Preface

The book is devoted to the assessment of the quality of numerical results produced by computers. It addresses the central question:

How does finite precision affect the convergence of numerical methods on the computer, when convergence has been proved in exact arithmetic ?

The book describes the principles according to which the general picture of stability and convergence in Theoretical Numerical Analysis, established with exact arithmetic, can be distorted on the computer by finite precision. It shows that such a distortion should be expected for *reliable* software only in the neighbourhood of *singularities*. It also shows that the distortion can be severely amplified by certain parameters in the problem, such as high nonnormality for matrices.

The book gives a rigorous theory of computability in finite precision by setting in a unified framework techniques which belong to areas traditionally kept far apart, such as Functional Analysis familiar to mathematicians, and Numerical Software, more familiar to engineers. The presentation combines the use of three principles:

i) the Lax principle of equicontinuity to prove convergence when, ideally, the number of digits in the arithmetic tends to infinity,

ii) the Wilkinson principle of backward error analysis to grade the quality of algorithms and numerical methods with respect to the available machine precision and/or the level of data uncertainty, and

iii) the principle of robust convergence with respect to perturbations in the data.

The fact that numerical stability in exact arithmetic can become conditional to finite precision arithmetic is illustrated on a variety of examples which realize a cross-section of applied numerical analysis. An in-depth treatment is given for the most basic problems in Scientific Computing. The unified survey of normwise/componentwise error analysis for linear algebra (systems, eigenvalues, least squares) and roots of polynomials that is presented, appears in a

book for the first time. Special attention is given to the influence of spectral instability on the robustness of the convergence of methods, with emphasis on nonnormality. The treatment takes advantage of modern programming environments (graphical facilities and numerical software tools) to perform experimental mathematics near a singularity by means of the toolbox PRECISE developed under MATLAB and which is available electronically.

By bridging the traditional gap between applied mathematicians and software developers, this book will provide a better understanding of the mathematical principles that underlie the stability of computations on a computer, and, in particular, a better understanding of the role of rounding errors. The book will help to disseminate the essential knowledge about backward error analysis beyond the circle of software developers.

The book is intended for applied mathematicians involved in intensive numerical computations, specially those working on extreme problems at the edge of chaos. It is also designed for engineers dealing with numerical simulations, who have to assess the quality of their software. Though concise, the text is written in an informal, non intimidating style which is enhanced by a variety of illustrative computer plots.

The book has been used as a text for advanced undergraduate and beginning graduate students at the University of Paris IX Dauphine. It has also served as a written support for several Advanced Training Courses for engineers organized by CERFACS.

Even though a book is a singularity in itself, like any human enterprise, it is a pleasant duty to acknowledge our numerous debts. We learnt a lot from the Founding Fathers of modern Scientific Computing, Turing and von Neumann, and from two children of theirs, Wilkinson and Henrici. As for our loans from their innumerable descendants, they are too many to be all cited. Out of friendship, we would like to single out the names of Serguei Godunov, Gene Golub, Velvel Kahan, Beresford Parlett, Pete Stewart, and in the younger generation, those of Mario Arioli, Jim Demmel, Nick Higham and Nick Trefethen.

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