. The comparison of the results for the two surfaces is used to conclude on the nature of the acoustic sources. The acoustic field has been calculated on 180 observation points located at 1.85 m of the airfoil to yield a well resolved directivity. The PSD estimates were obtained using the Welsh’s periodogram method. Each signal was split in 10 parts with an overlapping of 33% in order to reduce the variance of the PSD estimates.

VI. Validation of the solver

This section assesses the ability of the compressible LES unstructured code AVBP to predict the noise generated by the rod-airfoil configuration. The near-field and the acoustic results are compared with the experimental ones. They are also compared with the more complete numerical results of Berland \textit{et al}\textsuperscript{10} and Eltaweel & Wang,\textsuperscript{11} which are obtained with two different methodologies presented in Sec. III.

A. Overview of the flow field

Figure 4 exhibits the vortex shedding behind the rod. The transition to turbulence of the wake occurs in the shear layer just after the separation. The turbulent wake develops leading to a wide range of turbulent structures. As stated in Sec. II, large turbulent structures break down while impacting on the leading edge.
Smaller eddies go along the airfoil sides and undergo a distortion. Outgoing acoustic waves seems to be mainly emitted from the impingement zone at the airfoil leading edge. No reflections are noticeable from the computational domain boundaries.

The porous FW-H surface $S_{\text{Porous}}$ is shown by the dashed-dotted line in Fig. 2 and is placed outside the hydrodynamic region, as highlighted in Fig 4(a). It is open downstream to avoid the crossing of the wake through the integration surface and to avoid the generation of spurious waves as shown by Casper et al.\textsuperscript{33}

![Global view of the computational domain and position of the porous FW-H surface](image1)

![Zoom on the rod-airfoil configuration](image2)

**Figure 4.** Snapshot of the dilatation field (in grayscale) in central plane and isovalue of the Q-criterion field ($Q = 10^8 \text{ s}^{-1}$).

**B. Near-field results**

Velocity profiles and turbulent intensities have been compared at the three axial positions shown in Fig. 1. They are compared with the experimental results, with the compressible LES of Berland et al.\textsuperscript{10} and with the incompressible LES of Eltaweel & Wang.\textsuperscript{11} The asymmetric pattern of the experimental curves is most likely caused by the misalignment of the rod with the airfoil.

A fair agreement of the mean velocities and of the turbulent intensities with the experimental data is obtained. Some discrepancies in the velocity deficit in the wake of the cylinder exhibited in Fig. 5(a) and in Fig. 5(e) are observed. However, this behavior seems to be found in nearly all the numerical results obtained yet on this configuration.
The turbulent intensity based on the mean streamwise velocity fluctuations at the three slices [A], [B] and [C] (see Fig. 1) are shown in Fig. 5(b), in Fig. 5(d) and in Fig. 5(f) respectively. The overall agreement is pretty good compared to the experiment and slightly better than the other numerical simulations.

Figure 5. Comparison of mean velocity and turbulent intensities of the CANONICAL TTGC_35 case with experiment and other numerical simulations\textsuperscript{10,11} at the three axial positions [A], [B] and [C] (see Fig. 1).

The PSD of the streamwise component of the velocity at the point P in Fig. 1 is shown in Fig. 6. The numerical results are matching the experimental data. Both the fundamental tone related to the vortex
shedding and the broadband component of the spectra are resolved.

![Figure 6. PSD of the streamwise velocity component at the point P (see Fig. 1) for the CANONICAL_TTGC_35 case compared with experimental data and results from Eltaweel & Wang.](image)

C. Radiated acoustic field

Acoustic levels from the numerical results are corrected using Kato’s formula and are compared with the experimental data. Precisely, a special care has to be considered for the fundamental frequency peak associated with the vortex shedding phenomenon. As the spanwise extent of the domain is lower than the pressure spanwise coherence length on the rod, Eq. 11 has to be used to correct the fundamental frequency peak.

The acoustic PSD are shown in Figs. 7(a-c) for three different microphone positions. The numerical results match the experimental ones fairly well at all angles. The discrepancies at low frequencies may be imputed on installation effects of the experiment which are not accounted for in the simulations. A pretty good collapse between the numerical porous and solid surfaces is also observed. The Strouhal peak at $St = 0.19$ is perfectly resolved for the two source surfaces. The results from the solid surface also show some discrepancies at high frequencies. The latter are slightly over-predicted, which can be caused by the omission of the volume sources, as shown by Greschner et al.

When comparing the overall sound pressure levels in Fig. 7(d), a maximum of the directivity is found for $110^\circ$ mainly caused by the under-prediction of the vortex shedding peak. A possible cause is the diffraction effect induced by the nozzle which will modify the directivity of the primary source.

VII. Influence of the spanwise length

The influence of the spanwise length on the accuracy of the solution is assessed in this section. As shown by Jacob et al., the pressure spanwise correlation length is 6.5d for the shedding frequency: the related vortical structures have an average spanwise extent of 6.5d and relevant LES simulations should consider an appropriate computational domain to hamper any side effect. Moreover, the correction of the sound pressure level is straightforward for this span as it does not need the knowledge of the spanwise coherence length. Therefore, a simulation with a spanwise length of 7d is considered to investigate the influence of the spanwise length on the accuracy of the solution and to assess the need of a larger span length for LES computations. The CANONICAL_TTGC_70 mesh is obtained by merging two CANONICAL_TTGC_35 meshes. It leads to a mesh of 8.5 million nodes.
Figure 7. Acoustic results for the CANONICAL_TTGC_35 case compared with experimental data and results from Berland et al.\textsuperscript{10} and Eltaweel \& Wang.\textsuperscript{11}

A. Near-field results

Velocity profiles and turbulent intensities have been compared at the same axial positions [A], [B] and [C] in Fig. 8(a). The results for the cases CANONICAL_TTGC_35 and CANONICAL_TTGC_70 match very well in the three planes. CANONICAL_TTGC_70 predict a slightly better velocity deficit at slice [A]. Some differences are also observed on the turbulent intensity profiles at positions [B] and [C]. In the finer grid region, CANONICAL_TTGC_70 is slightly more accurate.

B. Acoustic results

The PSD of the far-field acoustic pressure at 1.85 m for the angular position 60\degree, 90\degree and 120\degree are shown respectively in Figs. 9(a-c) after having applied the Kato’s corrections on the spectra. The results obtained for the two spanwise extents give very similar spectra. Yet the larger span now predicts the lower frequency part of the spectra better and agree well with the experiment at the shallower angles of 60\degree and 120\degree. The comparison of the OASPL between the two cases is shown in Fig. 9(d). Only small differences are noticeable at the high angles.

Therefore, the sound radiated by a rod-airfoil configuration can be computed on a reduced spanwise length lower than the spanwise coherence length provided Kato’s corrections to the spanwise coherence
Figure 8. Comparison of mean velocity and turbulent intensities for the CANONICAL_TTGC_70 and the CANONICAL_TTGC_35 cases with experiment at the three axial positions [A], [B] and [C] (see Fig. 1).

length are applied to yield the far-field acoustic spectra. There is no differences for the flow statistics, and given the knowledge of the coherence length, it is possible to correct the acoustic results efficiently.
Figure 9. Acoustic results for the CANONICAL TTGC 70 and the CANONICAL TTGC 35 cases compared with experimental data.

VIII. Realistic setup - Influence of a misalignment

The experimental setup showed a slight misalignment of the rod and the airfoil (−2 mm in the y-direction). However, the influence of this bias on the near-field sources and on the far-field acoustic pressure has never been completely addressed. Boudet compared U-RANS simulations of the canonical and the actual shifted set-up. Negligible differences were noticeable in the near-field results. So accounting for the bias was not considered necessary. The LES calculations were then made on a symmetric case. Peth et al. took into account a bias of $\Delta y = 0.1d$ which was half the bias reported by Jacob et al. Nevertheless, no comparison was made between the computed case and the canonical one and the influence of the displacement has not been fully checked since then. The aim of this section is therefore to check the influence of the displacement of the rod and the validity of the approximation made by nearly all the numerical studies done on the rod-airfoil configuration so far.

A. Near-field results

Velocity profiles and turbulent intensities have been compared at the three axial positions shown in Fig. 1 with the experiment and with the symmetric case in the Fig. 10.

The asymmetric pattern is recovered in the plane [A] for the mean velocities and for the turbulent intensity as shown respectively in Fig. 10(a) and in Fig. 10(b). The shape of both mean velocities and turbulent intensities are also slightly improved at the other two axial positions on the airfoil. Yet, the levels and peak values remain different from the experiment, and the experimental shift does not fully explain the
observed discrepancies.

Figure 10. Comparison of mean velocity and turbulent intensities for the SHIFTED_TTGC_35 case and the CANONICAL_TTGC_35 with experiment and the canonical configuration at the three positions [A], [B] and [C] (see Fig. 1).
B. Radiated acoustic field

The PSD of the acoustic pressure at 1.85 m for the angular positions 60°, 90° and 120° are shown in Fig. 11(a-c) respectively. The results are similar to those obtained for the canonical configuration, both for the tonal peak and the broadband component.

The directivity plotted in Fig. 11(d) shows small differences at low and high angles, but they remain very small compared to the mismatch made with the experiment. Thus a small bias in the cross-stream direction cannot fully explain the observed discrepancies.

![Figure 11. Acoustic results for the SHIFTED_TTGC_35 and the CANONICAL_TTGC_35 cases compared with experimental data.](image)

IX. Conclusion

The present study has shown the reliability of a computation using a compressible unstructured LES solver with a FW-H acoustic analogy to predict the airframe noise from a rod-airfoil configuration. The compressible LES on an unstructured mesh indeed allows the high-fidelity resolution of the acoustic sources and their propagation in the near-field for this particular geometry with a limited number of nodes. The FW-H analogy is then able to extrapolate the pressure field in the far-field region at a small computational cost with neither any dissipation nor dispersion. This hybrid method is able to predict both the tonal and the broadband components of the sound generated by this rod-airfoil configuration.

Both solid and porous formulations of the FW-H analogy have been used on this configuration as the Mach number is small enough to neglect the quadripolar source contributions. They show similar results
with some discrepancies in the low and high frequencies. At high frequencies, the quadrupolar term becomes significant and the agreement is better with the porous formulation than with the solid one.

A comparison of the configurations with two different spanwise lengths, one lower and one greater than the spanwise coherence length for the shedding frequency, has been carried out. The effect on the mean flow field is somehow limited and a good agreement on the PSD is retrieved provided a proper correction is applied.

The role of the misalignment of the rod with the airfoil has also been investigated. Although a better agreement can be obtained for the near field results, its influence seems to be limited on the acoustic field. Slight differences can be noticed on the directivity but do not fully recover the discrepancies found between the canonical case and the experiment. The latter are most likely caused by the diffraction effects of the nozzle in the actual set-up.

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